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"A TEXTBOOK OF SOUND"

FOR B.A. & B.Sc. STUDENTS OF
INDIAN UNIVERSITIES

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by

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PREFACE TO THE THIRD EDITION

The authors have felt greatly encouraged at the increasing popularity of the book in the various Indian Universities and are extremely grateful to the teachers and the students concerned.

The present edition has been carefully revised and enlarged to make the book uptodate so as to include the curricula of our Universities upto the degree stage.

The Fourier's analysis, Doppler's principle and Piezo electric effect have been comprehensively treated. Several important additions have been made in the body of the text, notably (*i*) Sir. W. H. Bragg's work on the objectivity of the combination tones (*ii*) Musical instruments of common interest both in the East and of the West (*iii*) Mathematical treatment that energy is not transferred in a stationary wave. (*iv*) Motion of a point on a vibrating string. (*v*) Acoustical Impedance, velocity amplitude and Resonance Catastrophe. (*vi*) Interest of sound pressure amplitude and limits of audibility and (*vii*) The microphone, The telephone and the loudspeaker.

Several new solved examples and a large number of questions with answers have been added at the end of each chapter in the text with a view to help the students to apply their knowledge to calculations.

20th March, 1952.

D. R. KHANNA
R. S. BEDI

PREFACE TO THE FIRST EDITION

This book is intended to meet the requirements of the students preparing for a pass degree of the various Indian Universities and to serve as a link between the elementary and the advanced Treatises on Sound. It is the outgrowth of class lectures delivered to B.A. and B.Sc. students of the Punjab University over a number of years. A systematic and logical development of the subject has been throughout kept in view and more stress has been laid on the explanation of the Physical Principles of the subject rather than on arranging a set of mathematical equations. Accordingly both the graphical and the analytical methods involving the use of Calculus have been made use of as found convenient to achieve that purpose. The authors have felt that without the use of mathematics of a somewhat advanced type, the explanations could not be adequate and understandable but of course the mathematical attainments of the students at this stage have been taken into consideration.

Another important feature of the book is that a large number of class demonstrations have been incorporated and they are uniformly spread over the text.

Certain Articles of the book have been marked with an asterisk (*). They are of a rather higher standard than the remaining articles and may be omitted by the students at a first reading for it has been aimed that the subject matter should not lack continuity and sufficiency without them.

In recent years the subject of sound has attracted a good deal of attention on account of the important role it is going to play in the advancement of science in general. We know that the modern acoustical research has brought in its train a large number of technical and industrial applications of the subject. So a full chapter has been devoted to it but only elementary details have been given to serve as a mere guide and with a view to inspire interest in the subject.

A large number of graded exercises and questions set at the various University examinations have been given at the end to enable the students to judge the grasp of the fundamental principles.

The authors will be grateful to the reader for pointing out errors or omissions and for suggestions to improve the book.

March 1946
LAHORE

D. R. KHANNA
R. S. BEDI

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A TEXTBOOK OF SOUND

S CHAPTER I Sound

SOUND AND SIMPLE HARMONIC MOTION

1. Sound, its Meaning and Origin.—The word *Sound* conveys a double meaning, it refers to (*i*) the *mental sensation* perceived by the ear, and (*ii*) the cause responsible for that perception, namely, the physical phenomenon external to the ear—the *wave motion* which excites the auditory nerves.

Every one knows that sound is due to the vibratory movement of some body, although the movement may be so slight and rapid that it is not visible. A bell, a tuning-fork, the strings of a violin, the air inside an organ pipe, the reed of the harmonium etc., are all in a state of tremor when producing sound. The vibratory motion of a bell or a tuning-fork can be exhibited by bringing a light pith ball pendulum suspended by means of a thread close to them. The pith ball will be thrown off violently. Fig. 1 illustrates an optical method of demonstrating the vibratory motion of a fork. Light from an arc lamp, *A*, after passing through a narrow hole, is allowed to fall on a small mirror, *m*, attached with wax to a prong of the fork, *T*. After reflection it is received by a revolving mirror* *M*, which throws it on a distant screen *S*. By means of a convex lens, *L*, interposed in the path of the light between the hole and the mirror *m* a well-defined image of the hole is

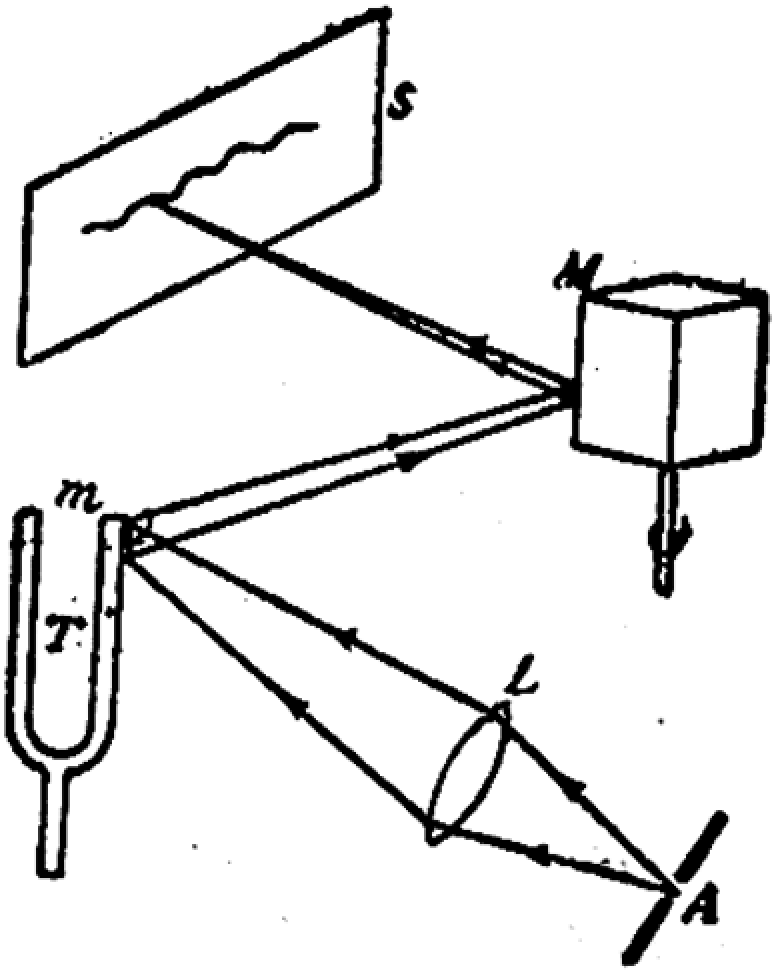

Fig. 1.

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*It is a hollow box with its four side faces made of plane mirrors and mounted so as to be capable of rotation about a vertical axis passing through its centre.

obtained on the screen. On rotating the revolving mirror, the spot of light will appear to be drawn out into a horizontal straight line. If now the fork is bowed, it emits a musical sound while the spot of light describes a periodic curve on the screen. It is a *sine curve.* On stopping the prong both the sound and the vibrations die out simultaneously.

The vibration of the air inside an organ pipe can be shown by bringing a light paper pan having some fine dry sand on it close to its end. It will be noticed that the sand particles dance about when the pipe is sounding. Thus we see that in every case the body producing sound is in a state of vibration. Most sounding bodies, unless their motion be very great, execute a particular type of periodic motion called **simple harmonic motion** or their vibration consists of a combination of simple harmonic motions and hence the importance of this type of motion in the study of Sound.

2. Simple Harmonic Motion.—Let a particle, P (Fig. 2), move with uniform speed, v, round a circle and let N be its projection on the diameter BB'. As P moves round the circle in the anti-clockwise direction starting from A, the point N moves from O to B, B to B' and back again to O. This oscillatory motion of N about the centre O is **simple harmonic motion**. It is *the projection of uniform circular motion upon any diameter* of the circle.

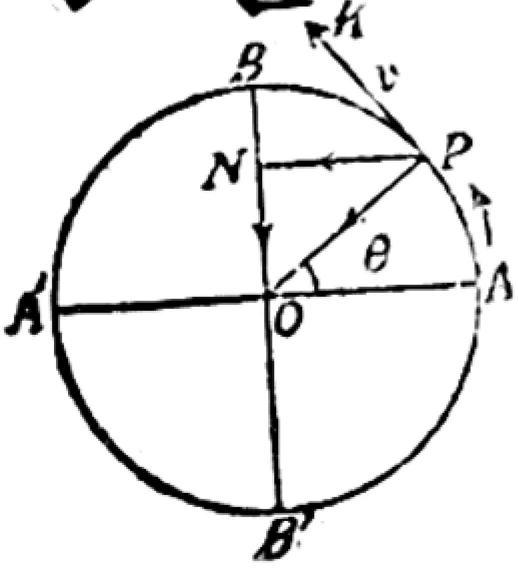


Fig. 2.

Let a be the radius of the circle and θ the angle at any instant which OP makes with OA, the line joining the initial positions of P and N. The angular velocity ω of P is $\frac{v}{a}$ and the displacement of N from O is given by

$$y=a \sin \theta$$

If t seconds be the time in which OP describes the angle θ, then $\theta=\omega t$ and we may write

$$y=a \sin \omega t \quad \ldots \quad \ldots \quad (1)$$

The radius a of the circle of reference or the maximum displacement of N on either side of O is called its *Amplitude.*

3. Velocity, Acceleration and Periodic Time of N.—The velocity of N is the component of the velocity of $P \parallel$ to BB'. Now the velocity P is v along the tangent PK. Its component $\parallel$ to BB' *i.e.* $\perp$ to PN is $v \sin \angle KPN = v \cos \theta$, since θ and $\angle KPN$ are complementary. Therefore the velocity of N is $v \cos \theta$

$$= a\omega \cos \theta = a\omega \sqrt{1 - \frac{y^2}{a^2}} = \omega \sqrt{a^2 - y^2}. \quad \ldots \quad (2)$$

The acceleration of N is the component along NO of the centripetal acceleration of P. Since P is moving in a circle, the acceleration of P is $\frac{v^2}{a} = a\omega^2$ directed towards the centre, therefore the acceleration of N

$$\begin{aligned} &= -a\omega^2 \sin \theta \\ &= -\omega^2 (a \sin \theta) \\ &= -\omega^2 y \end{aligned}$$

$-$ sign indicates that it acts opposite to the direction in which y increases.

Using the language of differential calculus, by differentiating equation (1) with respect to time we have for the velocity of N

$$\frac{dy}{dt} = a\omega \cos \omega t.$$

Differentiating again, we get the acceleration of N,

$$\begin{aligned} \frac{d^2y}{dt^2} &= -a\omega^2 \sin \omega t \\ &= -\omega^2 y \quad \ldots \quad \ldots \quad (3) \end{aligned}$$

or

$$\frac{d^2y}{dt^2} + \omega^2 y = 0 \quad \ldots \quad \ldots \quad (3a)$$

where ω^2 is constant.

This is the "differential equation" of S. H. Motion. Every y satisfying it describes Simple Harmonic Motion to be defined as follows :—

A body is said to move in *Simple Harmonic Motion* if the acceleration $(\omega^2 y)$ is always directed towards a fixed point in the centre and is proportional to the distance from it.

The periodic time T is equal to the time occupied by P in moving once round the circle. Since the angular velocity is ω, the periodic time $T=\frac{2\pi}{\omega}$ and the frequency f *i.e.*, the number of vibrations/sec$=\frac{\omega}{2\pi}$.]

Also from (3) we learn that ω^2 is the acceleration when the particle is at a unit distance from the centre. We have, therefore

$$T=2\pi/\sqrt{\text{acceleration per unit displacement.}}$$

$$=2\pi/\sqrt{\frac{\text{acceleration}}{\text{displacement}}}=2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}} \quad \ldots \quad (4)$$

The time period is independent of the amplitude *i. e.*, the vibrations are *isochronous**, whatever the nature of the path, straight or curved.

It may be interesting for the student to acquaint himself with the mathematics of solving the differential equation of S. H. M. Equation ($3a$) cannot be integrated directly.

Let $v=\frac{dy}{dt}$, so that $\frac{d^2y}{dt^2}=\frac{d}{dt}\left(\frac{dy}{dt}\right)$

$$=\frac{dv}{dt}=\frac{dv}{dy}\cdot\frac{dy}{dt}=v\frac{dv}{dy}$$

Equation ($3a$) therefore becomes

$$v\frac{dv}{dy}=-\omega^2 y$$

or

$$vdv=-\omega^2 ydy$$

Integrating we have

$$\tfrac{1}{2}v^2=-\frac{\omega^2}{2}y^2+C,$$

where C is a constant of integration

For extreme displacement

$$y=a \quad \text{or} \ -a \ \text{and} \ v=0$$

$$\therefore \quad 0=-\frac{\omega^2a^2}{2}+C \quad \text{or} \quad C=\frac{\omega^2a^2}{2}$$

---

*Vibrations whose periods do not alter are called 'isochronous' but note that it is not necessary that isochronous vibrations are necessarily Harmonic.

Whence,
$$\tfrac{1}{2}v^2=\frac{\omega^2a^2}{2}-\frac{\omega^2y^2}{2}$$
or
$$v^2=\omega^2(a^2-y^2)$$
or
$$v=\omega\sqrt{a^2-y^2}$$

This is equation (2) above

Writing $\frac{dy}{dt}$ for v, we have

$$\frac{dy}{dt}=\omega\sqrt{a^2-y^2}$$
or
$$\frac{dy}{\sqrt{a^2-y^2}}=\omega\,dt.$$

Integrating it

$$\text{Sin}^{-1}\frac{y}{a}=\omega t+\phi$$

ϕ being a constant of integration determinable by initial conditions.

It may be put as

$$y=a\sin(\omega t+\phi)$$

Expanding the equation, we get

$$y=a\cos\phi\sin\omega t+a\sin\phi\cos\omega t$$

where $a\cos\phi$ and $a\sin\phi$ are just new constants. Let us represent them by A and B.

Clearly then our equation could be expressed by the formula

$$y=A\sin\omega t+B\cos\omega t$$

and in special cases either A or B may be zero.

Example 1. *If a point executes S. H. Motion of period 4 seconds and amplitude 2 cm., find its maximum velocity and that at half its full displacement. Also find the acceleration at the turning points and when the displacement is $\frac{3}{4}$ cm.*

Using the notation above,

$T=4$ seconds.

$a=2$ cm.

Since $T=\frac{2\pi}{\omega}$. $\therefore \quad \omega=\frac{2\pi}{T}=\frac{2\pi}{4}$.

$$=\pi/2.$$

By equation (2) above, the velocity of the particle

$$=\omega\sqrt{a^2-y^2}$$

The velocity is max. when $y=0$.

So $v_{\max}=\omega\, a$

$$=\frac{\pi}{2}\times 2=\pi \text{ cm./sec.}$$

Velocity at half its full displacement is obtained by putting $y=1$ cm. in the equation (2) above.

So the velocity$=\omega.\sqrt{2^2-1^2}$.

$$=\frac{\pi}{2}.\sqrt{3}=\frac{\sqrt{3}}{2}\pi \text{ cm./sec.}$$

Also acceleration $\frac{d^2y}{dt^2}=-\omega^2 y$.

At the turning points $y=a$.

$\therefore$ acceleration$=\left(\frac{\pi}{2}\right)^2\times 2=\frac{\pi^2}{2}$cm./sec^2.

When the displacement is $\frac{3}{4}$ cm., the acceleration$=\omega^2.\frac{3}{4}$

$$=\left(\frac{\pi}{2}\right)^2.\frac{3}{4}=\frac{3\pi^2}{16}\text{cm./sec.}^2$$

Example 2. *A body is describing S. H. M. of amplitude 1 ft. its velocity while passing through the neutral point is 10 ft./sec. What is its frequency ?*

$$y=a\sin(\omega t+\varphi)$$

Differentiating

$$v=\frac{dy}{dt}=a\,\omega\cos(\omega t+\varphi)$$

The maximum speed possessed by the body when it passes through the neutral point is clearly.

$$v_{\max}=a\,\omega.$$

Hence $$\omega = \frac{v_{\max}}{\text{amplitude}} = \frac{10 \text{ ft./sec}}{1 \text{ ft.}} = 10 \text{ sec.}^{-1}$$

and $$f = \frac{\omega}{2\pi} = \frac{10}{2\pi} \text{ sec.}^{-1}$$

4. Elasticity and S. H. Vibrations.—Whenever a system of forces is applied to change the volume or the shape of a body, we observe that after the removal of the impressed forces, the body returns to its original volume or shape by virtue of the internal reacting forces called into play in it. This property of a body is called its *Elasticity*. No object is, however, perfectly elastic and the measure of this property, therefore, varies with the nature of the object. The resisting force called into play is termed the *stress*, and the deformation caused in the body on account of the relative displacement of its particles is called *strain*.

Simple strains are of three kinds : (*a*) *change of length strain*, (*b*) *change of volume strain*, and (*c*) *change of shape strain.*

Robert Hooke (1635-1703) discovered the fundamental law existing between the resisting force and the resulting deformation of an elastic body. According to this law "*stress is always proportional to the strain*, whatever the type of the strain" ; *i.e.*, the ratio $\frac{stress}{strain}$ is constant. The value of this constant for any substance is taken as the measure of its '*elasticity*'. *The* law is, however, true for small displacements and holds within the *elastic limits**

Most of the sound vibrations are due to this property of elasticity possessed by bodies. The vibrations of the prongs of a tuning fork, the motion of the string of a violin and that of the air inside an organ pipe, are all due to this property. Let us see how ? When a body is deformed, it acquires potential energy equal to the amount of work done in deforming it. On releasing it, the forces due to elasticity begin to

---

*In the case of solids there are limits beyond which the deformation must not go if the body is to recover its original state. If, for instance, the forces exceed a certain value the body acquires a permanent deformation and will not recover its original state. It is then said to be deformed beyond the '*limits of Elasticity*'.

restore it to the equilibrium position with increasing velocity and a transformation of potential into kinetic energy takes place. When the body reaches its mean position, the whole of its potential energy is converted into kinetic, and there is momentarily no deformation and no elastic force. But on account of the momentum acquired, the body swings over to the other side and a conversion of kinetic energy to potential starts till at the extreme end of the swing the whole of energy is potential again. The body is momentarily at rest and then springs back. If there were no dissipation of energy, the body will continue to vibrate to and fro with undiminished amplitude. But in actual practice the energy is dissipated in a number of ways ; a part of the energy of vibration is transferred to the air in the form of waves of compressions and rarefactions to be discussed later ; a part is spent in overcoming internal friction and is converted into heat and so on. As a consequence of these losses the vibrations gradually die down till the whole of energy is dissipated and ultimately the body comes to rest.

It is obvious from Hooke's law that the force bringing every particle back towards the mean position is proportional to its displacement from that mean position. Thus the vibrations executed by a body under the action of the elastic forces satisfy the characteristics of a Simple Harmonic motion described above and hence the importance of such elastic vibrations in the theory of Sound.

5. Period of Vibration of a Body under the action of Elastic Forces.—Consider a particle which is vibrating under the action of an elastic force. If M be the mass of the particle and F_y the force which acts on it when it has a displacement y, then by Newton's second law of motion

$$F_y = M\frac{d^2y}{dt^2}, \text{ where } \frac{d^2y}{dt^2} \text{ denotes the acceleration.}$$

Since in a Simple Harmonic motion acceleration is proportional to displacement, the force must also be proportional to the displacement so that

$$F_y \propto y \text{ or } F_y = -Fy,$$

where F is the restoring force at a unit displacement. Hence

$$M\frac{d^2y}{dt^2} = -Fy, \text{ or } \frac{d^2y}{dt^2} = -\frac{F}{M}y.$$

$$\therefore \; T=\frac{2\pi}{\sqrt{F/M}}=2\pi\sqrt{\frac{M}{F}} \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

$$=2\pi\sqrt{\frac{\text{mass}}{\text{Restoring force at a unit displacement}}}.$$

6. Some Illustrations of Simple Harmonic Motion.—We have seen above that in simple Harmonic motion the force is always directed towards a central or mean position and varies as its distance from that position. From equation (5) the periodic time

$$T=2\pi\sqrt{\frac{M}{F}}$$

i.e., it increases with the mass M and decreases with the elastic constant F (F is the restoring force at a unit displacement from the mean position). This relation is of wide application in the study of sound vibrations. The following are a few typical examples of simple Harmonic vibration : (*i*) Simple Pendulum, (*ii*) Motion of a liquid in a U-tube, (*iii*) Oscillations of a gas in a cylinder, (*iv*) Motion of a body suspended by a spring. For (*i*) and (*ii*), see a book on Properties of Matter. we shall consider here illustrations (*iii*) and (*iv*) and calculate the period of vibration in each case.

7. Oscillations of a gas in a cylinder.—Consider a cylinder C with a frictionless air-tight piston P (Fig. 2*a*). If the piston is forced in, so that the gas is compressed and then released, it will be pushed out by the springing action of the compressed air. Its momentum will carry it beyond the position of rest, the air inside then expands causing a deficit of pressure. It becomes unable to balance the weight of the piston and the external pressure. Hence the piston is driven in again in excess of its normal position and the process is repeated at regular intervals. Let us calculate the period of such oscillations.

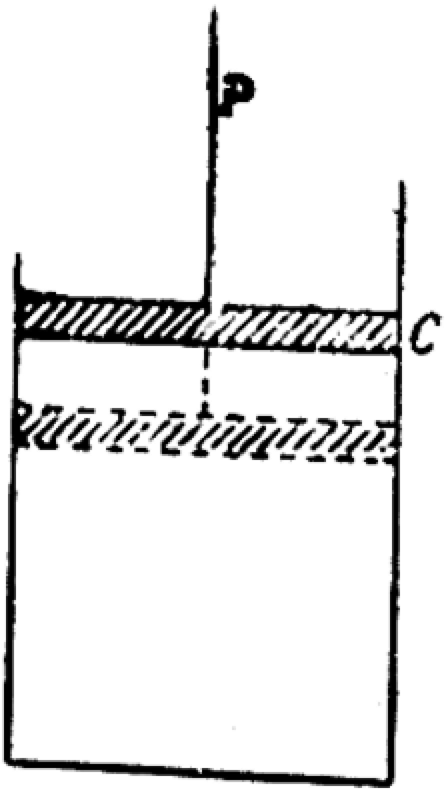


Fig. 2*a*.

Suppose p and v are the pressure and volume respectively of the air inside the cylinder. Then since for an isothermal change

$$pv = \text{a constant}$$

we have, by differentiating

$$p\,dv + v\,dp = 0$$

or

$$p = -\frac{v\,dp}{dv}.$$

If A be the cross section of the piston, the excess force on the piston for a change in volume dv is

$$F = A\,dp = -Ap\frac{dv}{v}.$$

If y be the displacement of the piston and l the length of the portion of the cylinder below the piston, then $dv = Ay$ and $v = Al$

$$\therefore \quad F = -Ap.\frac{Ay}{Al} = -\frac{Apy}{l}.$$

If M be the mass of the piston, its acceleration$=F/M$ or

$$-\frac{Apy}{lM}.$$

Here the acceleration is proportional to displacement, so the motion is Simple Harmonic and hence the periodic time is given by

$$T = 2\pi \Big/ \sqrt{\frac{Ap}{lM}} = 2\pi\sqrt{\frac{lM}{Ap}} \quad \ldots \quad \ldots \quad (6)$$

8. Motion of a Body suspended by a Spring.—Suppose we have a heavy body of mass M suspended by a light spiral spring, S. In Fig. 3, the spring and the mass hang in a state of equilibrium. Let a force be applied to stretch the spring displacing the mass M a distance y. After being released the mass M moves up and down in Simple Harmonic motion with periodic time.

$$T = 2\pi\sqrt{\frac{M}{F}}$$

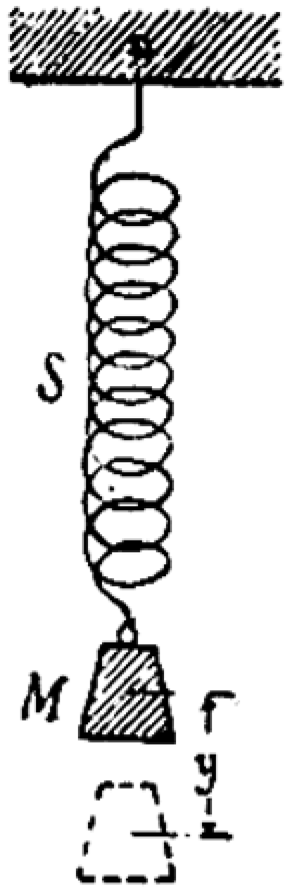


Fig. 3.

where F is the force required to stretch the spring through 1 cm. To find this add an additional mass m and note the extension. Let it be x, then x is found to be proportional to m since Hooke's law is obeyed. The force for a displacement of 1 cm. $=\frac{mg}{x}$.

Putting this value, we have

$$T=2\pi\sqrt{\frac{M}{mg/x}}=2\pi\sqrt{\frac{Mx}{mg}} \quad \dots \quad (7)$$

$$=2\pi\sqrt{\frac{b}{g}}$$

where b is the extension produced by the original mass M.

Example. *A spring is hung vertically and loaded with a mass of 4 oz. A force of 2 lbs. stretches it 12 in. Find its frequency of oscillation, its period, its amplitude and its phase constant.*

Using the relation,

$$T=2\pi\sqrt{\frac{M}{F}},$$

we have $M=4$ *oz*, assuming the spring to be so light as to have negligible mass itself

$$F=\frac{2\text{lb. } wt}{12 \text{ in.}}=\frac{64 \text{ poundals}}{1 \text{ ft.}}=64 \text{ poundals per ft.}$$

$$\therefore \quad T=2\pi\sqrt{\frac{4}{64}}=\frac{\pi}{2} \text{ seconds.}$$

The frequency $f=\frac{1}{T}=\frac{2}{\pi}$ sec.$^{-1}$

The amplitude and phase cannot be determined from the data provided. They are found from the initial conditions of the motion. For instance if the mass was released at $t=0$, with a displacement of 4 inches, we find that $A=4''$ and $\phi=\pi/2$.

9. Phase.—We have seen above (Art. 2) that the displacement of N, is $y=a \sin \omega t$, on the assumption that we start counting time when P crosses the x-axis. If instead of measuring time from this particular instant, it is measured from an instant when P is at P_0, where $\angle AOP_0=\phi$, the displacement, ON, is given by $y=a \sin (\omega t-\phi)$, since $\angle AOP=(\omega t-\phi)$. The angle $(\omega t-\phi)$ is called the **phase** of the vibration while ϕ is termed the '*epoch*' or '*the initial phase*'.

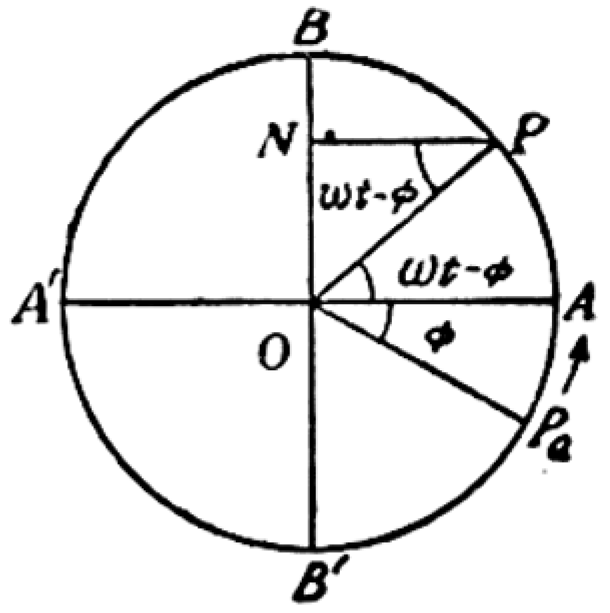


Fig. 4.

If, however, we start counting time when P has already crossed the x-axis and its initial position is on the other side of A. Calling this initial angle ϕ as before, the displacement at time t is given by $y=a \sin (\omega t+\varphi)$, so $(\omega t+\phi)$ is the phase of the vibration.

By the *phase* of a harmonically vibrating point at any instant we mean its state as regards its position and its direction of motion at that instant. It is the value of the angle AOP and denotes what stage of the cycle the particle has reached at that instant. If it is zero, the particle is at the position of zero displacement and moving towards B. If it is $\pi/2$, the particle is at the position of the maximum positive displacement and so on.

The term 'phase difference' between two $S.\ H.$ Vibrations indicates how much the two vibrations are out of step with each other. Thus if two vibrating points pass simultaneously through their undisplaced positions in the same direction, they are at the instant of such passage in the same phase. If, however, the direction of one of them is reversed, all else remaining the same, they would then be in opposite phase. It is convenient sometimes to express **phase** *of a harmonically vibrating point as a fraction of the time period which has elapsed since the point last passed through its mean position in the positive direction.*

We conclude from the above discussion that the three main characteristics of S. H. motion are its period, its amplitude and its phase embodied in the equation of its displacement

$$y=a \sin (\omega t-\phi) \qquad \cdots \qquad \cdots \qquad (8)$$

10. Energy of a Particle in S. H. Vibration.—Consider the differential equation for Simple Harmonic motion

$$\frac{d^2y}{dt^2}=-\omega^2 y$$

If m be the mass of the particle executing this motion, the force required to maintain the displacement $y=m\omega^2 y$. The work done by the force for a small displacement dy is equal to

$$F dy=m\omega^2 y\, dy.$$

Hence the total work for displacement y which also measures the Potential Energy (P) of the particle

$$=\int_0^y m\omega^2 y\, dy=\tfrac{1}{2}m\omega^2 y^2$$

Now the velocity of the particle at displacement y is equal to

$$\frac{dy}{dt}=\frac{d}{dt}\,[a\sin(\omega t-\phi)]$$

$$=a\,\omega\cos(\omega t-\phi)$$

The Kinetic Energy $(K)=\frac{1}{2}m\left(\frac{dy}{dt}\right)^2$

$$=\tfrac{1}{2}m\omega^2a^2\cos^2(\omega t-\phi)$$
$$=\tfrac{1}{2}m\omega^2a^2[1-\sin^2(\omega t-\phi)]$$
$$=\tfrac{1}{2}m\omega^2a^2\left[1-\frac{y^2}{a^2}\right]$$
$$=\tfrac{1}{2}m\omega^2[a^2-y^2].$$

Hence the total Energy $(E)=P+K=\frac{1}{2}m\omega^2y^2+\frac{1}{2}m\omega^2(a^2-y^2)$

$$=\tfrac{1}{2}m\omega^2a^2=2m\pi^2a^2/T^2 \qquad \ldots \qquad \ldots \qquad (9)$$

Here we assume that no dissipative force is at work and therefore from the law of conservation of energy the total energy at any instant of the motion of the particle is constant.

11. Graphic Representation of Simple Harmonic Motion.—Let us draw the displacement curve of the particle N moving in S. H. Motion *i.e.*, a curve showing the relation between the displacement of N and the time which has elapsed since it was at O. N is the projection of the uniform motion of P in the reference circle $ABA'B'$ on the diameter BB'. Take C as the origin, the time being represented along CG while the displacement of N from the centre O at any instant is given by the corresponding ordinate. Thus at any time $t(=Ca)$ when the rotating particle is at P, where $\angle AOP=\theta=\omega t$, the position of N on the y-axis is represented by p, where $ap=ON$. If a series of points corresponding to the position of N at various instants during one complete period of its vibration are determined in this way and joined by a smooth curve, the graph shown in Fig. 5 is obtained. It enables us to find the position of the particle at any time, for the distances measured along the horizontal line from C are proportional to the time elapsed from the moment when the particle was passing through its equilibrium position upwards and the ordinate at the point corresponding to any given moment shows the displacement at that moment. The curve is a *Sine Curve.*

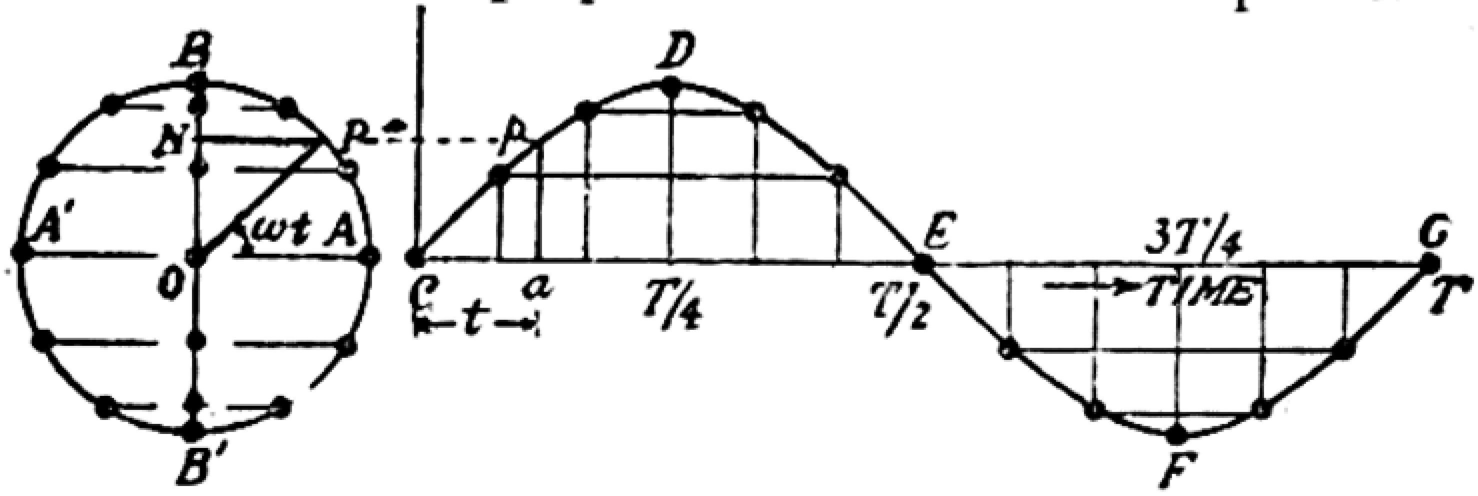
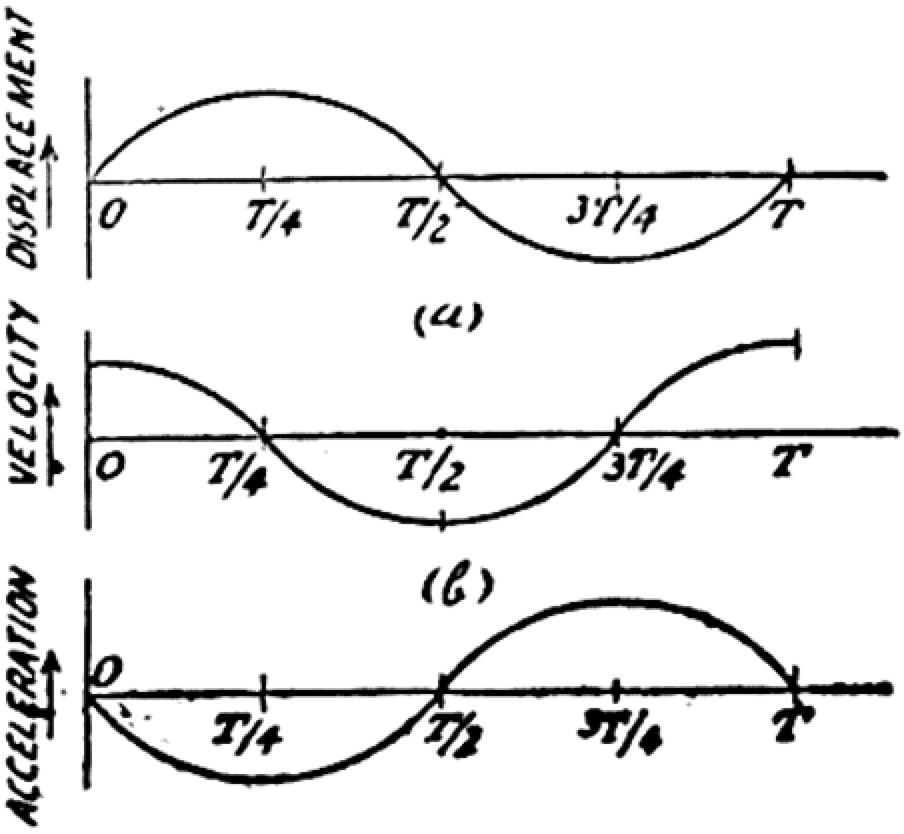


Fig. 5.



(a)

(b)

(c)

Fig. 6.

12. Relation of Velocity and Acceleration to Displacement.—We saw in Art. 3 that the velocity of the particle is given by $v=a\omega\cos\theta$. It is a cosine curve with amplitude $a\omega$. It will be displaced a quarter of period to the left as compared with the displacement curve, for the velocity is maximum when the particle is passing through its mean position *i.e.*, when the displacement is zero and *vice versa.* In the figure ordinates above the time axis denote the velocities upwards and ordinates below the time axis velocities downwards.

Also the acceleration of the particle was shown to be equal to

$$-\omega^2 y = -\omega^2 a \sin \theta.$$

It is similar to the displacement curve, only its amplitude is different and it is acting in a direction opposite to y increasing. All these curves are shown in Fig. 6.

CHAPTER II

COMPOSITION OF SIMPLE HARMONIC MOTIONS

13. A point may be under the influence of two or more simple harmonic vibrations at the same time. It will then describe a motion which is the resultant of the two or more vibrations as the case may be. It does not mean, however, that at any one instant the point can be moving with respect to a given frame of reference, with more than one velocity or in more than one direction. Conventionally we say that the point is executing two or more simple harmonic vibrations simultaneously.

It is understood that Harmonic Motion, like all motions is relative. Let us take the case of the reciprocating motion of the piston of the engine of a rolling ship which moves to and fro in such a way that its distance from the end of the cylinder varies harmonically so it is vibrating in simple harmonic vibration with respect to the cylinder. But as the ship travels on, the movements of the piston with respect to the earth may be very complex.

If then a point P is executing S. H. vibration with respect to a point Q, while Q is moving harmonically with respect to the earth, the movement of P with respect to the earth, is said to be the resultant of two harmonic motions or more concisely P may be said to be executing both movements simultaneously. When a point is executing two rectilinear simple harmonic vibrations at one time, its motion need not be either rectilinear or harmonic. If the periods of vibrations are incommensurable, it even loses the distinctive characteristics of a vibration. We shall discuss the various cases in the following sections.

13a. Composition of Simple Harmonic motion with Uniform motion in a Straight Line.—Let us study the curve traced out by a particle which is under the influence of a simple Harmonic motion in one line and uniform motion in a perpendicular direction. To obtain the resultant curve in the laboratory, take a tuning-fork and attach a small wire or a style to one of its prongs. Make the tuning-fork to vibrate by bowing across it and hold it in such a position that the end of the style lightly touches against a smoked glass plate. The style will trace out on the plate a straight line whose length will define the extent of the vibration. If now the fork or the plate is drawn uniformly in the direction of the length of the fork, we shall get a sine curve on the plate, the direction of motion of the fork or the plate serving as the axis of the curve. Equal distances along the axis will correspond to equal intervals of time and distances measured perpendicular to the axis will give the displacement of the prong from the mean position.

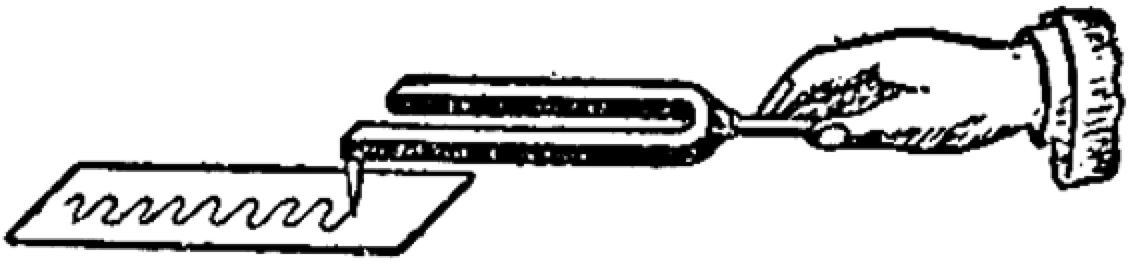


Fig. 7.

This method of exhibiting the movements of the prongs of a fork is used for the accurate determination of the frequency of the fork or small time intervals (*see* Art. 202).

14. Composition of S. H. Motions in a Straight Line.—If two or more simple harmonic motions are acting on a particle simultaneously, it shall execute a resultant vibration which is obtained by compounding the component vibrations. We shall consider a simple case :—

Two Simple Harmonic Vibrations of equal periods and hence of the same angular velocity ω but of different amplitudes.

GEOMETRICAL TREATMENT.—Suppose P and Q are the particles revolving round O with uniform angular velocities, then their projections on BOB' correspond to two S. H. Motions of equal periods. Let ϕ_1 and ϕ_2 be the epochs and a_1, a_2 the amplitudes of the

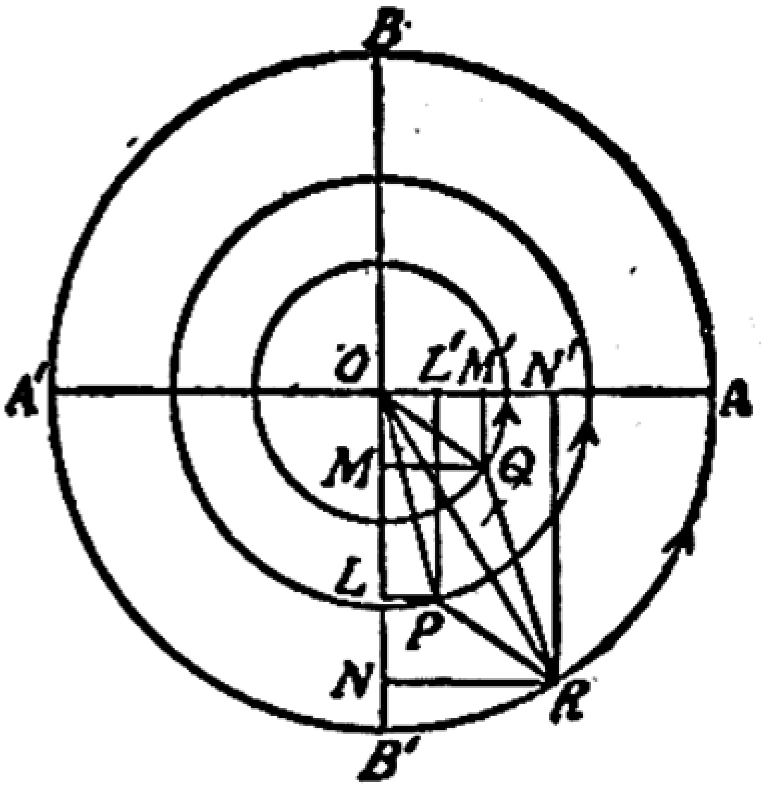


Fig. 8.

component vibrations. Since the angular velocities of P and Q are the same, the angle POQ will be constant; and hence if the parallelogram $OPRQ$ is completed, the resultant motion of the point will be represented by the motion of R since it will revolve with the same angular velocity as P or Q. The projection of the motion of R on BOB' will be the S. H. Motion resulting from the composition of two given motions. As the velocities of P, Q and R agree, the *period* of the resultant is the same as that of the component vibrations.

It is clear from the figure that

$$\angle POQ=\phi_1-\phi_2$$

and

$$OR^2=OP^2+OQ^2+2OP.OQ \cos \angle POQ$$

or

$$r^2=a_1{}^2+a_2{}^2+2a_1a_2 \cos (\phi_1-\phi_2).$$

The phase angle ROA $(=\theta)$ of the resultant vibration is given by

$$\tan\theta=\frac{RN'}{ON'}=\frac{ON}{ON'}=\frac{OL+LN}{OL'+L'N'}=\frac{OL+OM}{OL'+OM'}=\frac{a_1 \sin\phi_1+a_2 \sin\phi_2}{a_1 \cos\phi_1+a_2 \cos\phi_2}$$

When there is no difference in epochs, *i.e.*,

$$\angle POQ=0, \cos 0=1.$$

$$\therefore \quad OR^2=OP^2+OQ^2+2OP.OQ$$

or

$$OR=OP+OQ$$

i.e.. the amplitude of resultant S. H. M. is equal to the sum of the amplitudes of the components.

When, in addition, OP and OQ are equal, we get a simple harmonic motion of double the amplitude.

When $\angle POQ=\pi$, and $OP=OQ$, $OR=OP-OQ$, there is no motion and the particle is at rest.

ANALYTICAL TREATMENT.—Let the component vibrations be respectively

$$y_1=a_1 \sin (\omega t-\phi_1) \quad \ldots \quad \ldots \quad (1)$$

and

$$y_2=a_2 \sin (\omega t-\phi_2) \quad \ldots \quad \ldots \quad (2)$$

The difference in phase between them is given by $(\phi_1-\phi_2)$.

The resultant displacement

$$y = y_1 + y_2 = a_1 \sin(\omega t - \phi_1) + a_2 \sin(\omega t - \phi_2)$$

$$= a_1 [\sin \omega t \cos \phi_1 - \cos \omega t \sin \phi_1] + a_2 [\sin \omega t \cos \phi_2 - \cos \omega t \sin \phi_2]$$

$$= \sin \omega t\, [a_1 \cos \phi_1 + a_2 \cos \phi_2] - \cos \omega t\, [a_1 \sin \phi_1 + a_2 \sin \phi_2]$$

Let us choose an angle θ such that

$$\left.\begin{aligned} r \cos \theta &= a_1 \cos \phi_1 + a_2 \cos \phi_2 \\ \text{and} \quad r \sin \theta &= a_1 \sin \phi_1 + a_2 \sin \phi_2 \end{aligned}\right] \quad \ldots \quad (3)$$

$$\therefore \quad y = r\, [\sin \omega t \cos \theta - \cos \omega t \sin \theta]$$

$$= r \sin(\omega t - \theta) \ldots \quad \ldots \quad \ldots \quad (4)$$

where r and $(\omega t - \theta)$ are the amplitude and the phase of the resultant vibration.

To obtain r, square and add the two components of equation (3) which gives

$$r^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos(\phi_1 - \phi_2).$$

Also
$$\tan \theta = \frac{a_1 \sin \phi_1 + a_2 \sin \phi_2}{a_1 \cos \phi_1 + a_2 \cos \phi_2}.$$

The equation (4) shows that the resultant motion is simple harmonic of the same period and in the same straight line with a phase $(\omega t - \theta)$.

Now let us consider some *Special Cases* :—

(*i*) If $\phi_1 - \phi_2 = 0$, or any integral multiple of 2π, the vibrations are in the same phase and $r^2 = (a_1 + a_2)^2$ or $r = a_1 + a_2$.

(*ii*) If $\phi_1 - \phi_2 = \pi$, the vibrations are in opposite phase and $r^2 = (a_1 - a_2)^2$ or $r = a_1 - a_2$. In this case the amplitude will be zero, provided $a_1 = a_2$ *i.e.*, the particle will remain at rest.

15. Composition of S. H. Motions at Right Angles : (Lissajous' Figures).—When a body has two simple harmonic motions at right angles to each other, they may be compounded and the resultant motion obtained is called a *Lissajous figure.* The nature of the motion or the curve traced out depends upon the amplitudes, frequencies and phase differences of the two constituents. We shall consider in the following sections some simple cases of the composition of two rectangular S. H. vibrations,

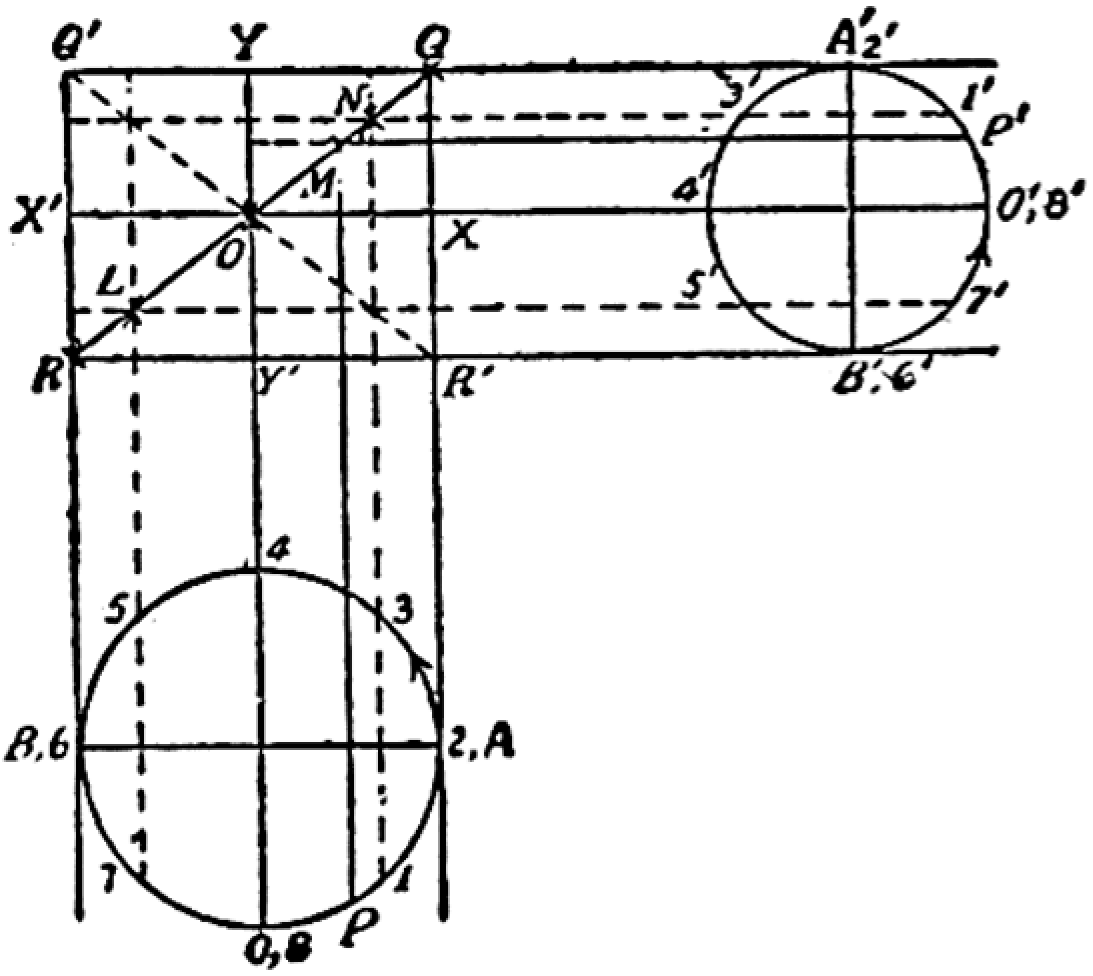


Fig. 9.

16. Composition of two Rectangular S. H. Vibrations of Equal Periods.—GRAPHICAL METHOD : (1) *Amplitudes unequal, Phase same.* Let one of the component vibrations take place parallel to AB, the horizontal diameter of the lower reference circle and the other ‖ to $A'B'$, the vertical diameter of the upper reference circle. (Fig. 9). Since the two vibrations are in the same phase they pass their mean position simultaneously. Upon AB and $A'B'$ draw reference circles of radii equal to the amplitudes a and b of the component vibrations and divided into equal number of parts, say 8. Each part will be travelled in the same time. We are compounding the component of the motion of P along AB or XX' and that of P' along $A'B'$ or YY'. The positions zero, zero on the respective circles coincide with the mean position O. This would show that the two S. H. Motions have no phase difference between them initially. From each point 0, 1, 2 etc., on the circle AB draw perpendiculars on XOX'. Similarly, in the case of circle $A'B'$ draw perpendiculars from 0′, 1′, 2′, on YOY'. At the moment of start both the perpendiculars intersect at O, the position taken up by the vibrating point. In the positions 1, 1′ respectively they intersect at N, in the positions 2, 2′ at Q and so on till after a complete time period, the point O is reached again. Thus the positions Q. N, O, L, R, are determined. By joining these points we get the Lissajous' figure. In this case the resultant path is a straight line QR, the diagonal of the rectangle of sides $2a$ and $2b$. It is a simple harmonic motion of the same time-period and phase, but amplitude.

$$r=\sqrt{a^2+b^2}.$$

If the first vibration were to start from 4 and the 2nd from O' then the path will be the other diagonal $Q'R'$ shown dotted in the figure.

(2) *Amplitudes unequal. Phase $\pi/2$ or $T/4$.* In this case the zero position in the circle $A'B'$ is kept the same while that in the circle AB is shifted forward by $\pi/2$. By drawing perpendiculars and proceeding exactly as in the preceding case, the locus is found to be an ellipse whose axes are XOX' and YOY' and which is inscribed within the rectangle $QQ'RR'$. The curve will be a circle if the amplitudes are equal. The student is expected to draw graphically the figure for this case.

(3) If the phase difference is $\pi/4$, we have the resultant as an oblique ellipse within the same rectangle. Fig. 10 illustrates this case. Here the numerals in the reference circle AB are shifted forward by $\pi/4$, those in $A'B'$ remaining in the same position as before.

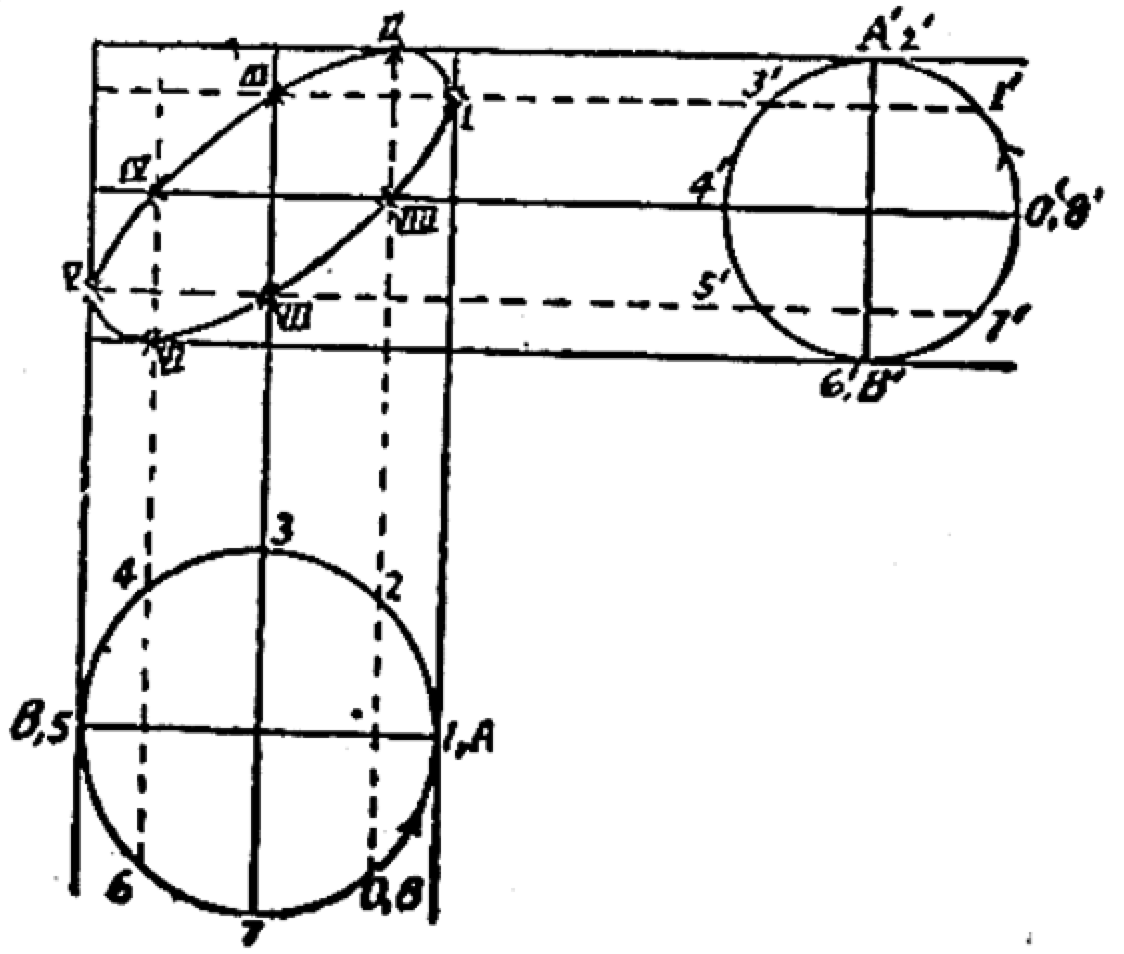


Fig. 10.

In Fig. 11 are shown the curves for different values of the phase difference varying from o to π. They are obtained by the method, described above.

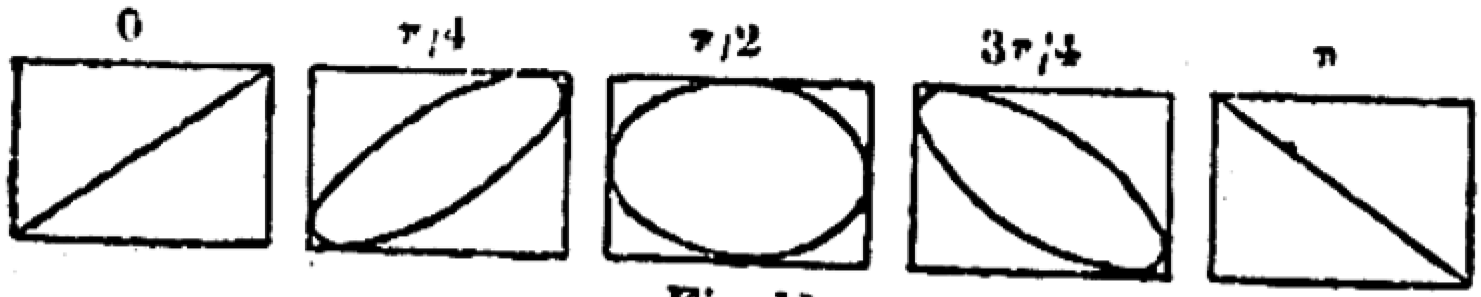


Fig. 11.

As the phase difference increases from π to 2π, the various figures are obtained in the reverse order thus completing a cycle of changes.

Analytical Treatment : (Periods 1:1).—Let the two erctangular vibrations be represented by

$$x = a \sin(\omega t + \phi) \quad \ldots \quad (1)$$

and

$$y = b \sin \omega t \quad \ldots \quad (2)$$

where a and b are the amplitudes of the component vibrations and the first vibration is ahead of the second by a phase angle ϕ. To obtain the equation of the locus *i.e.*, the path of the resultant vibration, we eliminate t between them.

Now,

$$x = a \sin(\omega t + \phi)$$

or

$$\frac{x}{a} = \sin(\omega t + \phi)$$

$$= \sin \omega t \cos \phi + \cos \omega t \sin \phi \quad \ldots (1a)$$

From (2) $\sin \omega t = \frac{y}{b}$ $\therefore$ $\cos \omega t = \sqrt{1 - \frac{y^2}{b^2}}$

Putting the value of $\sin \omega t$ and $\cos \omega t$ in equation (1a) we get

$$\frac{x}{a} = \frac{y}{b} \cos \phi + \sqrt{1 - \frac{y^2}{b^2}} \sin \phi$$

or

$$\frac{x}{a} - \frac{y}{b} \cos \phi = \sqrt{1 - \frac{y^2}{b^2}} \sin \phi$$

Squaring

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} \cos^2\phi - \frac{2xy}{ab} \cos \phi = \sin^2 \phi - \frac{y^2}{b^2} \sin^2 \phi$$

or

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} (\cos^2 \phi + \sin^2\phi) - \frac{2xy}{ab} \cos \phi = \sin^2\phi$$

or

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} \cos \phi = \sin^2 \phi \quad \ldots \quad (3)$$

which is the equation of an ellipse inclined to the axes of co-ordinates. This ellipse may be inscribed in a rectangle whose sides are $2a$ and $2b$.

Particular cases :—

(*i*) When $\phi = 0$, the equation (3) becomes

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} = 0, \text{ or } \left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0,$$

which represents a pair of coincident straight lines lying in the first and third quadrants represented by the diagonal shown in curve No. 1, Fig. 11.

It corresponds to the diagonal QOR of Fig. 9.

(*ii*) When $\phi = \pi/4$, $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{\sqrt{2}xy}{ab} = \frac{1}{2}$, which is an oblique ellipse (Fig. 11, curve No. 2).

(*iii*) When $\phi=\pi/2$, we have $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, which is the equation of the symmetrical ellipse shown in Fig. 11, curve No. 3. If $a=b$, this becomes circle $x^2+y^2=a^2$.

(*iv*) When $\phi=\pi$, $\left(\frac{x}{a}+\frac{y}{b}\right)^2=0$, representing the diagonal shown in Fig. 11, curve No. 5.

17. Composition of two Rectangular S. H. Vibrations of Time Periods nearly Equal.—When the two S. H. Vibrations are exactly of equal time-periods the elliptic paths referred to above remain perfectly steady. If, however, there is a slight difference in their time-periods, the form of the ellipse slowly undergoes a change on account of a gradual change in the relative phase ϕ of the two components. All these elliptic orbits lie within a rectangle of sides $2a$ and $2b$. Let us see the effect of a gradual change in ϕ. Starting with the phase in agreement *i.e.*, $\phi=0$, the ellipse coincides with the diagonal $\frac{x}{a}-\frac{y}{b}=0$, of the rectangle. As ϕ increases from 0 to $\pi/2$, the ellipse opens out to the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, passing through intermediate oblique positions. It closes up again ultimately coinciding with the other diagonal $\frac{x}{a}+\frac{y}{b}=0$, as ϕ increases from $\pi/2$ to π. As the phase difference varies from π to 2π, the reverse process takes place until the ellipse again coincides with the first diagonal. The half cycle of changes is shown in Fig. 11. It may be noted that the frequency of the complete cycle is equal to the difference of the frequencies of the two component simple harmonic motions.

18. Composition of two Rectangular S. H. Vibrations of Periods in the Ratio of 1 : 2.

GRAPHICAL METHOD. (1) *Amplitudes unequal, Like phase.* Here we shall proceed in the same manner as for equal periods. But as the period of one of the component vibrations say, along YOY' is double that along XOX', during the time the particle P' describes its journey once round the reference circle $A'B'$, the particle P of which the frequency is

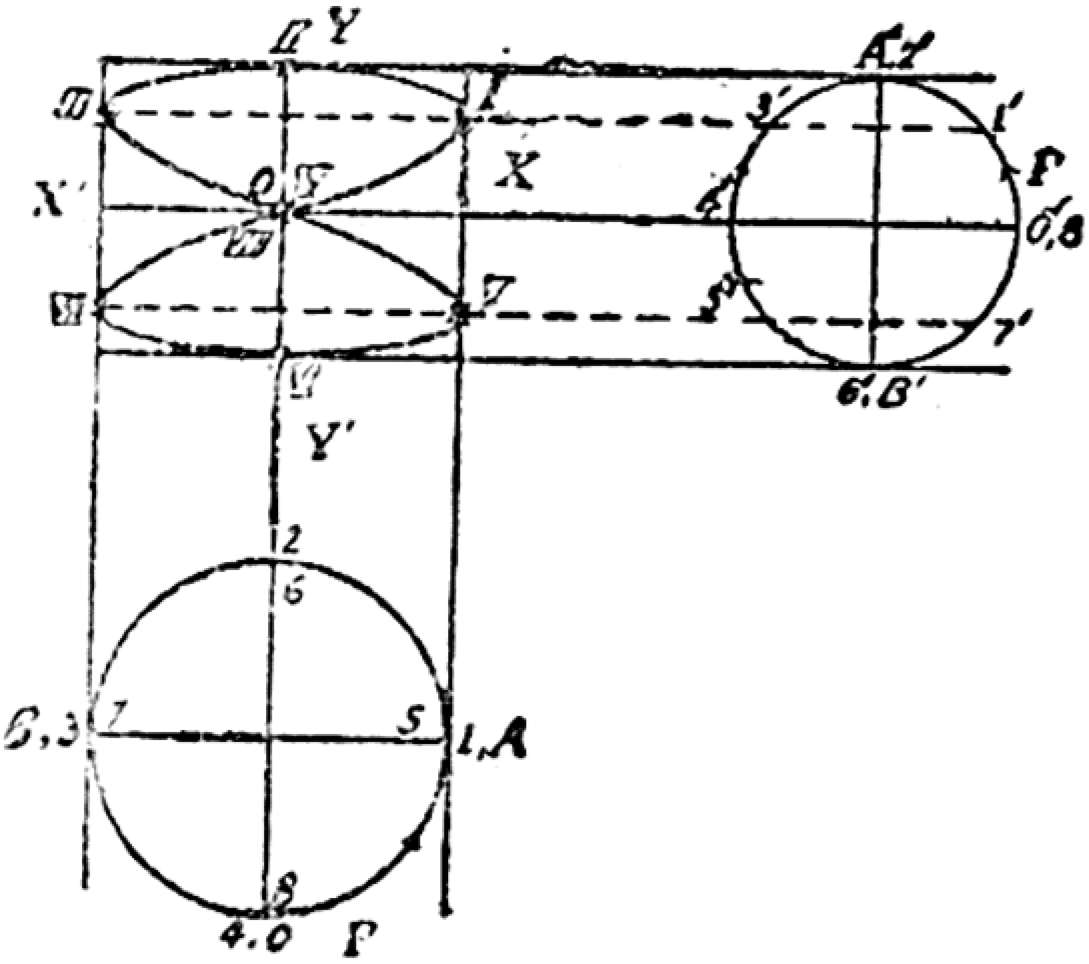


Fig. 12.

double describes two journeys. Divide the reference circle $A'B'$ into 8 equal parts and the circle AB into 4 equal parts and put numerals as indicated in Fig. 12. Both the vibrations start from their central positions simultaneously *i. e.*, they are in like phase. As before to get the position of the vibrating point draw perpendiculars on XOX' and YOY' from the corresponding points on the circles of reference and determine the points of intersections I, II, III etc. By joining these points we obtain the resultant curve. This is a figure of 8.

(2) Fig. 13 illustrates the case for an initial* phase difference corresponding to a quarter of the smaller period, the zero of circle AB has been shifted to 1, while that in the circle $A'B'$ it is in the same position. On proceeding as above we find that the resultant path is a parabola.

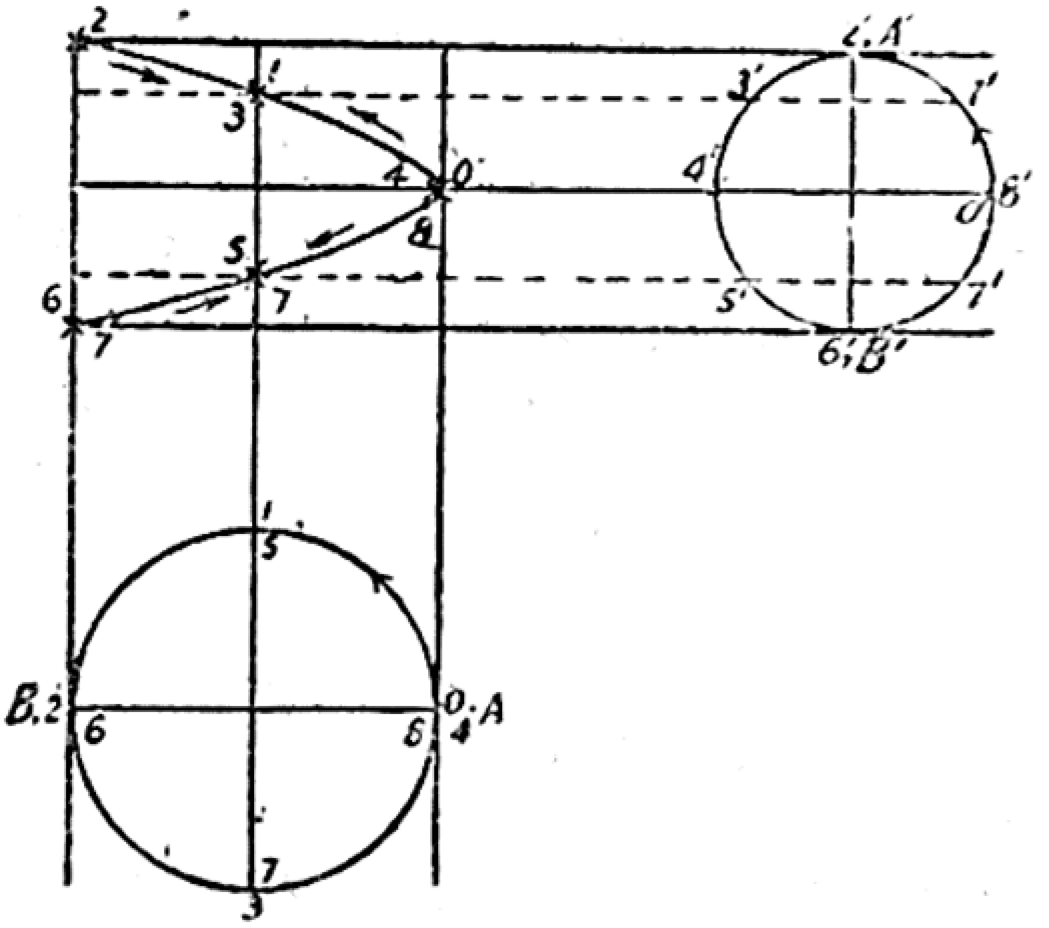


Fig. 13.

The student is expected to draw graphically figures for different initial phases. These figures for initial phases varying from 0 to π of the smaller time period, are shown below in Fig. 14 (*a*). If the ratio of time-periods is not exactly 1 : 2, the curve will change gradually forwards and backwards through the series of Fig. 14 (*a*).

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*It should be noted that, here, since the periods are quite different the phase is not constant though the particles may start in the same phase, they immediately come into phases differing from each other hence the term 'initial' phase difference.

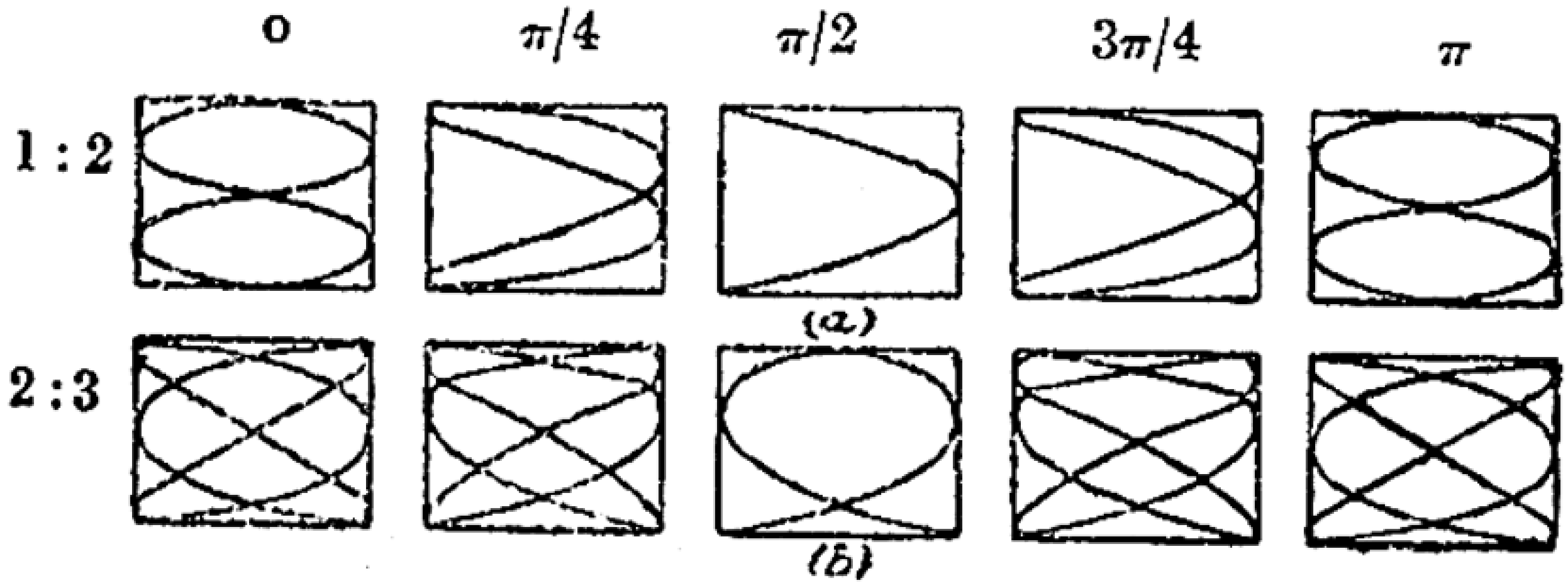

Fig. 14.

ANALYTICAL METHOD. **(Periods 1 : 2) :—**

The component vibrations are represented by

$$x=a\sin(2\omega t+\phi) \qquad \ldots \qquad \ldots \qquad (1)$$

and

$$y=b\sin\omega t \qquad \ldots \qquad \ldots \qquad \ldots \qquad (2)$$

(*i*) First introduce phase difference $\phi=0$, in (1) so we get

$$x=a\sin 2\omega t+2\,a\sin\omega t\cos\omega t.$$

or

$$\frac{x}{a}=2\sin\omega t\cos\omega t$$

Putting $\sin\omega t=\frac{y}{b}$ and $\cos\omega t=\sqrt{1-\frac{y^2}{b^2}}$ from (2),

we have

$$\frac{x}{a}=2\,\frac{y}{b}\sqrt{1-\frac{y^2}{b^2}}$$

or

$$\frac{x^2}{a^2}=\frac{4y^2}{b^2}\left(1-\frac{y^2}{b^2}\right)=-\frac{4y^2}{b^2}\left(\frac{y^2}{b^2}-1\right)$$

or

$$\frac{x^2}{a^2}+\frac{4y^2}{b^2}\left(\frac{y^2}{b^2}-1\right)=0, \qquad \ldots \qquad \ldots \qquad (3)$$

which represents a figure of 8.

(*ii*) If $\phi=\pi/2$, we get

$$\frac{x}{a}=\sin\left(2\,\omega t+\frac{\pi}{2}\right) \text{ from (1)}$$

$$=\cos 2\omega t$$

$$=1-2\sin^2\omega t$$

or

$$\frac{x}{a}=1-\frac{2y^2}{b^2} \qquad \left[\because \sin\omega t=\frac{y}{b}.\right.$$

or $$\frac{2y^2}{b^2}=1-\frac{x}{a}=-\frac{x-a}{a}.$$

$$\therefore \quad y^2=-\frac{b^2}{2a}(x-a)$$

representing the equation of a parabola with vertex at $(a, 0)$. A more general treatment of this case is given in the following section.

19. A general Analytical Treatment of the Composition of two Rectangular S. H. Vibrations of periods in the ratio of 1 : 2.—

The component vibrations are represented by

$$x=a \sin (2\omega t+\varphi) \quad \ldots \quad \ldots \quad (1)$$

and $$y=b \sin \omega t \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

To obtain the equation of the locus, eliminate t between (1) and (2)

$$\frac{x}{a}=\sin (2\omega t+\phi)$$
$$=\sin 2\omega t \cos \phi+\cos 2\omega t \sin \phi$$
$$=2 \sin \omega t \cos \omega t \cos \phi+(1-2 \sin^2 \omega t) \sin \phi \ldots (1a)$$

From (2) $\sin \omega t=\frac{y}{b}$ and $\cos \omega t=\sqrt{1-\frac{y^2}{b^2}}$

Putting these values in equation (1a), we have

$$\frac{x}{a}=\frac{2y}{b}\sqrt{1-\frac{y^2}{b^2}}\cos \phi+\left(1-\frac{2y^2}{b^2}\right)\sin \phi$$

or $$\left[\frac{x}{a}-\left(1-\frac{2y^2}{b^2}\right)\sin \phi\right]=\frac{2y}{b}\sqrt{1-\frac{y^2}{b^2}}\cos \phi$$

Squaring both sides, we get

$$\left[\frac{x}{a}-\left(1-\frac{2y^2}{b^2}\right)\sin \phi\right]^2=\frac{4y^2}{b^2}\left(1-\frac{y^2}{b^2}\right)\cos^2 \phi$$

or $$\left[\left(\frac{x}{a}-\sin \phi\right)+\frac{2y^2}{b^2}\sin \phi\right]^2=\frac{4y^2}{b^2}\cos^2 \phi-\frac{4y^4}{b^4}\cos^2 \phi$$

or $$\left(\frac{x}{a}-\sin \phi\right)^2+\frac{4y^2}{b^2}\sin \varphi\left(\frac{x}{a}-\sin \varphi\right)+\frac{4y^4}{b^4}\sin^2 \varphi$$

$$=\frac{4y^2}{b^2}\cos^2\varphi-\frac{4y^4}{b^4}\cos^2\varphi$$

or $$\left(\frac{x}{a}-\sin\varphi\right)^2+\frac{4y^2}{b^2}\times\frac{x}{a}\sin\varphi-\frac{4y^2}{b^2}\sin^2\varphi+\frac{4y^4}{b^4}\sin^2\varphi$$

$$=\frac{4y^2}{b^2}\cos^2\varphi-\frac{4y^4}{b^4}\cos^2\varphi$$

Transposing we get

$$\left(\frac{x}{a}-\sin\varphi\right)^2+\frac{4y^2}{b^2}\times\frac{x}{a}\sin\varphi$$

$$=\frac{4y^2}{b^2}(\sin^2\varphi+\cos^2\varphi)-\frac{4y^4}{b^4}(\sin^2\varphi+\cos^2\varphi)$$

or $$\left(\frac{x}{a}-\sin\phi\right)^2+\frac{4y^2x}{b^2a}\sin\phi=\frac{4y^2}{b^2}-\frac{4y^4}{b^4}$$

or $$\left(\frac{x}{a}-\sin\phi\right)^2+\frac{4y^2}{b^2}\left(\frac{y^2}{b^2}+\frac{x}{a}\sin\phi-1\right)=0 \qquad \ldots \quad (3)$$

which is the general equation for a curve having two loops for any phase difference and amplitude.

Particulars cases :—

(*i*) When $\phi=0$, or π, the equation (3) becomes

$$\frac{x^2}{a^2}+\frac{4y^2}{b^2}\left(\frac{y^2}{b^2}-1\right)=0 \qquad \ldots \quad \ldots \quad \ldots \quad (4)$$

the same as equation (3) of the last section representing the fig. '8'.

(*ii*) When $\phi=\pi/2$, we obtain

$$\left(\frac{x}{a}-1\right)^2+\frac{4y^2}{b^2}\left(\frac{y^2}{b^2}+\frac{x}{a}-1\right)=0$$

or $$\left(\frac{x}{a}-1\right)^2+\left(\frac{2y^2}{b^2}\right)^2+4\frac{y^2}{b^2}\left(\frac{x}{a}-1\right)=0$$

or $\left[\left(\frac{x}{a}-1\right)+\frac{2y^2}{b^2}\right]^2=0$, representing two coincident parabolas each of which has an equation $\left(\frac{x}{a}-1\right)+\frac{2y^2}{b^2}=0$

or
$$\frac{2y^2}{b^2} = -\left(\frac{x}{a} - 1\right) = -\frac{(x-a)}{a}$$

$$\therefore \qquad y^2 = -\frac{b^2}{2a}(x-a) \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

If the time-periods depart slightly from the ratio 1 : 2, the form of the curve slowly varies in the manner indicated by the changes of ϕ in equation (3) above. Typical curves of this character have already been shown in Fig. 14 (*a*).

20. Composition of two Rectangular S. H. Vibrations of periods in any Commensurable ratios.—The graphical and analytical methods explained above are general and applicable to rectangular vibrations in any commensurable ratios like $m : n$ reduced to lowest terms. The period of description of the resultant curve will be mn, the L. C. M. of the separate periods because when one attains its extreme value n times and the other m times, the origin or the position of the starting point is reached. The series of curves in Fig. 14 (*b*) shows the forms for time-periods in the ratio 2 : 3 and phases varying from 0 to π.

When the two periods are incommensurable, we cannot represent them by two integral numbers. The particle executing the resultant vibration will never come back to the starting point but will go on describing an endless curve.

21. Experimental Methods of obtaining Lissajous' Figures.—

(1) **Optical Method.**—*A* and *B* are two tuning forks whose vibrations may be maintained electrically. Each fork has a small mirror attached to the side of a prong. They are arranged so that *A* is vertical and *B* horizontal. *A* beam of light from a small powerful source, say, a circular hole illuminated by an arc lamp, strikes the mirror of the vertical fork, is reflected to the mirror of the horizontal fork and thence brought to focus on a screen *S* by means of the lens *L*. Then if the

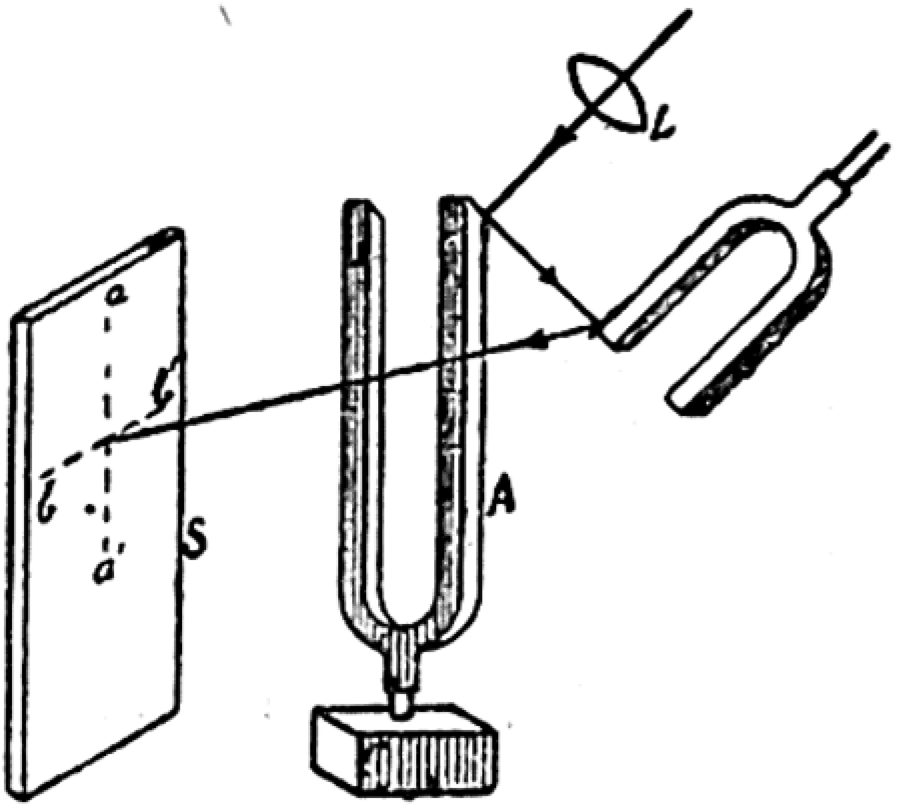

Fig. 15.

horizontal fork vibrates, a horizontal bright line *bb'* is traced on the screen and if the vertical fork vibrates only, a vertical bright line *aa'* is obtained. Both these motions represent two S. H. Vibrations at right angles to each other. When both the forks vibrate together the vibrations of the spot of light are compounded. The spot will describe the locus of the resultant motion. If the ratio of their periods is a simple one, Lissajous' figures will be seen on the screen. A better arrangement is to observe them through a telescope. This enables us to use a small source of light and obtain sharper images. The method is usually used for comparing the frequencies of two forks. If the ratio of frequencies is 2 : 1, a figure of 8 may be obtained. It will gradually pass through the various phases already discussed if the ratio between the frequencies is approximately 2.

(2) **Helmholtz's Vibration Microscope.**—The instrument was originally designed by Helmholtz to study the vibrations of a violin string. It consists of a horizontal tuning fork *B* generally electrically maintained of known frequency with a lens *L* fixed to one of its prongs and having its axis perpendicular to the plane of vibration. The lens serves as an objective to a microscope, the upper part of which is held separately by a vertical stand. Another tuning fork of unknown frequency, is held vertically such that its vibrations are at right angles to that of *B*. A dot *D* is marked on the top of one prong of the fork *A* and is kept below the microscope. When both the forks vibrate, the motion of the dot is the resultant of the rectangular S. H. vibrations. If the ratio between the frequencies is expressible by simple whole numbers, stable Lissajous' figures may be observed, otherwise the figure changes too rapidly to be recognized. If the periods are exactly equal, any one of curves in Fig. 11. may result. If the periods are not quite equal then all the possible forms dependent on phase difference are successively seen, while one motion gains a period over the other. The comparison of the frequencies of the forks are made as explained in Art. 22.

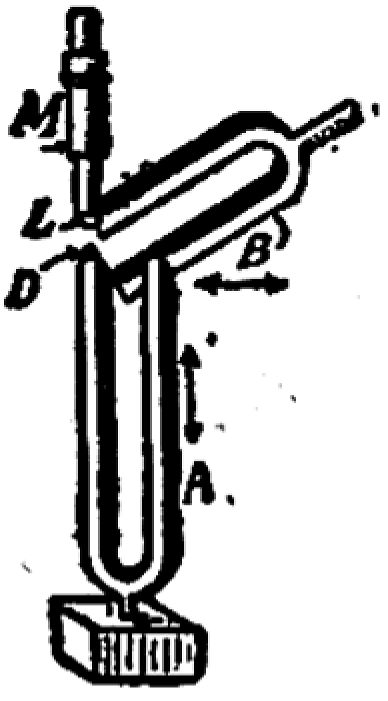

Fig. 16.

For further use of the Vibration Microscope see Art. 54 (*a*)

(3) **Wheatstone's Kaleidophone.**—Consider a rod of rectangular section fixed at one end in the jaws of a vice. This shall have different periods of vibration in two directions at right angles to each other. On drawing aside the free end obliquely to the plane of the jaws *i.e.*, in a direction not parallel to either of the sides of the rectangular rod, both the vibrations will be executed simultaneously, and if a small bright bead is placed at the top, it will describe one of the Lissajous' figures.

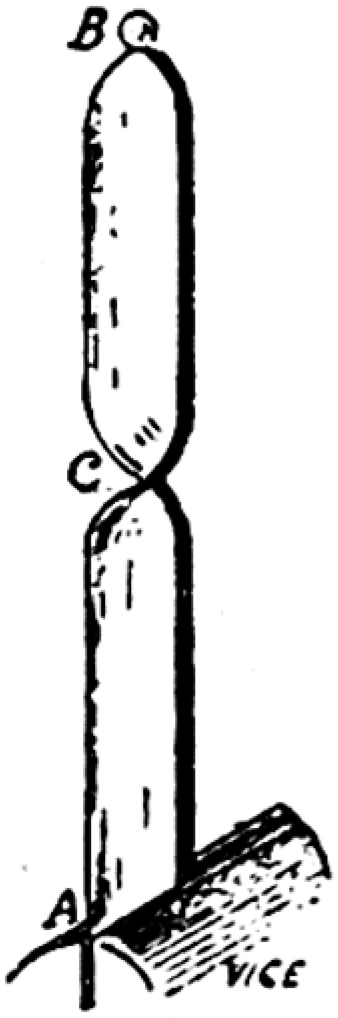

Fig. 17.

A simple device is the *Kaleidophone*. Two thin strips of steel are joined end to end, their planes being at right angles to each other. Or we may simply take a straight strip *AB* of steel 5 to 6″ in length, heat a small portion near the middle point to redness in a blow pipe flame and twist it while in the flame, so that the planes of the two halves are mutually at right angles (Fig. 17). The lower end is fixed in the jaws of a vice. If the other end *B* is vibrated *parallel to the plane of the jaws*, the lower half will remain straigth and the time period will depend upon the length and stiffness of the upper half. In case the end *B* is displaced at *right angles to the jaws*, the upper half will remain straight and the lower half will vibrate, the time-period depending upon the length and stiffness of the lower half. By clamping the lower half at different places, the two vibrations may be made to have *any desired frequency ratio.* As before if a bright silver bead is fixed at the top, and the rod is displaced obliquely to the plane of the jaws of the vice, Lissajous' figures will be described by the bead their shape depending upon the frequency ratio already adjusted. On account of persistence of the visual impressions, the bright bead will seem to trace out a continuous curve.

For observing the tracks in the methods described above we make use of that property of the eye known as *persistence of vision*, viz., that any impression produced on the retina lasts for about $\frac{1}{16}$ to $\frac{1}{10}$ second, so the vibrations should be rapid. We give below two devices whereby we can have a permanent trace and the vibrations may be slow.

(4) **Blackburn's Pendulum.**—A laboratory form of the apparatus consists of a weight R suspended by three strings meeting at E where it is supported by another string $PADEDC$. This second string passes through holes at A and C in a wooden bar AC fixed at the top of a framework and is attached at the top to a screw plug P which regulates its length below AC. By adjusting the position of a ring at D the length DM can also be altered at will. The system is equivalent to two pendulums with lengths BM and DM. If it is displaced in the plane of the paper it will vibrate as a pendulum of length DM. If displaced at right angles, the length of the pendulum considered is BM. Let $DM=l_1$ and $BM=l_2$, then the time-periods of the component vibrations are

$$t_1=2\pi\sqrt{\frac{l_1}{g}} \text{ and } t_2=2\pi\sqrt{\frac{l_2}{g}}.$$

Dividing one by the other, we have

$$\frac{t_1}{t_2}=\sqrt{\frac{l_1}{l_2}}$$

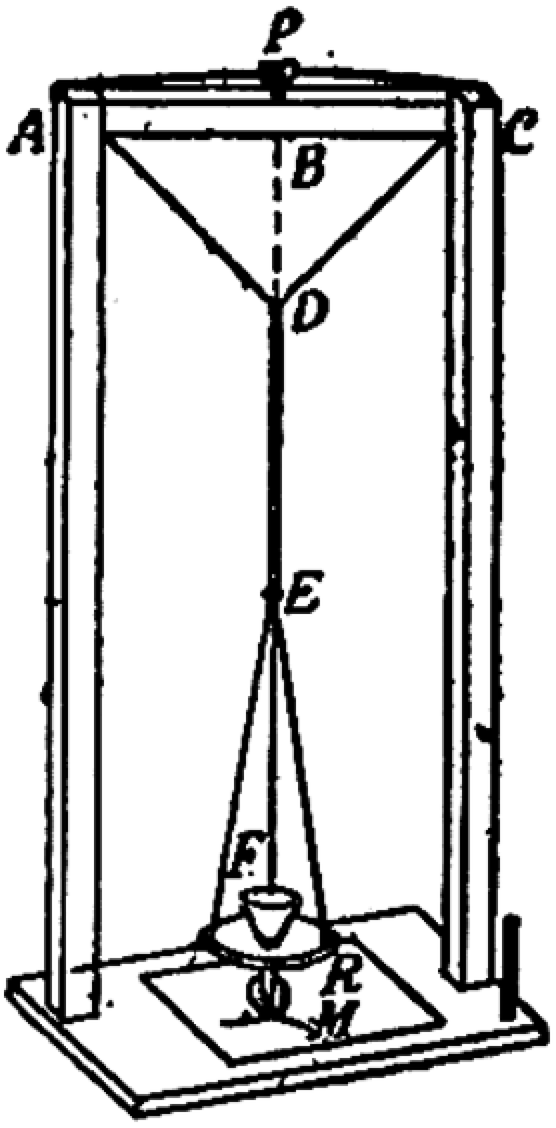

Fig. 18.

If a funnel F filled with sand is placed in a hole in the weight R and the weight is displaced obliquely, the sand running out will trace one of the Lissajous' figures, in which the time periods of the component vibrations will be directly proportional to the square root of the lengths.

If the length of the pendulum DM, say, is $\frac{1}{4}$ of the length of the pendulum BM *i.e.* $l_1=l_2/4$, then from the relation above we have,

$$\frac{t_1}{t_2}=\sqrt{\tfrac{1}{4}}=\tfrac{1}{2}$$ giving us a figure of 8. The resultant motion will gradually pass through various phases shown in fig. 14 (*a*) if the ratio between time periods is not exactly 1 : 2.

(5) **Tisley's Harmonograph.**—This is a very interesting device to show composition of rectangular vibrations ; P and Q represent two pendulums suspended so as to swing in planes at

right angles to each other (Fig. 19) A and B are two levers joined together at J by a flexible joint with a pen P attached to the joint. The other ends of the levers are hinged at the top of each of the pendulums which slightly extend above the points of supports. Either pendulum swinging without the other would give the pen a very nearly harmonic rectilinear motion. When both the pendulums are swinging together the motion of the pen will be compounded of the two motions and the pen traces a series of Lissajous' figures.

In all these methods it is rather difficult to adjust the periods of vibration quite exactly to any simple ratio, so the curve described always changes slowly. In addition, on account of friction and damping the amplitudes of the constituent vibrations decrease gradually, so that interlacing of the curves occurs and very beautiful patterns are produced.

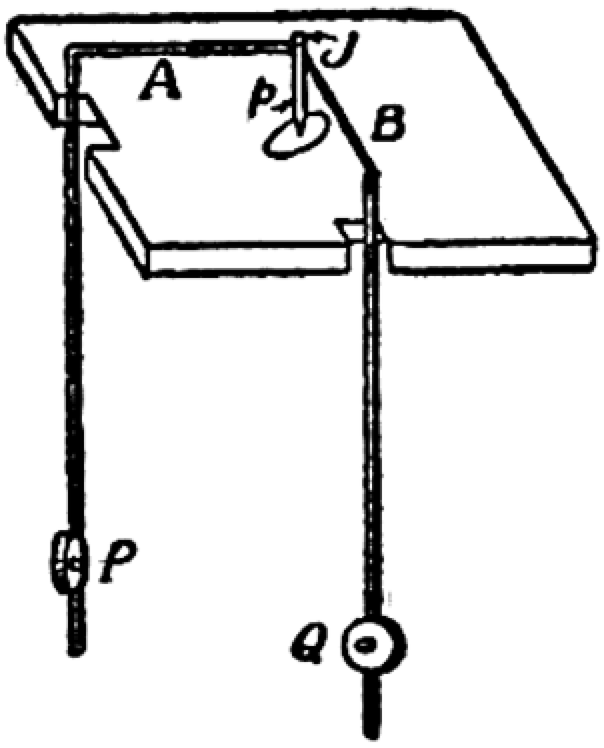

Fig. 19.

22. Uses of Lissajous' Figures in Acoustical determinations.—

(1) *Measurement of the ratio between the time-periods of the component vibrations*—One practical application of Lissajous' curves is that they enable us to find by inspection the vibration ratio of its constituents. For instance in the first curve of Fig. 14 (*b*), the curve touches the horizontal side of the bounding rectangle twice and the vertical side three times. It follows that the vibrating point makes three vibrations in the horizontal direction in the time it takes to make two in the vertical direction or the ratio of periods is 2 : 3. The same result could be obtained by drawing horizontal and vertical lines across the Lissajous' curves. In the above case the horizontal line cuts the curves 4 times and the vertical line 6 times, the ratio of periods is 4 : 6 or 2 : 3. In general if these lines cut a curve, say m and n times respectively, the required ratio is $m : n$.

(2) *Comparison of the frequencies of two forks which are nearly in unison with each other.*—Let the upper fork in the optical method of obtaining Lissajous' figures have a frequency

slightly higher say $n+b$, where n is the frequency of the lower fork and b is a small fraction. The sharpness of the upper fork could be tested by attaching a small piece of the wax to a prong of the fork, and noting the changes in the figures which are slowed down showing that the upper fork is sharp. Having done this, start obtaining the Lissajous' figures, the curve obtained will be approximately an ellipse but soon one fork will be found to gain over the other, the phase changing gradually ; the curves will pass through a series of changes shown in Fig. 11 from No. 1 to 5 and back again to No. 1, when the phases agree once more. This cycle of changes will be repated. If t seconds be the time for this cyclic change, then $bt=1$ or $b=\frac{1}{t}$. Hence the upper fork is of frequency $n+\frac{1}{t}$.

Example 1. *Let the lower fork be of frequency 100 and let one cycle of changes be completed in 50 seconds. Then the upper fork gains one vibration over the other in 50 seconds i.e., the frequency of the upper fork is greater by $\frac{1}{50}$ of a vibration.*

$$b=\frac{1}{t}=\frac{1}{50}$$

$$\therefore \quad n+b=100+\frac{1}{50}=100+\cdot02=100\cdot02.$$

Example 2. *Two tuning forks A and B of approximately equal frequencies are employed to obtain Lissajous figures and it is observed that the figures go through a cycle of changes in 15 seconds. On slightly loading A with wax the figures go through the cycle in 10 seconds. If B has a frequency of 200, what is the frequency of A before, and after loading ?*

Before loading the fork A gains or loses one vibration over B in 15 seconds i.e., the frequency of A is greater or less than B by $\frac{1}{15}$ of a vibration

$$\begin{aligned} A &= B\pm\tfrac{1}{15} \\ &= 200\pm\tfrac{1}{15} \\ &= 200\cdot066 \text{ or } 199\cdot934 \end{aligned}$$

Now loading A flattens the fork A and since it makes the changes in the Lissajous figures quicker the fork is too flat so the frequency before loading is 199·934 and after loading it is $200-\frac{1}{10}=199\cdot9$.

(3) *Comparison of the frequencies of two forks which are very nearly an octave apart.—*

For a nearly octave interval Lissajous' figure varies from a figure of 8 to a parabola and *vice versa.* In this case the time t in which the upper fork executes one vibration more or less than double the number made by the lower, is the time taken to complete the cycle of changes once *i.e.*, the figure changes from a parabola with its vertex in one direction, through a figure of 8, to a parabola with its vertex in the opposite direction and back again. As before $b=\frac{1}{t}$.

∴ The upper fork has frequency $2n\pm b$.

Example 1. *Let the frequency of the lower fork be 100. Suppose the cycle of changes is found to be completed 5 times in 20 seconds. i.e., time to complete one cycle of changes is 4 seconds. Thus the upper fork will execute 1 vibration more or less than double of the lower in 4 seconds i.e., its frequency is greater or less than double of the lower by* $\frac{1}{4}$ *of a vibration.*

So $$b=\frac{1}{t}=\frac{1}{4}.$$

∴ The upper fork has frequency
$$\begin{aligned}&=2n\pm b\\&=200\pm\tfrac{1}{4}\\&=200{\cdot}25 \text{ or } 199{\cdot}75.\end{aligned}$$

We can find whether we are to take the higher or lower number by sticking a very small piece of wax on the upper fork. This flattens the fork and if it makes the changes in the Lissajous' figures slower the fork is too sharp and *vice versa.*

Example 2. *Lissajous figures are produced with two tuning forks whose frequencies are approximately in the ratio 2 : 1. It takes 6 seconds to go through a cycle of changes. On loading slightly the fork of the higher pitch, the period of cycle is raised to 10 seconds. If the frequencies of the lower fork is 150, what is the frequency of the other fork before and after loading.*

The upper tuning fork executes 1 vibration more or less than double of the lower in 6 seconds *i.e.*, its frequency is greater or less than double of the lower by $\frac{1}{6}$ of a vibration.

Since the frequency of the lower fork is 150, that of the upper fork

$$
\begin{aligned}
&=2\times 150\pm\tfrac{1}{6}\\
&=300\pm\tfrac{1}{6}\\
&=300{\cdot}166 \text{ or } 299{\cdot}934
\end{aligned}
$$

Since on loading the upper fork the changes in the Lissajous figures are slowed down, the upper fork must be flatter viz. its frequency before loading is $300\frac{1}{6}$ and after loading $300\frac{1}{10}$.

For practical purpose the fork whose frequency is required should be taken, as it is. No mirror should really be attached to it for that would change its frequency. Hence instead of the optical method, Helmholtz's Vibration Microscope should be used. The fork whose frequency is to be determined is placed vertically, below the microscopic objective.

The *Lissajous' Figures* are specially made use of in tuning two forks accurately to some consonant interval, for even if the interval is so nearly accurate that the figure has not completed its cycle of changes before the vibrations stop altogether, it is easy to judge whether the change is taking place or not. The fork can be made quicker by filing it near the ends of the prongs or slower by filing it at the bend till a Lissajous' figure corresponding to the required vibration ratio is obtained.

CHAPTER III

WAVE MOTION : TRANSVERSE AND LONGITUDINAL

23. Wave Motion.—The concept of wave motion is very fundamental in physics. It is one of the most important methods of transfer of energy. We now know that it is chiefly with its help that not only sound energy but various other forms of energy are transmitted from one place to another, for instance light, heat, wireless, X-radiation and γ-radiation. We shall begin by explaining how waves are produced in a medium.

In Chapter I we discussed the S. H. Motion of a single isolated particle or a point in a body in the hope that it might lead to a useful method of describing wave motions. Let us now consider a number of points in an elastic medium so that one of them cannot move without disturbing its neighbours. Thus a displacement of any particle will produce a disturbance which is handed on from particle to particle till all of them have suffered greater or less displacements. This disturbance is termed **Wave Motion.** It is then caused by the simultaneous periodic motions of a series of particles of an elastic medium but in which phase differs regularly from one particle to the next. It may be noted that if energy is to be transmitted through any medium by means of waves, that medium must satisfy three independent conditions which may be stated thus :

(1) The medium must possess '*Elasticity*' *i.e.*, there must be a tendency for the medium to return to its original condition after being disturbed.

(2) The medium must be capable of storing up energy *i.e.*, it must have the property of '*Inertia*'.

(3) The frictional resistance must not be so great as to damp the oscillatory movement. In our present discussion we shall assume that such resistance is negligible.

There are two distinct types of wave motions with which we are mainly concerned in the study of sound : (*i*) **Transverse wave** in which each particle of the medium executes simple harmonic motion about its mean position of rest in a direction at right angles to the direction in which the wave advances; for instance ripples on the surface of water, vibrations in strings, etc.; (*ii*) **Longitudinal wave** in which each particle of the medium executes S. H. Motion about its mean position of rest in the line along which the wave is travelling though it may be in opposite direction along that line, such as waves transmitted along a stretched spring by compressing its one end and then releasing it and those set up in air by the vibrations of a sonorous body.

In the following few sections, we shall deal with the formation and propagation of the *progressive* type of transverse and longitudinal wave motions which are characterised by the *continuous transference in the same direction of a condition of altered relative position of adjacent particles by similar movements performed by consecutive particles one after the other.*

Such a type of wave motion is called **Progressive Undulation.**

24. Transverse Wave.—In order to understand the formation and propagation of a transverse wave let us consider a series of particles arranged in a row AB and equidistant from one another when they are at rest. Suppose all these particles execute S. H. Motions of equal amplitudes and time—periods along lines at right angles to AB but in which the successive phases differ by $\pi/6$ or $T/12$, *i.e.*, each particle is affected $T/12$ seconds later than its predecessor, T being the periodic time of vibration. The various stages in the formation of the wave form are shown in Fig. 20, each stage corresponding to an increase of time $T/12$ starting with time

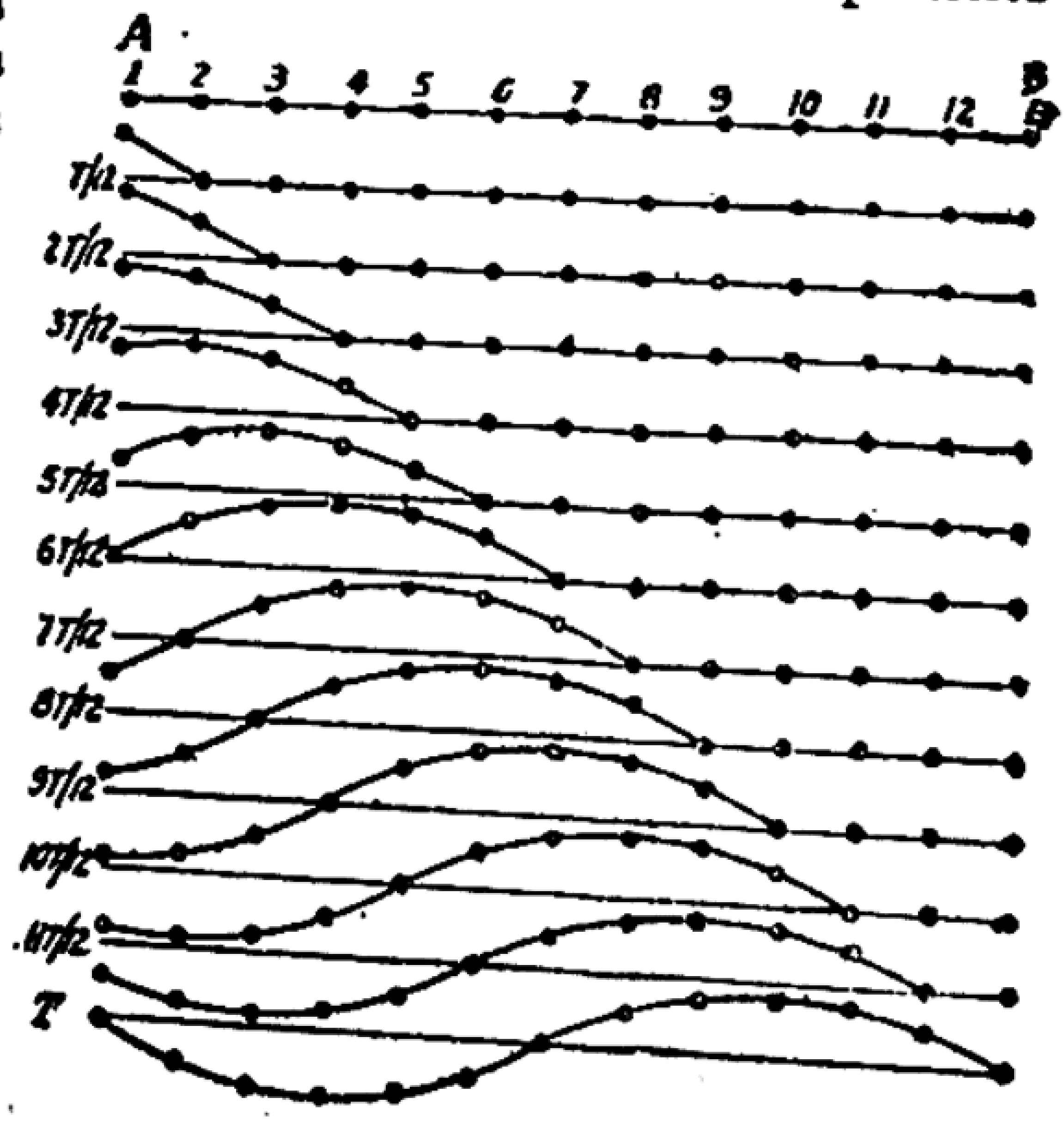

Fig. 20.

$t=0$. To clearly picture how the wave form progresses, motion of all these particles at any subsequent time may be represented by those of 12 particles moving at equal distances round a reference circle in the anti-clockwise direction, the radius of the circle being equal to the common amplitude of the vibrations. The displacement of any particle for a particular instant is given by the ordinate of the corresponding particle in the circle [Fig. 21 (*a*)], hence the curve drawn through the positions of the particles at any instant will be a harmonic curve. This is shown by the continuous curve of Fig. 21 (*b*). From this it is clear that particles No. 1 and 13 are in the same phase of disturbance and so are particles No. 2 and 14 and so on. As their phases differ by a whole period, they will have the same displacement and will be moving in the same direction. In Fig. 21 (*b*) the dotted curves indicate the positions of the particles at two successive intervals of $T/12$ each, starting from the position shown in thick line. It will be noticed that both the dotted curves may be obtained from the first by displacing it towards the right and hence as the motion goes on, the curve connecting the particles appears to move steadily towards the right. These curves are called **waveform curves**. They are similar to the displacement curves already discussed but remember that whereas the displacement curve represents the position of one particle at different times in its motion, the abscissae representing the time, the present curves are really instantaneous pictures representing the positions of all the disturbed particles at one particular instant, the abscissae being the distances of the mean positions of the particles measured from some fixed point.

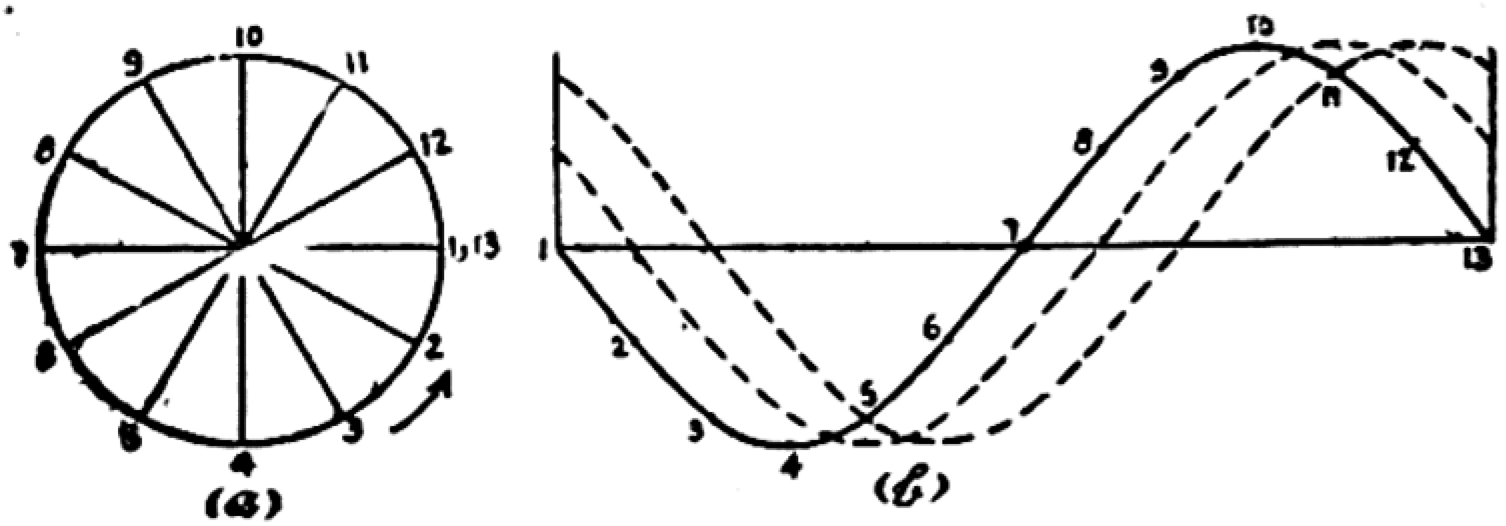

Fig. 21.

The succession of these waveform curves produces the effect of a wave motion. The student is familiar with this type of wave motion—*ripples* produced on the surface of still water when a stone is dropped in a pond. As soon as the stone strikes the surface of water, it causes a depression, water tries to recover its original position (due to elasticity) and consequently it flows into the depression, but on account of the momentum acquired it overshoots the mark with the result that a depression is followed by an elevation which is again followed by a depression and so on. Due to cohesion the neighbouring particles are also affected which repeat the motion of their predecessors. The resultant effect is that although the particles move up and down, a wave passes across the surface of water in the form of crests and troughs. A particle is said to be at the *crest* of a wave when it is at its maximum positive displacement and is at a *trough* when the displacement has maximum negative value, the distance from crest (or trough) to the next crest (or trough) is called a *wavelength*. It is the distance moved by the wave in one complete time period or the distance between two consecutive particles which are in the same phase. We understand from above that a **wave motion** may be regarded *as a disturbance which is caused by the particles of a medium executing definite periodic vibrations about their mean positions,* the phase varying regularly from particle to particle and with a definite velocity depending upon the density and the elastic constants of the medium.

It is worth noticing that as the wave advances into the undisturbed portion of the liquid, each portion of the wave itself becomes, in effect, the source of a disturbance which is passed on to the portion of the medium in its front exactly as the original disturbance caused by the stone passed outward by the first wave. This seems to explain the *self propagating* nature of the wave motion.

25. Ripple Tank.—The propagation and properties of wave motion in general and those of transverse waves in particular, can be beautifully illustrated by means of an arrangement called *Ripple* tank. In its simplest form it can be devised in any Physical Laboratory. A shallow rectangular tank with gradually sloping sides* is fitted with a glass bottom and levelling screws. It is partly filled with water and levelled on a table with a hole underneath for illuminating it by a strong source of light at the bottom. The light projected vertically through the water is received on an inclined mirror which reflects it horizontally on the screen. If now a rod is made to dip smartly in the surface of water, ripples will be caused travelling outwards to the boundary of the vessel in circular form and their shadows obtained on the screen. On placing a few small pieces of cork on the surface of water, they will be seen to rise and fall and not propagate outwards with the wave motion.

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*To avoid reflection and consequent interference with the direct waves.

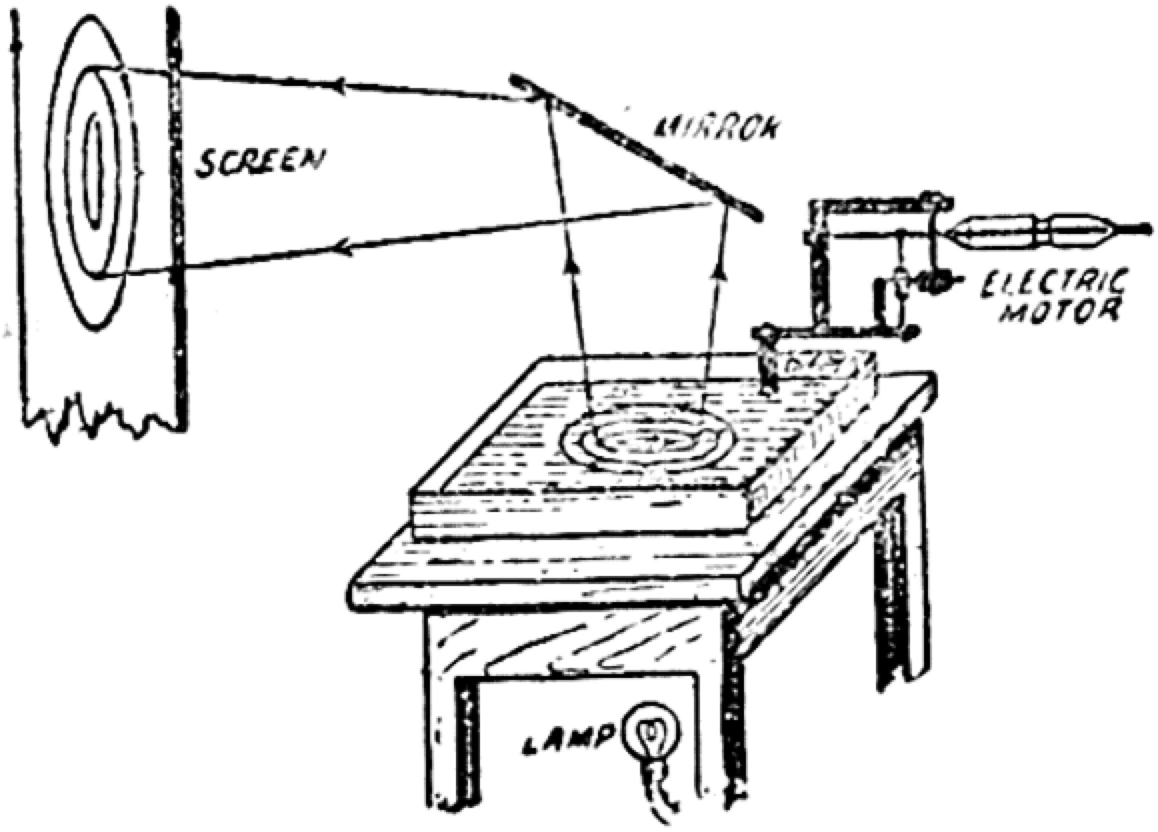

Fig. 22. (Ripple Tank)

To produce ripples constantly an electrically maintained tuning-fork of low frequency is used. A prong of this fork carries a light frame-work to which is attached a wire dipping in the water in the tank. When the tuning-fork vibrates circular ripples are produced. If two wires are attached to the frame-work, two simultaneous systems of circular waves are set up and interference effects may be conveniently studied and photographed. To generate plane waves, a flat wooden strip with a sharp edge and of moderate length is attached across the two wires so that its edge just dips in the surface of water. In another form of the apparatus by R. W. Pohl the device for producing ripples is a vibrating lever operated by an eccentric fixed to the shaft of a motor. The lever is arranged to have a large amplitude and low frequency.

For circular waves a metal rod with a tapering end is attached to the end of the lever and for plane waves the metal rod is replaced by a wooden strip as before. On the screen circular waves would appear as bright concentric circles while the plane waves appear as a series of illuminated parallel lines.

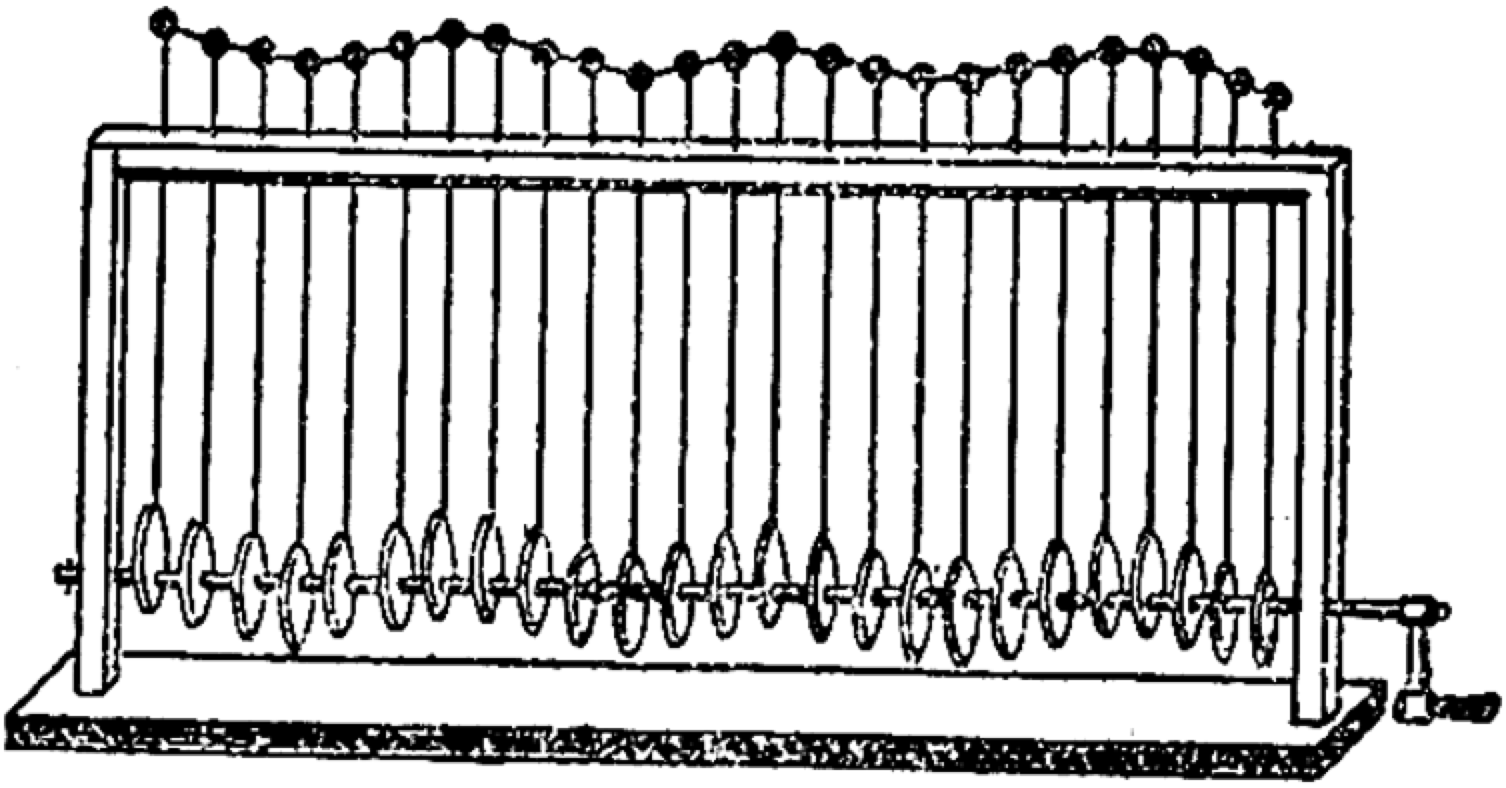

Fig. 23. Transverse Wave Machine.

26. Wave Apparatus.—A suitable model to demonstrate transverse wave and to clearly bring out the distinction between the motion of the wave and the motion of the medium is shown in Fig. 23. It consists of a series of vertical rods passing through holes in a wooden bar forming the top of a framework. Each rod carries a small ball at its top end while its lower end rests on the circumference of a series of eccentric discs fitted to a shaft which can be turned by a handle. On turning the handle each ball performs an up and down motion in a vertical path whereas the waves represented by the instantaneous positions of these balls appear to pass along the top of the model horizontally.

✓**27. Transverse Waves in a String.**—Transverse waves may be conveniently demonstrated by means of a cord about 15 ft. long. Hold its one end *A* in the hand and fasten the other end to a hook *B* in a wall. Stretch the cord so that *AB*

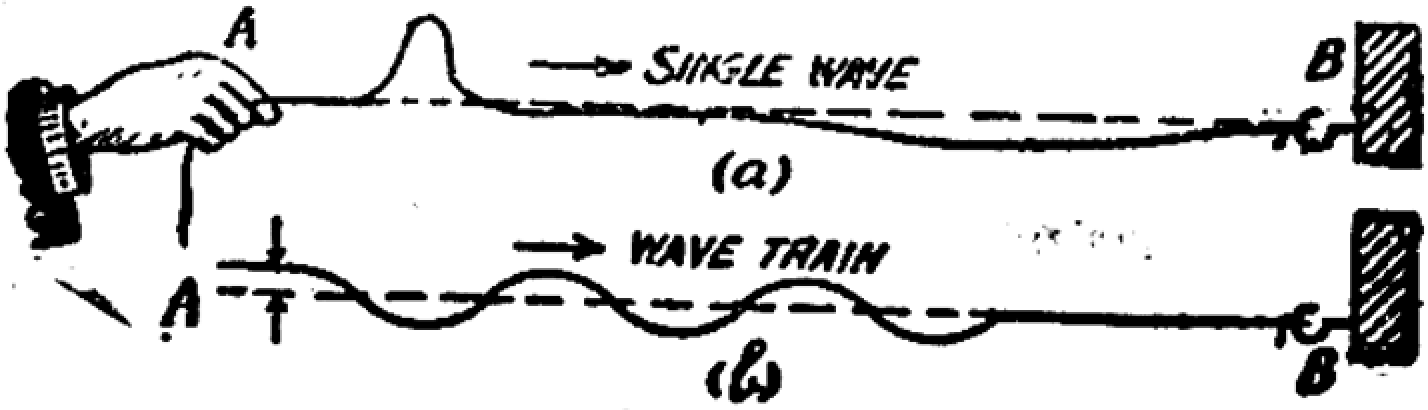

Fig. 24.

is horizontal. When the hand is moved up and down rapidly a number of loops consisting of crests and troughs travel along the cord as indicated in Fig. 24 (*b*). Each element of the string executes the same motion as the end *A* but the relative phases are different for the phase of any given particle in the string depends upon its distance from the end *A* and the velocity with which the wave propagates.

Fig. 25 shows what may be an instantaneous picture of it. Each element of the rope is moving up and down with the same frequency. At *a*, *e* and *i*. the particles have reached their crests while those at *c* and *g* have reached their troughs. At points lying between *a* and *c*, the particles have various displacements but all are moving upwards and those between *c* and *e* are moving downwards and so on. Looking at this figure and thinking of these motions, one can see that in a short time particles at *a* will have moved downward and those at *b* will have moved up and formed a crest with the result that a crest will have moved towards the right. In the same time *c* has moved up and *d* down so that the trough also moved towards the right. Thus by a transverse vibratory motion of the cord the crests and troughs travel towards the right. Note there is no motion of the cord towards the right. It is an important fact in a wave motion that the medium is not carried along with the wave but there is always a transfer of an energy in the direction of the wave. The forces and motions of the particles are always in such direction that each particle is doing work on the particle in front of it.

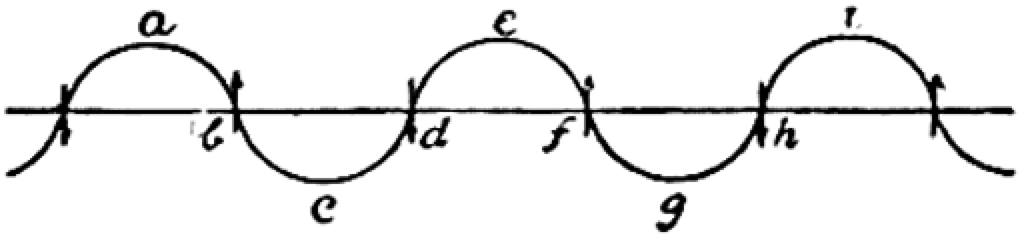

Fig. 25.

Referring to Fig. 25 again it will be seen that the segment of the cord at '*e*' is pulling upon '*f*' in the direction '*f*' is moving, hence it is doing work on the element '*f*'. The segment at '*e*' also exerts a force on '*d*' but '*d*' is not moving in the direction of the force and hence does not receive energy from '*e*'. Energy is transmitted only in the forward direction. We shall study more about these waves in Chapter IV.

Other illustrations of transverse vibrations are the vibrations of the prongs of a fork, vibrations in reeds, etc.

28. **Longitudinal Wave.**—As said above in this type of wave motion particles of a medium travel backwards and forwards in the line in which the wave travels : the particles periodically approach or recede from one another forming alternate compressions and rarefactions in the body. Here also as in the transverse wave the vibrating particles move through a small distance about their mean positions but the energy is carried forwards in the medium. To get a general idea of longitudinal vibrations of air particles, imagine a hollow cylinder *AB* closed at one end and fitted at the other end with an airtight piston *P* which can easily move to and fro within it. If the piston is rapidly moved towards the right, a thin layer of air *CD* close to the piston is suddenly compressed. In relieving its strain it will move forwards to the right compressing the next layer *DE* and soon a pulse of compression will travel onwards in the medium. Let us next imagine that the piston is rapidly moved towards the left. The layer of air *CD* will then be rarefied and its pressure will fall. Air will flow in from *DE* to restore the pressure of *CD* and *DE* will be rarefied and the same will happen to layer after layer. Thus a pulse of rarefaction will travel towards the right.

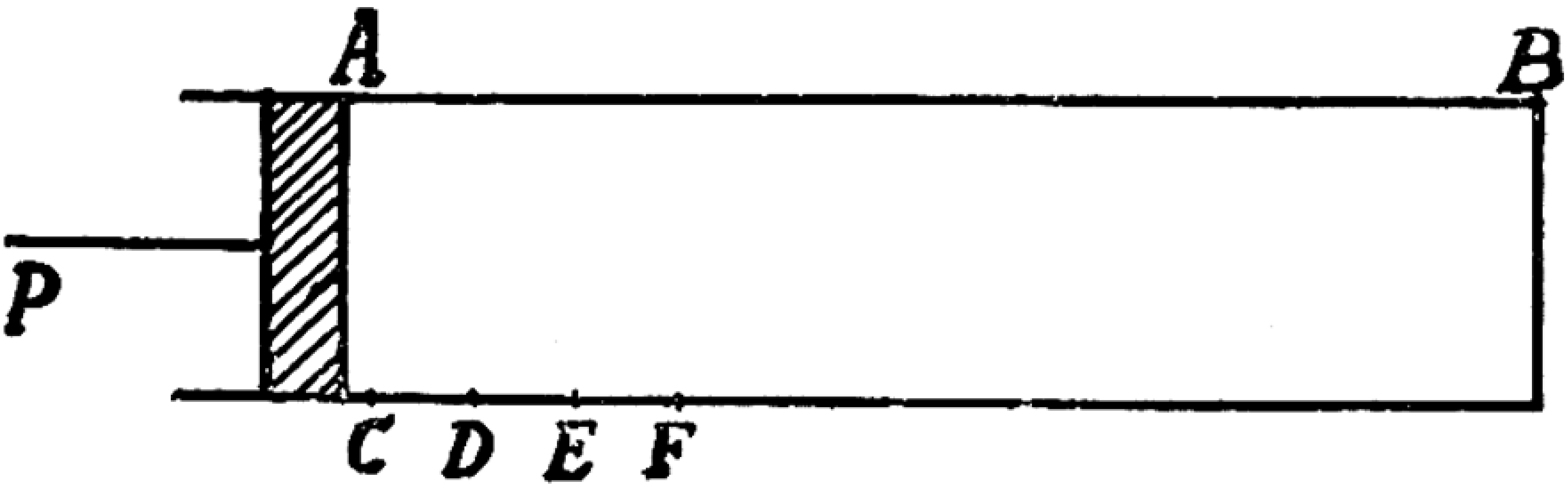

Fig. 25 (a).

If the piston is rapidly moved to and fro, a train of waves consisting of alternate condensations and rarefactions will travel along the cylinder, the particles of air vibrating to and fro parallel to its axis. It is for this reason that waves of condensation and rarefaction are called '*Longitudinal waves*' to distinguish them from waves described earlier in the chapter, where the individual particles vibrate in lines transverse to the direction in which the waves travel.

Besides the distinction already made out between longitudinal and transverse waves, it may be noted that in longitudinal progressive wave, the condition transferred is always one of altered relative *distance* between successive particles whereas in transverse progressive wave the condition transmitted is an altered relative *direction*, *i.e.*, slope and not condensation or rarefaction. In both cases the altered relative position exists when the particles are moving and when the particles come to rest they are in their original positions relative to neighbouring particles though not in space. In both cases the velocities with which the particles at different points are moving are proportional to the difference between their actual and their ordinary relative positions.

The longitudinal waves are of great importance in the theory of Sound as they are the only type of waves possible in air, the medium by which the sound waves are generally conveyed from the place of their origin to the listener. Transverse waves are not at all propagated in a medium like air for there is no cohesion between the particles of a gas and therefore elastic forces responsible for transverse vibratory motion arising on account of *shear* (*i.e.*, change in shape strain) cannot be brought into play. The only type of strain that gives rise to elastic forces in a gas is the volumetric strain and therefore only compressional waves *i.e.*, waves involving compressions and rarefactions of the parts of the medium can be propagated in air. The sound waves though of much longer wavelengths are invisible like light waves and hence cannot be seen as we see transverse waves in a vibrating string or ripples on the surface of water. They, however, give rise to certain phenomena which can be easily demonstrated (Chapter IV). We shall for the present illustrate them by a mechanical analogy.

29. Mechanical Analogy.—Consider a number of spherical balls resting in a frictionless V-shaped groove and connected by springs, Fig. 26 (*a*) If the first ball is moved towards the second, the spring between the two is compressed and the latter therefore will begin to move. But the motion of the second ball will start only after the first ball has approached it ; for if the two moved exactly together there will be no compression of the spring between them and consequently no force exerted on the second ball to move it. As the second ball moves forward, there is the compression of the second spring. In recovering its shape it compresses the third ball and so on along the entire row, the balls being set in motion, one after the other, as the wave of compression reaches spring after spring. Now if the first ball is drawn away from the second, the spring connecting the first and the second is stretched. In recovering it causes the motion of the second ball towards the left, the second spring is stretched thereby, the motion is therefore communicated through the whole row of balls as a wave accompanied by stretching or expansion of springs. If now the end ball is made to oscillate regularly backwards and forwards, a series of waves will be sent along the row of balls consisting of compressions and rarefactions as shown in Fig. 26 (*d*) Note that each ball in turn will oscillate as the preceding one, but there is a regular phase difference, each ball lagging behind its predecessor. This is precisely the kind of motion which is set up in air by a rapidly vibrating body like a tuning fork or a bell, and which excites in our ears the sensation of sound. Such *compressional* waves produced in air are called **sound waves.**

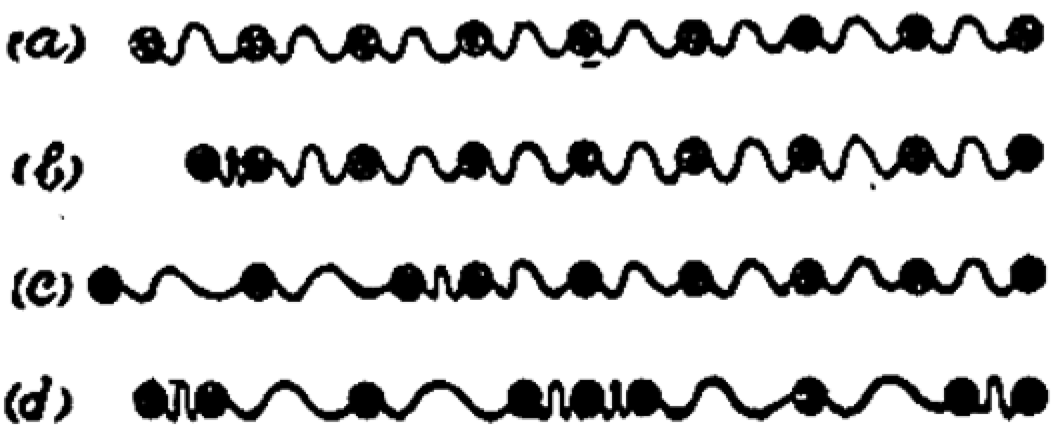

Fig. 26.

30. Formation of Longitudinal Wave.—Just as in the case of a transverse wave discussed in Art. 24, let us consider a series of particles arranged in a row AB and equidistant from one another when they are at rest. Suppose all the particles execute S H. Motions of equal amplitudes and time-periods to and fro like the layers of air in the cylindrical tube of Fig. 25 (*a*) so that each particle is affected $T/12$ seconds later than its predecessor. Fig. 27 shows 12 stages separated by this interval of time ($=T/12$) in the formation and the course of a longitudinal wave, the last stage indicating the positions of the particles after the complete time period. To clearly understand the details of motions of particles or layers in a sound wave let us proceed as below :—

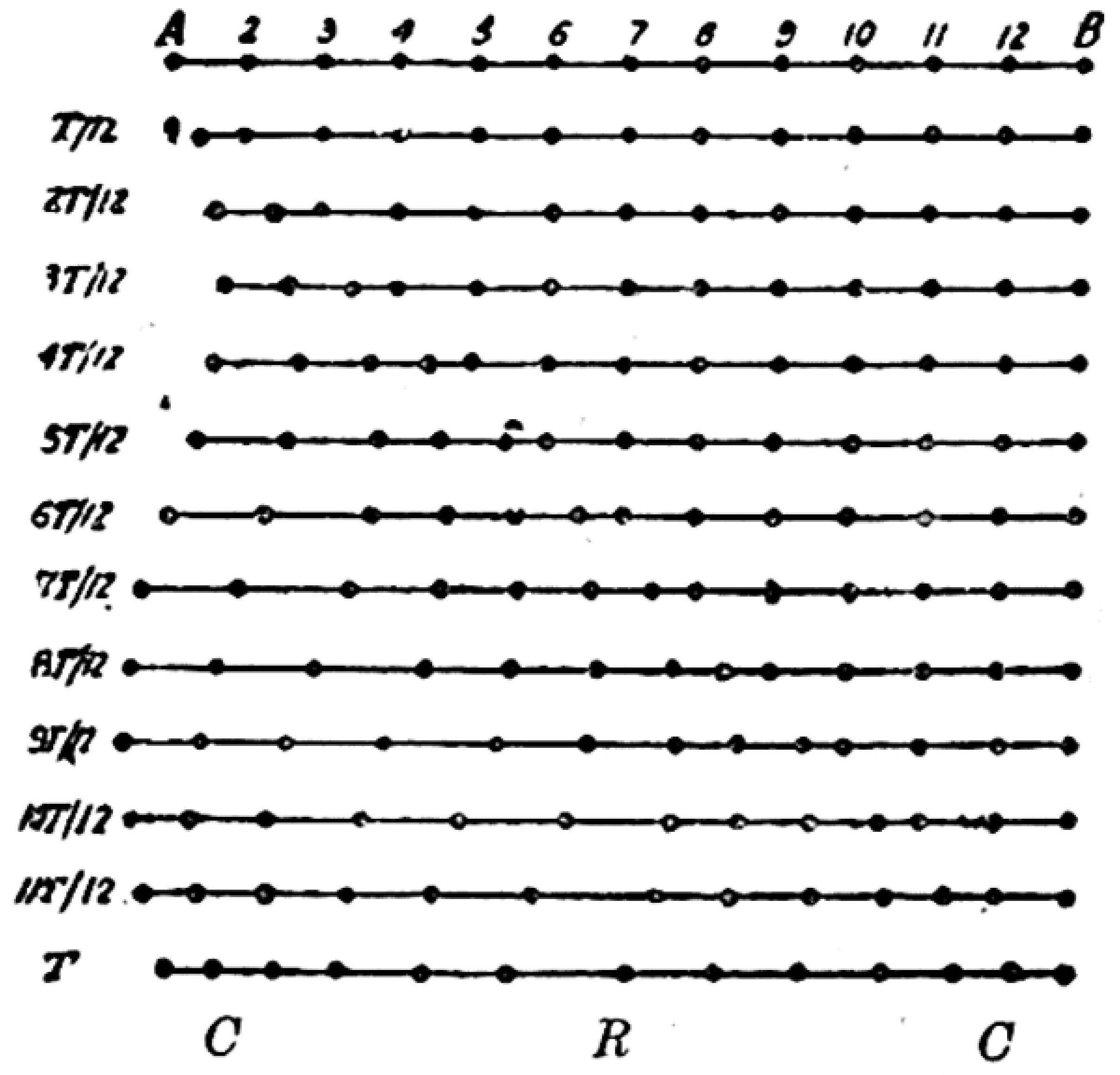

Fig. 27.

In the row *AB* (Fig. 28) thirteen particles are shown in their undisplaced positions each as a small dot at the centre of a short straight line which represents the path in which it oscillates. When a sound wave is passing over these particles, each will execute a S. H. Motion in the line *AB*, the period and amplitude being the same for all, but the phase of each particle is $T/12$ second behind that of the preceding one. Suppose the particle No. 1 is passing through its mean position from left to right at the instant we are considering then the particle No. 2 will also be moving from left to right but will pass its equilibrium position one-twelfth of a period later. Similarly

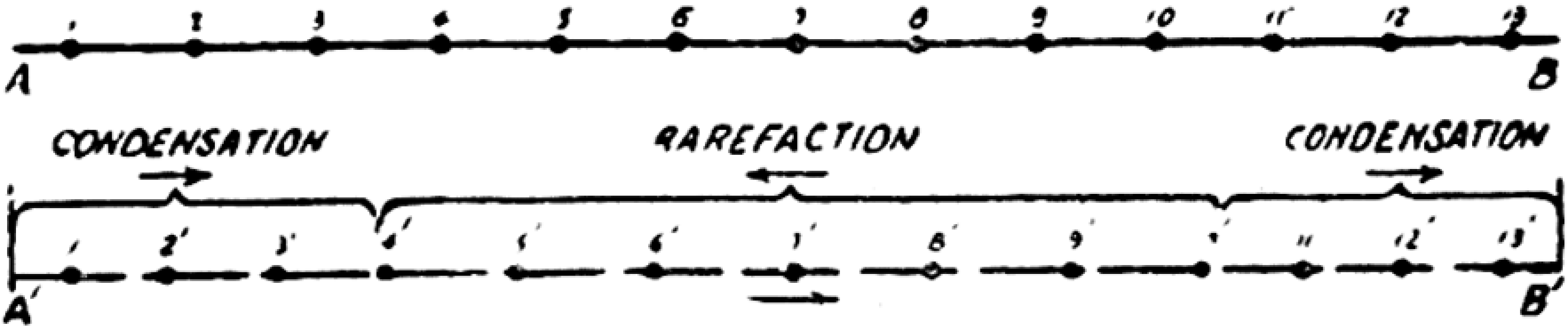

Fig. 28.

the particle No. 3 will be $\frac{1}{8}$th of a period behind, No. 4 will be a quarter of period behind, just at the left end of its swing and so momentarily at rest. Particle No. 5 will be coming up to its left hand limit so it will be moving towards the left. Proceeding in this way the positions and the directions of motion of the various particles can be determined. They are shown in the row *A'B'*. The arrows above the row *A'B'* indicate the directions of motion of the particles and the single arrow below it the direction of propagation of wave. It will be seen that particles No. 1 and 13 are in the same phase and accordingly the distance between them is a wave-length. The particles from 1 to 3 and 11 to 13 are moving towards the right and all from 5 to 9 towards the left, 4 and 10 are at the ends of their swings, while 1, 7 and 13 are passing through their mean positions. Although any consecutive pair of particles are having the same phase difference, yet their velocities at any instant are different. They are having their greatest velocities when passing through their mean positions and zero velocities at the ends of their swings.

31. **Condensations and Rarefactions.**—Moreover, in the above form of the wave motion we find that the distances between the adjacent particles alter so that the particles are alternately crowded together and spread out. A point where at any instant the crowding together is maximum is called a **Condensation,** while a point where the distance, between the particles is more than their normal distance, is called a **Rarefaction.** Referring to Fig. 28 we find that any pair of particles from 1 to 4 are less than their normal distance apart and consequently there is a condensation in this region. The condensation is greatest at 1 and diminishes as we go towards 4, whereas a pair of particles from 4 to 10 are more than their normal distance apart and constitute a region of rarefaction with their centre at 7. There are intermediate points where the medium is neither condensed nor rarefied. These are the points which are at the ends of their paths of vibration and are momentarily at rest, as at 4 and 10.

Note that in the condensed region the particles are moving forward in the direction in which the wave is advancing; in the rarefied region they are moving opposite to the direction of propagation of the wave.

32. Propagation of a Sound Wave.—A sound wave is propagated and conveyed to the ear by means of the intervening layers of air. Consider a vibrating tuning-fork (Fig. 29). Let us confine our atttention to the right hand prong only. When it moves towards the right, it compresses the layer of air in front of it and as a consequence the pressure of this layer will be greater than the adjacent layers. It tends to relieve the strain thus created, by compressing them. These, in turn, hand on the compression. Thus a pulse of compression will travel onwards to the right. Again when the prong moves towards the left, a rarefaction is started. These follow one another and as the fork vibrates, compressions and rarefactions are sent out in regular succession. These waves at last reach the ear and set the tympanic membrane in vibration. It is this physical movement of the membrane which is ultimately transmitted *via* a system of bones and cords to the brain and causes the mental sensation called Sound.

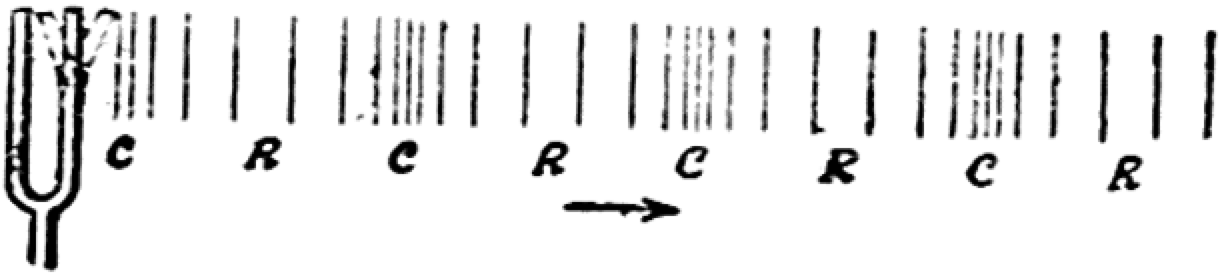

Fig. 29.

33. Experimental Demonstration of a Longitudinal Wave.—To make the propagation of a longitudinal wave visible, let us take a long spiral spring suspended horizontally by fine threads as shown in Fig. 30. If one end of the spring is suddenly pushed inwards a little, the turns near the end are compressed. This part, in recovering, will compress that in front of it and so on, a wave of compression will be seen travelling along the spring to the other end. If now the end of the spring is pulled out a little, the distance between the turns will increase and this rarefied state, which we call a *rarefaction*, will also be seen to travel along the spring. On imparting a harmonic vibration of, say, an

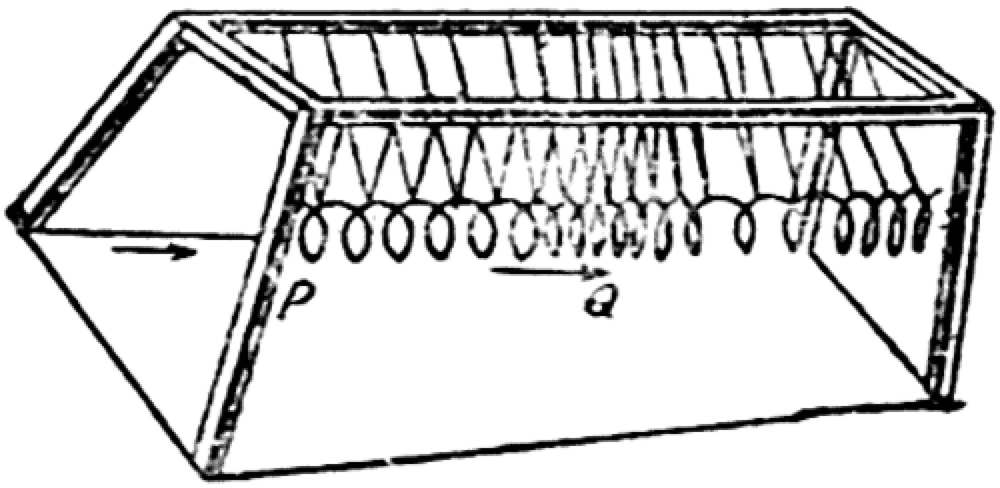

Fig. 30.

amplitude 1" to one end, a series of condensations and rarefactions will travel along the spring. Each turn of the spring will be seen to execute a to-and-fro vibration similar to the vibration that the end was made to perform, and each turn will begin its vibration a little later than its left hand neighbour.

34. **Crova's Disc.**—A further illustration of a compression wave travelling in air is obtained by means of **Crova's Disc.** It may be constructed as follows :

In the centre of a large piece of cardboard draw a small circle and divide its circumference into eight equal parts. With each of these points of division, taken as centres in regular succession, describe circles with radii increasing successively by a constant amount greater than the distance between the successive centres. Draw a large number of circles filling the entire cardboard piece, going as many times round the circle of centres as may be necessary. Mount the disc on a horizontal axis passing through the centre and hold before it another piece of cardboard with a slit cut in it as shown in Fig. 31. If now the Crova's disc is rotated, the portions of the circles observed

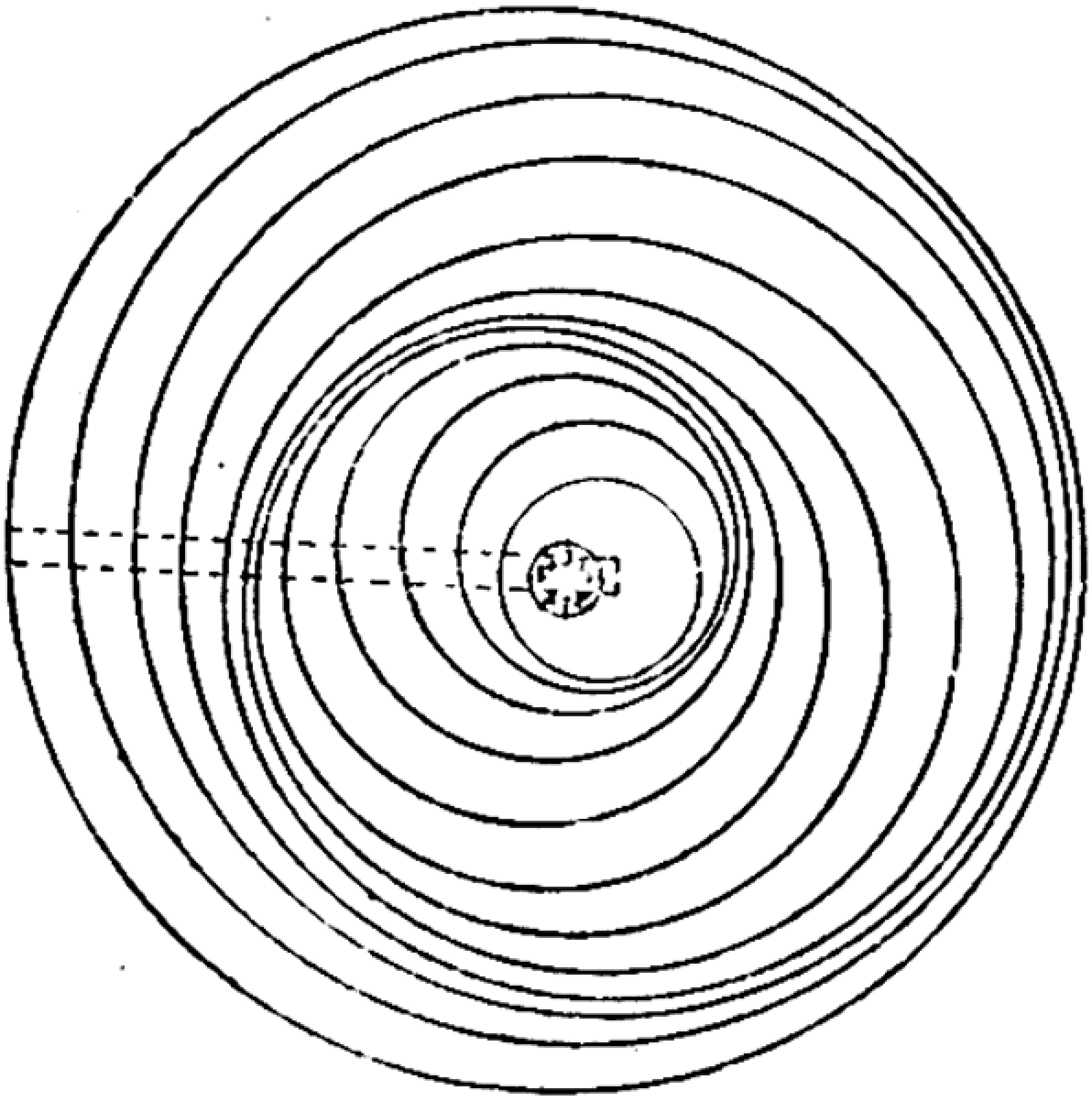

Fig. 31.

through the slit which are practically straight lines close at some places, and wider apart at others, will be seen to vibrate, to-and-fro, and waves of compressions and rarefactions will be seen to travel along the slit.

35. Plane and Spherical Waves.—We shall now explain what are *plane* and *spherical* waves.

Suppose all points in a plane are constrained to execute similar motions just like air particles in layers inside the cylindrical tube of Art. 28. We saw that all the particles in any one of these layers were at any instant displaced from their positions of equilibrium by the same amount and were in the same phase. Such a form of disturbance transmitted through an elastic medium is said to constitute a *plane wave train.* The plane perpendicular to the direction of wave transmission to which the disturbance has just reached is termed the *wave front.* It is the locus of all the adjoining points that are at the moment in the same phase of vibration. It is evident that in a plane wave train, the wave fronts are all plane and the amplitude is always constant. This would mean that the flow of energy is the same through any cross-section. It is for this reason that a parallel beam of sound or light with plane wave fronts travels over a long distance without loss of energy.

The more usual type of waves with which we have to deal in practice are '*spherical*' in nature. They are produced when a single point in the medium is constrained to execute S. H. Motion continuously. Suppose a small sphere expands and contracts harmonically. When it expands the spherical layer of air near its surface is compressed. In releasing its strain it compresses the layer just on its outside, which in turn hands on the compression to the next layer and so on the compression spreads outwards when the sphere contracts a wave of rarefaction is sent outwards following in the rear of compression. The periodic expansion and contraction produces a series of compressions and rarefactions spreading out in all directions in the medium with the same velocity so that the wave front assumes a spherical form with the pulsating sphere as centre. Further, it is clear from symmetry that the direction of vibration of any particle of air is along the radius of a system of spherical wave fronts.

The intensity* of spherical waves, however, does not remain constant as in plane waves. Obviously, it must be so for the total amount of energy crossing any spherical shell with the source as centre must be the same whilst the area of the shell being proportional to the square of its radius increases rapidly. Hence the quantity of energy passing per sq. cm. of the surface of any shell is inversely proportional to the square of the radius of the shell or the intensity of sound varies inversely as the square of the distance from the source. Since the energy is proportional to the square of the amplitude, we conclude that in a spherical wave amplitudes are inversely proportional to the distance from the centre of disturbance.

36. The Properties of Longitudinal Progressive Waves :—

(1) Each particle is vibrating to-and-fro with the same amplitude and period in the case of plane waves but in the case of spherical waves the amplitude varies inversely as the distance from the centre of disturbance.

(2) The particles in the wave do not vibrate simultaneously, but there is a progressive change in the phase as we go from one particle to the next.

(3) The to-and-fro motion of the particles produces a movement of condensations and rarefactions in the direction in which a wave propagates.

(4) In the condensation the particles are moving forward in the direction of the advance of the wave, whereas in the rarefaction they are swinging in the opposite direction.

(5) Each particle may be regarded as exerting a force on the one in front of it. In the condensation the elastic bond between the particles will be pushing the particles in front forwards, but in a rarefaction the particles in front will be pulled backwards. Since the particles move in the direction of these forces, work is being done on the particles ahead, *i.e., there is always a transfer of energy in the direction a wave travels.*

37. Representation of the Longitudinal Wave as a Sine-Curve.—We have seen above that in the case of longitudinal wave the particles move to-and-fro in the direction of propagation of the wave, and therefore remain in a straight line even when disturbed by a wave motion. Condensations and rarefactions are formed which correspond to crests and troughs in the transverse wave motion. But, here, the row of particles will never take the shape of a sine-curve as they do in the case of transverse waves. We can, however, for convenience, represent diagrammatically the longitudinal vibration also by a sine-curve. Refer to Fig. 28, in which 1, 2, 3, 4, etc. are the undisturbed positions of the particles to start with, and 1′, 2′, 3′, 4′, etc. are their positions a time period later when a sound wave is passing over them from left to right. If now we erect at the mean or the undisturbed position of each particle a perpendicular upward or downward according as the displacement of the particle is towards the right or the left, and having a height equal to the displacement of the particle from its mean position, then by joining the extremities of these ordinates we shall get a curve which is a Sine Curve. As the ordinates denote the displacements of the particles so the curve is called the *Displacement Curve.* It is shown in Fig. 32. It may be noted that the points where the curve cuts the axis correspond

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*It may be defined as the energy that crosses per sec one square cm. of a surface taken perpendicularly to the direction of propagation of the wave.

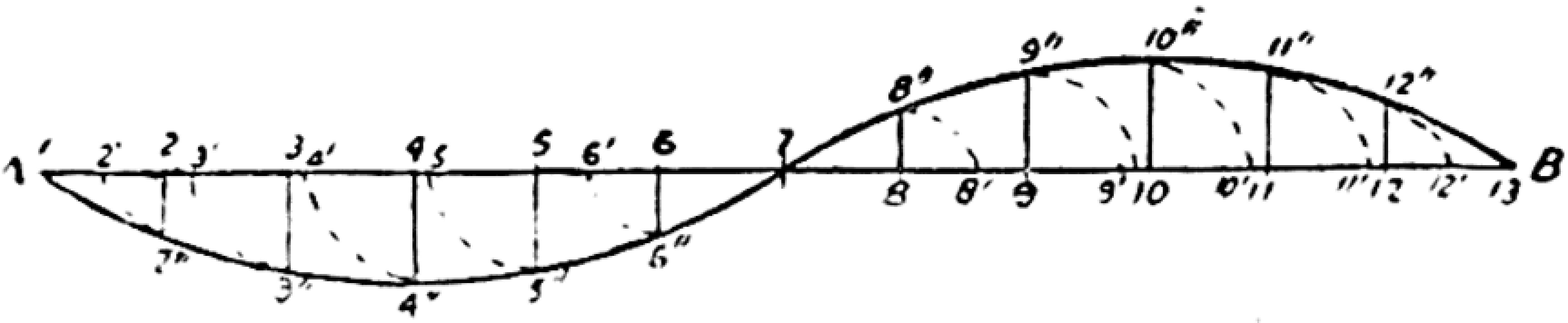

Fig. 32.

to the places where the particles are most crowded together or most spread out. The particle No. 7 is in the mean position, while its predecessor No. 6 is displaced towards the left by the amount 66′ and its succeeding particle No. 8 is displaced towards the right by the amount 88′, so the particles here are farther apart. Arguing in a similar way we can show that particles in the neighbourhood of 1 and 13 are condensed and closer together. Hence 7 is the centre of rarefaction and 1 and 13 are centres of condensation.

38. Condensation or Pressure Curve.—Very often it is convenient to study longitudinal vibrations by their *Condensation curve.* Let us now see how to draw this curve which represents the amount of compression or rarefaction at each point of the wave. To obtain the pressure curve, let us consider the displacement curve $OABCD$ of a longitudinal wave set up in a cylinder of unit cross-section, whose axis coincides with OBD. Then for two neighbouring points P and Q the displacements are Pp and Qq respectively. The volume of air between them is $PQ \times 1$ c.c. When the wave is in the position shown, the change in volume between P and $Q = (Qq - Pp) = rq$. Hence the volumetric strain or compression

$$= \frac{\text{change in volume}}{\text{original volume}} = \frac{rq}{PQ} = \frac{rq}{pr} = \frac{\delta y}{\delta x}$$

= the *slope* of the displacement curve.

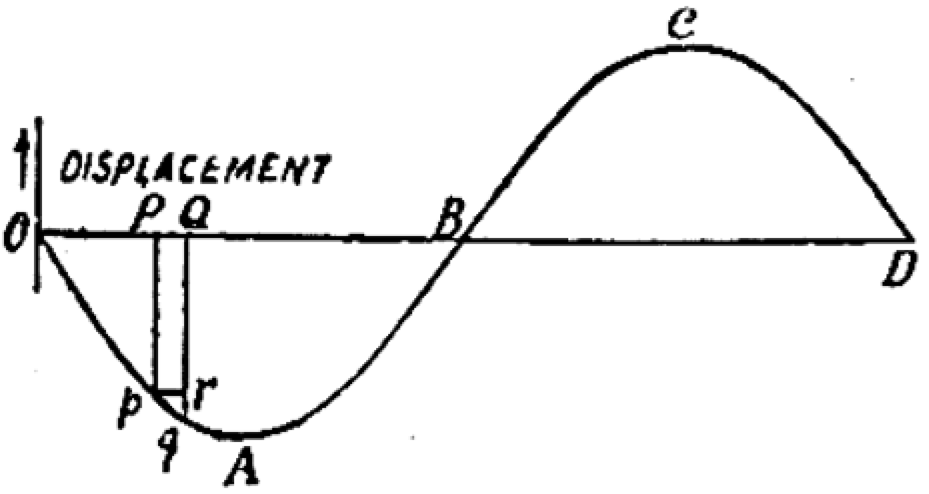

Fig. 33.

So to draw the pressure-curve, we measure the slopes of the displacement curve at different points and plot them to scale along the ordinates, the positions of the corresponding points being plotted along the axis of X. The ordinates may be above the axis when the pressure is above the normal (region of compression), and below the axis when the pressure is below the normal (region of rarefaction). Fig. 34 (a) represents the condensation curve.

39. Velocity Curve.—A curve showing the relation between the velocity of the particle and its position along the wave is called the *Velocity curve.*

Let the velocities in the direction in which the wave travels be represented by the upward ordinates and the velocities in the opposite direction by downward ordinates. Applying this convention to the series of particles in Fig. 32 we get the velocity curve as shown in Fig. 34 (b).

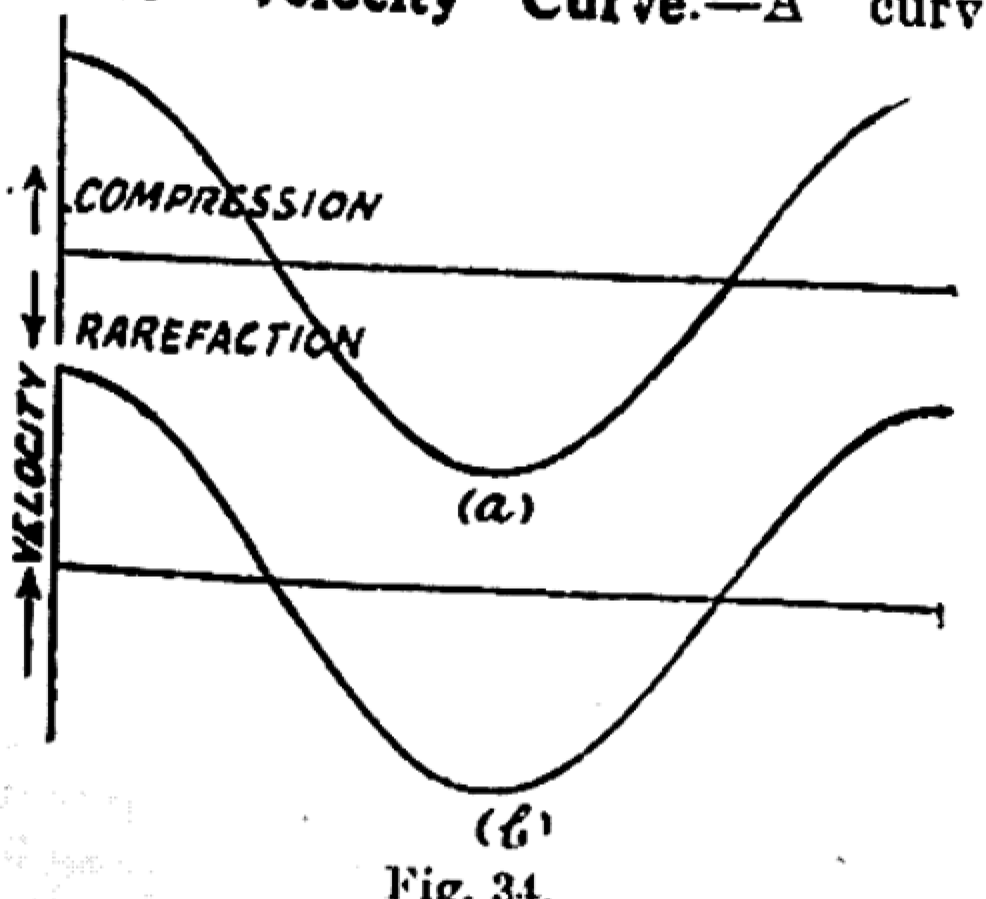

Fig. 34.

We find that the displacement, condensation and velocity curves of longitudinal waves are all *Sine Curves*. The velocity and condensation curves both agree in phase and are ahead of the displacement curve by one quarter period.

It may be clearly understood that these curves do not indicate directly these quantities in the directions in which they actually take place but merely represent them *conventionally*. It is for this reason that they are called the *Associated Curves* of a wave.

40. Relation between Wavelength and Velocity of Propagation of a Wave.—The velocity of a wave motion is the distance traversed by the crest or trough in the case of a transverse wave or the condensation or rarefaction in a longitudinal wave in one second. If T be the time period of vibration, λ, the wavelength and v, the velocity of propagation of the wave, we have

$$v=\lambda/T.$$

Now the number of vibrations a particle makes in one second is $1/T$. Let it be denoted by n. This, n, is called *frequency*. It also represents the number of complete waves passing a fixed point in one second. Then the above relation is expressed in the form,

$$v=n\lambda \quad \ldots \quad \ldots \ (1)$$

This formula is of great importance in acoustical calculations as it connects the three characteristics of a wave motion.

41. Equation of Simple Harmonic Wave.—We have already noticed that when a wave is passing over the surface of water, the particles of water are moving up and down their mean positions and if, at any instant, the instantaneous snapshot of the particles along the path of the wave were taken it will be seen that the surface presents a view like that shown in Fig. 35. It indicates that as the wave moves outwards, the

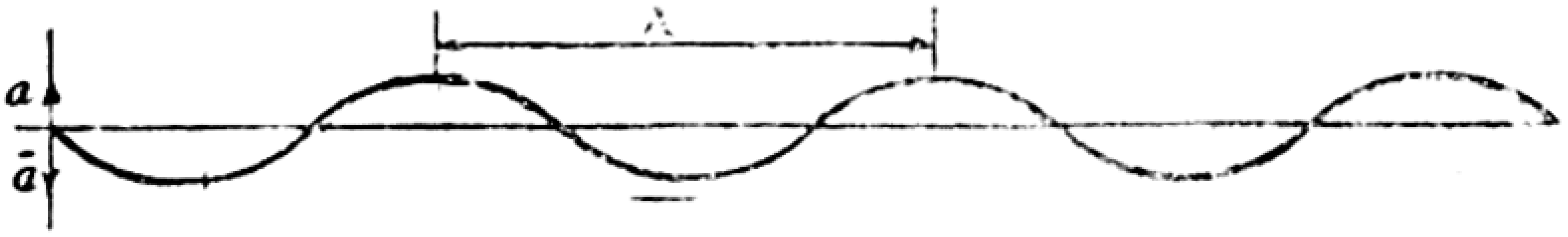

Fig. 35.

displacements of the various particles change according to a sine or cosine curve. As a matter of fact, what happens is that each particle of water executes S. H. Motion, only the different particles pass through their positions of maximum displacement one after the other. It is obvious, therefore, that if we could write the equation of a particle in such a manner that, at any particular instant, we could get the displacements of all the particles from it and the displacement of a single particle at any time, then that equation will represent a **Wave Motion**.

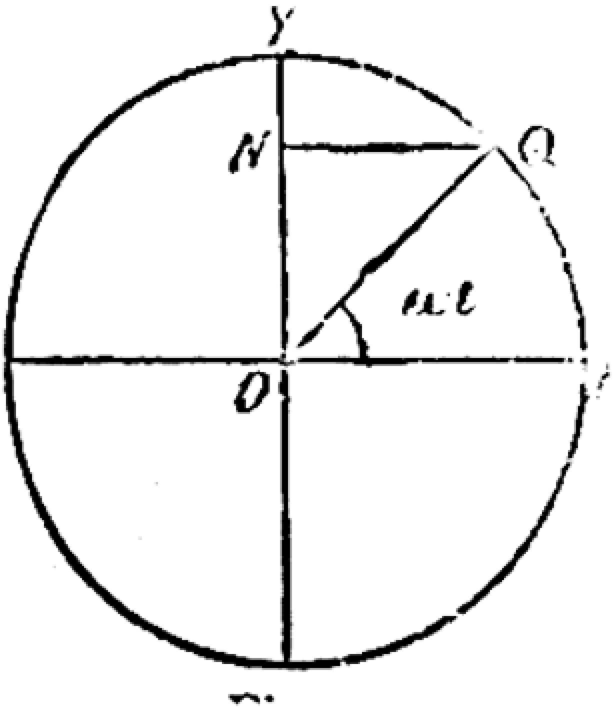

Fig. 36.

Suppose a particle O moving in S. H. Motion, acts as a source of wave train spreading out in a medium. Each particle of the medium is affected a little later than the particle preceding it and this delay *i.e.*, the difference increases from point to point along the wave. If we measure time from the moment that the particle O passes through its mean position. Its displacement at a given instant along the line OB, is given by

$$y=a \sin \omega t.$$

The displacement at the same instant of a particle P at a distance x from O towards the right can be written as

$$y=a \sin (\omega t-\phi). \qquad \ldots \qquad \ldots \qquad (2)$$

where ϕ is the phase lag, since it is evident that the particle situated towards the right of O must be always behind the particle at O in phase if the wave is travelling towards the right.

Since we do not know ϕ directly for the various particles, the equation in this form is not very useful. But if we could express ϕ in terms of the distance, the equation would involve those terms only which we can easily measure. It remains to find a way of evaluating the phase constant ϕ in terms of the wavelength and the distance x of the particle P from O.

We know that a change of 2π in phase occurs between two particles separated by a distance λ, hence we see that a change ϕ in phase corresponding to a distance x, is given by the relation

$$2\pi : \lambda :: \phi : x, \qquad i.e., \frac{2\pi}{\lambda}=\phi/x,$$

whence,

$$\phi=2\pi x/\lambda.$$

Substituting this value of ϕ in equation (2), above we get

$$y=a \sin\left(\omega t-\frac{2\pi x}{\lambda}\right)$$

$$=a\ \sin\ \left(\frac{2\pi t}{T}-\frac{2\pi x}{\lambda}\right) \qquad \left[\because\ \omega=\frac{2\pi}{T}.\right.$$

$$=a\sin 2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right) \qquad \ldots \qquad \ldots\ (3)$$

This is one form in which the equation of wave motion of the simplest type is represented.

Since we know that $v=n\lambda$,

or $v=\frac{\lambda}{T}$ or $\frac{1}{T}=\frac{v}{\lambda}$, where v is the velocity of the wave,

we can write the above equation as

$$y=a\sin\ 2\pi\left(\frac{vt}{\lambda}-\frac{x}{\lambda}\right)=a\sin\frac{2\pi}{\lambda}(vt-x) \qquad \ldots\ (4)$$

This equation gives us the complete form of the wave of amplitude a and being propagated with a velocity v in the positive x direction. If we keep t constant and plot the values of y for successively increasing values of x, we obtain the wave configuration in space as in Fig. 35. If on the other hand we keep x constant and allow t to increase we may calculate the transverse displacement at any instant of that particle which is vibrating in simple harmonic motion with a phase constant $-\left(\frac{2\pi x}{\lambda}\right)$.

The equation for a similar wave travelling in the negative x direction is given by

$$y=a\sin\frac{2\pi}{\lambda}(vt+x) \qquad \ldots\ (4a)$$

It may be noted that the constant velocity v with which the wave train proceeds has no relation to the velocity U of any of the mass particles in their vibratory motion. To obtain the value of U, we differentiate equation (4) with respect to t keeping x constant.

whence $$U=\frac{dy}{dt}=\frac{2\pi\ av}{\lambda}\cos\frac{2\pi}{\lambda}(vt-x) \qquad \ldots\ (4b)$$

$$U_{max}=\frac{2\pi\ av}{\lambda} \qquad \ldots \qquad \ldots \qquad \ldots\ (4c)$$

Similarly, $\frac{dU}{dt}$ gives the acceleration f of the particle at any instant in its transverse vibratory motion.

$$f = \frac{dU}{dt} = -\left(\frac{2\pi v}{\lambda}\right)^2 a \sin \frac{2\pi}{\lambda}(vt-x)$$

$$= -\frac{4\pi^2 v^2}{\lambda^2}\cdot y \qquad \dots (4d)$$

$$f_{max} = \frac{4\pi^2 v^2 a}{\lambda^2} \qquad \dots (4e)$$

In the following section we will show how wave velocity and particle velocity are related to each other.

Example 1.—*If the equation for a transverse wave in a stretched string is* $y=3 \sin 2\pi\left[\frac{t}{0{\cdot}04}-\frac{x}{40}\right]$ *with lengths expressed in cm. and time in seconds. Calculate the wave length, amplitude, frequency and the velocity of the wave. What is the maximum velocity and acceleration of a particle of the string?*

Expressing the given equation in the standard form

$$y=a \sin\frac{2\pi}{\lambda}(vt-x),$$

we have

$$y=3 \sin 2\pi\left[\frac{t}{0{\cdot}04}-\frac{x}{40}\right]$$

$$= 3 \sin 2\pi\left[25\, t-\frac{x}{40}\right]$$

$$= 3 \sin \frac{2\pi}{40}\left[1000\, t-x\right]$$

Comparing it with the same, we note that wave-length

$\lambda=40$ cm.

amplitude $a= 3$ cm.

velocity of the wave

$v= 1000$ cm./sec.

and frequency $n= \frac{v}{\lambda}=\frac{1000}{40}=25.$

From equation (4c)

$$\sigma_{max} = \frac{2\pi\, av}{\lambda}$$
$$= 2\pi\, an$$
$$= 2\pi \times 3 \times 25$$
$$= 150\pi \text{ cm/sec.}$$

From equation (4e)

$$f_{max} = \frac{4\pi^2\, v^2 a}{\lambda^2}$$
$$= 4\pi^2 n^2 a$$
$$= 4 \times 10 \times (25)^2 \times 3$$
$$= 75000 \text{ cm/sec}^2.$$

Example 2.—*A simple harmonic wave train of amplitude 1 cm and frequency 100 vibrations is travelling in the positive x direction with velocity 15 m/sec. What is the displacement y at x=180 cm. from the origin at t=5 sec, the wave-length, the particle velocity and the particle acceleration.*

Use the equation

$$y = a \sin \frac{2\pi}{\lambda}(vt - x)$$

Here,

$a = 1$ cm.

$v = 15m = 1500$ cm/sec.

$n = 100$

$$\lambda = v/n = \frac{1500}{100} = \mathbf{15cm}.$$

$x = 180.$

$$\therefore\ y = 1 \times \sin \frac{2\pi}{15}\Big[1500 \times 5 - 180\Big]$$
$$= \sin(2\pi \times 488)$$
$$= 0.$$

The particle velocity

$$U = \frac{dy}{dt} = \frac{2\pi a\, v}{\lambda} \cos \frac{2\pi}{\lambda}(vt - x)$$

Under the given conditions.

$$\sin \frac{2\pi}{\lambda}(vt - x) = 0$$

$$\therefore \cos\frac{2\pi}{\lambda}(vt-x)=1$$

and so

$$U=\frac{2\pi\, av}{\lambda}.$$
$$=2\pi\, an$$
$$=2\pi\times 1\times 100$$
$$=200\pi \text{ cm./sec.}$$

The particle acceleration

$$f=-\frac{4\pi^2v^2}{\lambda^2}y.$$

Since $y=0, f=0$.

42. Wave Velocity and Particle Velocity.—We have considered an instantaneous view of a wave-motion passing along different particles. To understand how a wave moves we have only to draw another graph corresponding to a different value of t. Let us suppose that the full line graph represents displacement of the particles at the time t_1 and dotted curve the displacement at the time t_2. Let the distance of A from O be x_1 and that of B, x_2, where the displacement is zero at times t_1 and t_2, respectively. It is clear from the equation that

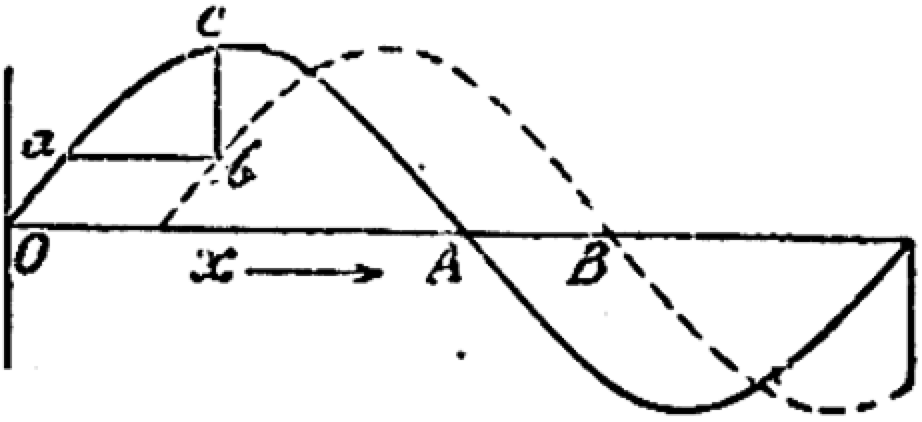

Fig. 37.

$$y=a\sin\frac{2\pi}{\lambda}(vt_1-x_1)=a\sin\frac{2\pi}{\lambda}(vt_2-x_2)$$

$$\therefore vt_1-x_1=vt_2-x_2$$

or

$$v=\frac{x_2-x_1}{t_2-t_1}=\frac{\delta x}{\delta t}$$

Particle velocity U is, obviously $=\frac{\delta y}{\delta t}$ where δy is the distance moved by the particle in a small interval of time δt.

Note that during the time the wave goes from a to b [or A to B] the particle c has come down to the position b. Hence,

$$\frac{\text{Particle Velocity}}{\text{Wave Velocity}} \text{ or } \frac{U}{v}=\frac{bc}{ab}=\frac{\delta y}{\delta x}$$

or
$$U=v\frac{\delta y}{\delta x} \qquad \ldots \qquad \ldots \quad (5)$$

where $\frac{\delta y}{\delta x}$ measures the slope of the displacement curve,

i.e., Particle velocity = Wave velocity × Slope of the displacement curve.

43. Differential Equation of Wave Motion.—We have already shown that for a harmonic wave the displacement of a particle at a distance x from the centre of disturbance, is given by

$$y=a \sin\frac{2\pi}{\lambda}(vt-x)$$

where a is the amplitude and v is the velocity of the wave.

The particle velocity, (Differentiating with respect to t)

$$\frac{dy}{dt}=\frac{2\pi v}{\lambda}a\cos\frac{2\pi}{\lambda}(vt-x) \qquad \ldots \quad (6)$$

and its acceleration,

$$\frac{d^2y}{dt^2}=-\frac{2\pi^2v^2}{\lambda^2}a\sin\frac{2\pi}{\lambda}(vt-x) \qquad \ldots \quad (7)$$

Also compression, (Differentiating with respect to x)

$$\frac{dy}{dx}=-\frac{2\pi}{\lambda}a\cos\frac{2\pi}{\lambda}(vt-x) \qquad \ldots \quad (8)$$

And rate of change of compression

$$\frac{d^2y}{dx^2}=-\frac{4\pi^2}{\lambda^2}a\sin\frac{2\pi}{\lambda}(vt-x) \qquad \ldots \quad (9)$$

From (6) and (8) $\frac{dy}{dt}=-v\frac{dy}{dx}$ ... ... (10)

Compare it with equation (5), Art. 42

From (7) and (9) $\frac{d^2y}{dt^2}=v^2\frac{d^2y}{dx^2}$ ... ... (11)

The equation (11) is the '*differential equation*' of wave motion.

44. Energy of a Progressive Wave.—We shall now consider in a general way the energy of wave motion and its *distribution* in place and time.

We know that in an advancing wave train new waves are being continually formed at the head, which means that energy is propagated along the wave train. The energy of new motion appearing at its head is supplied from behind. As a new length 'v' is set in motion every second, the energy transferred per second must be equal to the energy of a single particle multiplied by the number of such particles in length 'v'. The energy of the wave is partly kinetic and partly potential, the former depending upon the motion of the air particles relatively to the earth and the latter on the differences of pressure in different parts of the medium. As a wave passes over the particles of the medium, it sets them into harmonic oscillations about their mean positions. The particles possess kinetic energy except at the moment when they are at the ends of their swings. We also saw that in case of wave propagation in air there are regions of condensation and rarefaction in a wave and therefore there is *potential* energy everywhere except at a point where the particles are passing from a condensation to a rarefaction and *vice versa*.

The energy *distribution* is not uniform over the wave, for in a progressive wave there are points where velocity and compression are both zero and consequently at these points the air has no energy above what it would have if no waves were passing.

It is important to remember that while there is no transfer of the medium in the direction in which a wave is advancing, there is always a transfer of energy in the direction of the wave.

It will be shown in the next article that the total energy per unit volume of a medium through which sound waves are passing is $\frac{1}{2}\rho.\left(\frac{2\pi av}{\lambda}\right)^2$, where a is the amplitude, v the velocity of the wave, λ the wave length and ρ the density of the medium. The energy is (*i*) proportional directly to the square of the amplitude of the waves, and (*ii*) inversely proportional to the square of the wave length and further that the energy in a whole wave is, at any instant, *half kinetic and half potential.*

***45. Mathematical Treatment of Energy Distribution in a Progressive Wave.**—Let the wave be represented by

$$y_x = a \sin \frac{2\pi}{\lambda}(vt - x)$$

Differentiating it with respect to t, we obtain the velocity of an oscillating layer (x=constant) *viz* :

$$U_x = \frac{dy_x}{dt} = \frac{2\pi a v}{\lambda} \cos \frac{2\pi}{\lambda}(vt - x)$$

Its acceleration is given by

$$f_x = \frac{d}{dx}(U_x) = -\left(\frac{2\pi v}{\lambda}\right)^2 a \sin \frac{2\pi}{\lambda}(vt - x)$$

$$= -\left(\frac{2\pi v}{\lambda}\right)^2 y_x$$

If ρ be the density of the medium, the mass per unit volume is equal to ρ, hence kinetic energy per unit volume

$$(K) = \tfrac{1}{2}\,\text{mass} \times (\text{velocity})^2$$
$$= \tfrac{1}{2}\rho\,(U_x)^2$$
$$= \tfrac{1}{2}\rho\left(\frac{2\pi a v}{\lambda}\right)^2 \cos^2 \frac{2\pi}{\lambda}(vt - x) \quad \ldots \quad (12)$$

and its potential energy

$$(P) = \int_0^{y_x} \rho f_x \, dy_x = \rho.\left(\frac{2\pi v}{\lambda}\right)^2 \int_0^{y_x} y_x \, dy_x$$

$$= \tfrac{1}{2}\rho\left(\frac{2\pi v}{\lambda}\right)^2 y_x^2$$

$$= \tfrac{1}{2}\rho\left(\frac{2\pi a v}{\lambda}\right)^2 \sin^2 \frac{2\pi}{\lambda}(vt - x) \quad \ldots \quad (13)$$

The total Energy per unit volume

$$E = (K + P)$$

$$= \tfrac{1}{2}\rho\left(\frac{2\pi a v}{\lambda}\right)^2 \left[\cos^2 \frac{2\pi}{\lambda}(vt - x) + \sin^2 \frac{2\pi}{\lambda}(vt - x)\right]$$

$$= \tfrac{1}{2}\rho\left(\frac{2\pi a v}{\lambda}\right)^2.$$

Writing $\frac{1}{T^2}$ for $\frac{v^2}{\lambda^2}$, we have

$$E=\tfrac{1}{2}\rho\left(\frac{2\pi a}{T}\right)^2 \quad \dots \quad \dots \quad \dots \quad (14)$$

If we deal with a unit area of cross section of a parallel beam of radiation, this also represents the energy in unit length of the wave.

In such an advancing wave train a new length v is set in motion every second. Hence the energy transferred per second must be equal to energy contained in length v *i.e.*, the rate of flow of energy per unit time per unit cross section passing along the line of advance of waves or the energy current per unit area is given by

$$C=\tfrac{1}{2}\rho v\left(\frac{2\pi a}{T}\right)^2 \quad \dots \quad \dots \quad \dots \quad (15)$$

It should be noted that though K and P both vary with x and t, yet the total energy E and current C are both independent of x and t.

Let us now ascertain how the energy throughout the medium is *distributed* in the two forms of kinetic and potential.

From equation (12) we find that the maximum kinetic energy per unit volume

$$=\tfrac{1}{2}\rho\left(\frac{2\pi av}{\lambda}\right)^2,$$ the same as the total energy.

As an element recedes from its position of equilibrium, its kinetic energy diminishes to zero as the point of its maximum displacement is approached. In a wave train comprising of a number of waves, there will, at any instant be elements displaced to various extents and therefore possessing various fractions of the maximum kinetic energy. Consequently, the average kinetic energy per unit volume of the elastic medium in the path of wave train will be equal to half the above or to

$$\tfrac{1}{4}\rho\left(\frac{2\pi av}{\lambda}\right)^2.$$

The potential energy per unit volume is also

$$\tfrac{1}{4}\rho\left(\frac{2\pi av}{\lambda}\right)^2.$$

i.e., the energy of a progressive wave is at any instant *half kinetic and half potential.*

Example. *A musical instrument of frequency 256 per second is sending out plane waves of amplitude one thousandth of a cm. Find* (*a*) *the energy of radiation per c. c. and* (*b*) *the energy current per sq. cm.* (*Speed of sound=33200 cm. per second and density of air=0·00129 gm. per c.c.*)

(*a*) Using equation (14) we have

$$E=\tfrac{1}{2}\rho\left(\frac{2\pi a}{T}\right)^2$$

$$=\tfrac{1}{2}\rho.(2\pi an)^2 \text{ where } n \text{ is the frequency,}$$

Here, $\rho=0{\cdot}00129$ gm. per c.c.

$$a=\frac{1}{1000}\text{ cm.}$$

$$n=256/\text{sec.}$$

$$\therefore\quad E=\tfrac{1}{2}\times 0{\cdot}00129\times\left[2\pi\times\left(\frac{1}{1000}\right)^2\times 256\right]^2$$

$$=\frac{0{\cdot}00129\times 4\pi^2\times(256)^2}{2\times(1000)^2}$$

$=\mathbf{0{\cdot}00167}$ ergs. per c.c. nearly.

(*b*) Using the equation (15), we have the energy current

$$C=\tfrac{1}{2}\rho v\left(\frac{2\pi a}{T}\right)^2$$

$$=\tfrac{1}{2}\rho.\left(\frac{2a\pi}{T}\right)^2\times v$$

$$=E\times v$$

$$=0{\cdot}00167\times 33200$$

$=\mathbf{55{\cdot}4}$ ergs per sq. cm. per sec.

CHAPTER IV

TRANSVERSE VIBRATIONS OF STRINGS

46. The transverse vibrations of strings constitute one of the chief sources of *Musical Sounds*. The guitar, violin and piano represent three types of stringed instruments in which the strings are plucked, bowed and struck, respectively. Hence the importance of the study of the transverse vibrations of strings. By a *String* or a *Wire* in acoustics we mean a cord whose length is great as compared to its diameter, and which is perfectly uniform and flexible. It is stretched between two fixed points with a large tension. When plucked transversely and released, the various elements of the string vibrate to and fro in planes at right angles to the string and under suitable conditions emit a musical note. Here, the initial tension applied to the string, when in the position of equilibrium, is the only restoring force which we need consider, for the variations of tension, due to extra stretching on displacement, are so small as compared to it that they may be neglected.

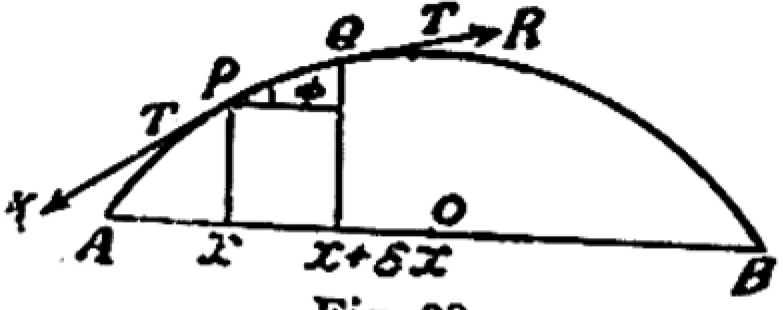

Fig. 38.

***47. Equation of Motion of a Vibrating String.**—Let us consider that a string AB is stretched between two points with a tension T. It is plucked at the centre O and when left to itself it commences to vibrate. The force tending to bring any element of the string back to its position of equilibrium is the component of tension T at right angles to AB. Assume that the gradient of the curve formed by the string in its displaced position is always so small that the tension T may be regarded as constant. The magnitude of the force acting on any element is proportional to the displacement and so the motion is Simple Harmonic. This vibration in effect results from transverse motions travelling in opposite directions along the string and successive reflections of the wave from the opposite fixed ends.

Take the x-axis of co-ordinates in the direction of the undisplaced string and y-axis in the direction of displacement perpendicular to the length of the string. Consider a small element PQ of the plucked string, δx in length, acted upon by the two end tensions each equal to T along the tangents PK and QR.

If $\phi+\delta\phi$ and ϕ are the angles of inclination of the tangents to the curve at the ends of the element δx, the difference in the tensions acting on the ends of the element in the direction of the y-axis will be

$$\begin{aligned} & T[\sin(\phi+\delta\phi)-\sin\phi] \\ &= T\cos\phi\;\delta\phi = T\;\delta\,(\sin\phi) \\ &= T\left[\frac{d}{dx}\,(\sin\phi)\right]\delta x. \end{aligned}$$

Now for small angles $\sin\phi=\tan\phi$ and $\tan\phi=\frac{dy}{dx}$, therefore the above equation may be written as

$$T\left[\sin(\phi+\delta\phi)-\sin\phi\right]=T\,\frac{d^2y}{dx^2}.\,\delta x.$$

If m be the mass of the string per unit length, the mass of the element $\delta x=m\,\delta x$, and its acceleration in the direction of y is $\frac{d^2y}{dt^2}$. Whence, by Newton's Second Law

$$m\delta x.\,\frac{d^2y}{dt^2}=T\,\frac{d^2y}{dx^2}.\delta x$$

or

$$\frac{d^2y}{dt^2}=\frac{T}{m}\cdot\frac{d^2y}{dx^2} \qquad \ldots \qquad \ldots (1)$$

This is the '*differential equation*' of motion of a vibrating string.

Comparing it with the differential equation of wave motion,

$$\frac{d^2y}{dt^2}=v^2\,\frac{d^2y}{dx^2}, \qquad \text{[Art. 43, equation (11)]}$$

we conclude that the velocity of a wave travelling along a string is given by

$$v^2=\frac{T}{m} \text{ or } v=\sqrt{\frac{T}{m}}$$

48. Velocity of Transverse Waves along a String.—Let us now develop, from elementary principles, an expression for the velocity v with which a transverse wave pulse travels along a stretched string. An example might be the pulse set up in a violin string by light plucking or that sent along a taut rope by giving one end a quick sidewise jerk. We shall assume that the string, wire or rope is perfectly flexible and uniform and that the displacements are small and we shall neglect the effect of reflections at the ends.

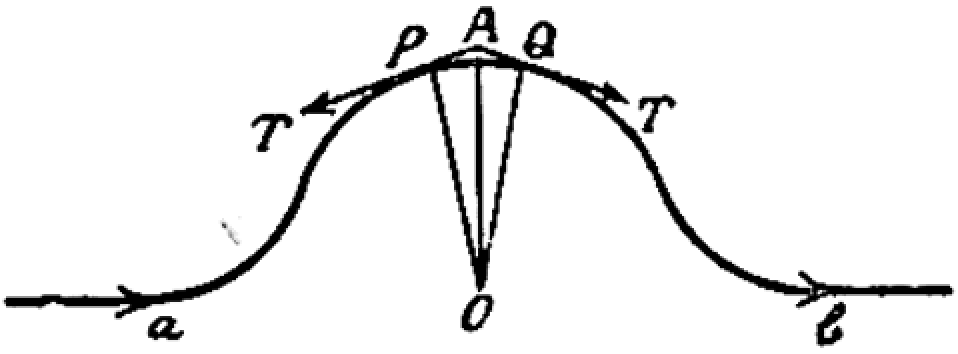

Fig. 39.

Suppose a wave pulse travels along a string from left to right with velocity v. Let us imagine the entire string to be moved from right to left with the same speed so that the wave form remains fixed while the particles comprising the string successively round the curve. Consider a small portion PQ of the string at the crest and having a length δx. It forms the arc of a circle of which O is the centre, such that $\angle POQ=\theta$. The mass of this portion of the string is $m\delta x$, if m be the mass per unit length, and since it is moving in a circle with speed v, the magnitude of the centripetal force acting on it is $m\delta x \frac{v^2}{R}$, where R is the radius of the circle. This force acts along AO, which bisects the angle POQ and is produced by the resultant of the tensions T and T, acting tangentially at the ends of the portion PQ. Resolving the tensions along AO and perpendicular to it, we find that the sum of the components of the tensions along AO is $2T \cos \hat{P}A\dot{O}=2T \sin\frac{\theta}{2}$, whereas the components perpendicular to AO being equal and in opposite directions cancel out. Since θ is very small we may write $\frac{1}{2}\theta$ for $\sin\frac{\theta}{2}$, and the resultant force due to the tensions is, therefore, equal to $T\theta$.

Hence for the equilibrium of the portion PQ, we have

$$T\theta=m\delta x \frac{v^2}{R}.$$

Now $\theta=\frac{\delta x}{R}$, $\quad \therefore\ T\frac{\delta x}{R}=m\delta x.\frac{v^2}{R}$, whence $$v^2 = \frac{T}{m} \quad \text{or} \quad v = \sqrt{\frac{T}{m}}.$$

Thus a transverse wave will travel along a string with velocity

$$v = \sqrt{\frac{T}{m}}, \quad \dots \quad \dots \quad \dots \quad (2)$$

where T is the stretching force and m is the mass per unit length. If T is in dynes and m in gm./cm. v is expressed in cm./sec. If T is in poundals and m in lbs./foot, v is in feet per second.

49. Frequency and Period of Vibration of a String.—Let us consider a string fixed at both ends. If it is plucked by the application of a simple harmonic force, two bumps start one in either direction, but on arrival at the fixed end it is unable to draw the fixed support to its side and thus it causes a rebound and bump is reflected. So a pulse starts in the reverse direction and goes to the other end, gets reflected there and arrives at the original point after traversing twice the length of the string. If during this time the point has executed one complete vibration, the harmonic force communicated to the point will find the incoming wave to be in phase with it. The wave will, therefore, be reinforced and vibrations will thus be maintained till the loss of energy due to friction and dissipation brings it to rest. Here, then during one time period the distance traversed by the waves is twice the length of the string so that $\lambda = 2l$, where l is the length of the string.

$$\text{Now } v = n\lambda, \therefore \; n = \frac{v}{\lambda} = \frac{v}{2l} = \frac{1}{2l}\sqrt{\frac{T}{m}} \quad \dots \quad \dots \quad (3)$$

This is the frequency of the lowest or the gravest mode of vibration and the note emitted is called the *Fundamental.*

The period of vibration of this fundamental mode of vibration is given by

$$T = \frac{1}{n} = 2l\sqrt{\frac{m}{T}} \quad \dots \quad \dots \quad \dots \quad (4)$$

If D be the diameter of the wire and ρ the density of the material of the wire, then

$$m = \frac{\pi D^2}{4}\rho$$

Putting this value of m in (3), we have

$$n = \frac{1}{2l} \cdot \sqrt{\frac{4T}{\pi D^2 \rho}}$$

$$= \frac{1}{Dl} \cdot \sqrt{\frac{T}{\pi \rho}} \qquad \ldots \qquad \ldots \quad (5)$$

Note that in using formula (3) l and m must be expressed in corresponding fundamental units (cm. and gm. or ft. and lb.) and T in dynes or poundals. For example, 15 cms. of a wire of mass 15 grams per metre under a tension of 5 kilograms would vibrate with a frequency of

$$n = \frac{1}{2 \times 15} \sqrt{\frac{5 \times 1000 \times 981}{15/100}}$$

$$= \frac{1}{30} \sqrt{\frac{5 \times 1000 \times 981 \times 100}{15}}$$

$$= 190 \cdot 6.$$

50. The Laws of Vibration of a String.—The laws of vibration of strings embodied in the equation (3) may be stated as follows :—

(1) For a given tension the frequency of a string varies inversely as its length so that if length is halved, pitch is doubled, *i.e.*,

$$n \propto \frac{1}{l}.$$

(2) For a string of given length and material, the frequency varies directly as the square root of the stretching force applied, so that if T is increased four times, n is doubled, *i.e.*,

$$n \propto \sqrt{T}.$$

(3) The frequency of vibration of strings of the same length and subjected to the same tension varies inversely as the square root of the mass per unit length of the strings, *i.e.*,

$$n \propto \frac{1}{\sqrt{m}}.$$

51. Experimental Verification by Sonometer.—The laws can be experimentally verified by a *Sonometer* or a *Monochord*. It consists, essentially of a thin metallic wire stretched across two bridges A and B on the top of a hollow, wooden sounding box about 1 metre long. One end of the wire is fastened to a peg at one end of the box. Its other end passes over a smooth frictionless pulley fixed at the other end of the box, and carries a scale pan so that it can be loaded to have any desired tension. There is a movable bridge C, between the two fixed bridges A and B, sliding over a *mm.* scale to adjust the length of the vibrating portion of the string. It is, sometimes, convenient to have a second string stretched by the side of the experimental string for comparison. This string can be tuned to any pitch by means of a pin that is twisted by a key so as to alter the tension.

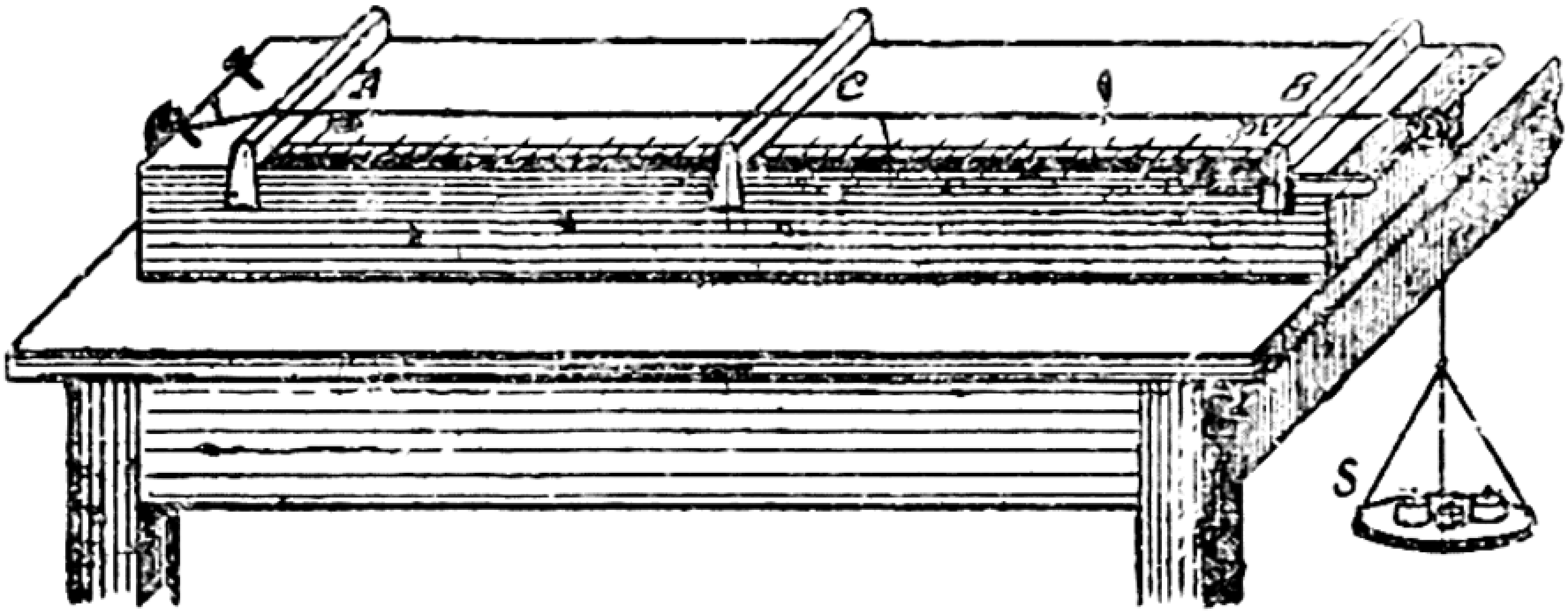

Fig. 40.

Expt. 1. *To verify that* $n \propto \frac{1}{l}$.—Attach a constant load to the wire passing over the pulley and adjust the movable bridge C, until a length, l, of the wire vibrates in *unison* with a tuning fork of known frequency. The vibration of the string can be detected by playing a *light* paper rider upon it. The tuning fork is struck and held with its stem pressing against the board. The position of the movable bridge is altered, till the paper rider placed at the centre of the vibrating wire which forms the antinode in the fundamental mode of vibration, is violently shaken and ultimately thrown off the wire when the tuning is exact.* Measure carefully the length, l, of the string. Repeat the experiment with other forks of known frequency, keeping the load constant. We will find that for each fork the product of the frequency and the corresponding tuning length *i.e.*, $n \times l$ is constant, thus showing that *the frequency varies inversely as the length of the wire.*

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*The equality of the pitch may also be tested by means of a wooden disc about 3″ in diameter, attached to one end of a short wooden rod whose axis is normal to the plane of the disc. The free end of the rod is placed on the sonometer board and the disc is pressed against the ear. If the adjustment is approximately correct, beats may be heard. The bridge C is shifted until the beats are very slow and ultimately disappear.

Expt 2. *To verify that* $n \propto \sqrt{T}$. Apply a known tension T_1 to the experimental wire and adjust the movable bridge C, until it vibrates in unison with a standard fork of frequency n_1, or with the comparison wire. For this purpose, the comparison wire is stretched with a definite tension so that when plucked by a violin bow, it emits a note of the pitch corresponding to that of the standard fork. When unison is obtained by the method explained in Expt. 1, measure the length, l_1, of the string. Vary the tension to T_2 and find the tuning length, l_2, repeating several times. Using the result obtained in the last Expt. *viz.*, nl=constant, we may calculate the frequency of the experimental wire if it were stretched under tension T_2 and its length remained l_1. It is given by $n_2 = n_1 \frac{l_2}{l_1}$. Proceeding in a similar way, the frequencies of the experimental wire, under different loads but with the constant length of the vibrating portion, may be determined. In all cases, we will find that

$$\frac{\sqrt{T_1}}{n_1} = \frac{\sqrt{T_2}}{n_2} = \frac{\sqrt{T_3}}{n_3} = \text{constant}.$$

i.e., the frequency of the wire varies directly as the square root of the stretching force.

Expt. 3. *To verify that* $n \propto \frac{1}{\sqrt{m}}$. Adjust the portion AC of the experimental wire when stretched with a given load to be in unison with the comparison wire or a standard fork as before. Let the tuning length be l_1. Remove the wire cutting off a portion of it, and weighing it to determine the mass m_1 per unit length. Next fix on the monochord another string of different diameter or material whose mass per cm. is m_2. and stretch it with the same load as in the first case. Adjust AC again to be in unison with the comparison wire or the standard fork and let the length be l_2. If the law be true and n be the frequency of the comparison wire or the standard fork,

then $$n = \frac{1}{2l_1}\sqrt{\frac{T}{m_1}} \text{ in the first case,}$$

and $$n=\frac{1}{2l_2}\sqrt{\frac{T}{m_2}} \text{ in the 2nd case,}$$

or $$\frac{1}{2l_1}\sqrt{\frac{T}{m_1}}=\frac{1}{2l_2}\sqrt{\frac{T}{m_2}}$$

or $$l_1\sqrt{m_1}=l_2\sqrt{m_2}$$

i. e., the product $l\sqrt{m}$ should be constant. The measurements taken above will be found to satisfy this relation and *to prove that* $n \propto \frac{1}{\sqrt{m}}$.

52. **Harmonics or Overtones of a String.**—We saw in §49 that if a string fixed at both ends is plucked or bowed at the centre it vibrates as a whole giving out its fundamental tone of frequency $n=\frac{1}{2l}\sqrt{\frac{T}{m}}$. Here the ends are permanently at rest, and the middle point of the string has maximum displacement. Besides it may vibrate in two segments or in three and so on, such that there are points marked N which are permanently at rest* between the extreme points as shown in Fig 41. Such points are called *nodes* while the points where the amplitude is maximum are *antinodes* or *loops*. The portion of the string between the two nodes is called ventral segment. These modes of vibration may be easily established by touching the string lightly at a point where a node is desired, and at the same time plucking it at a loop. For instance, if a

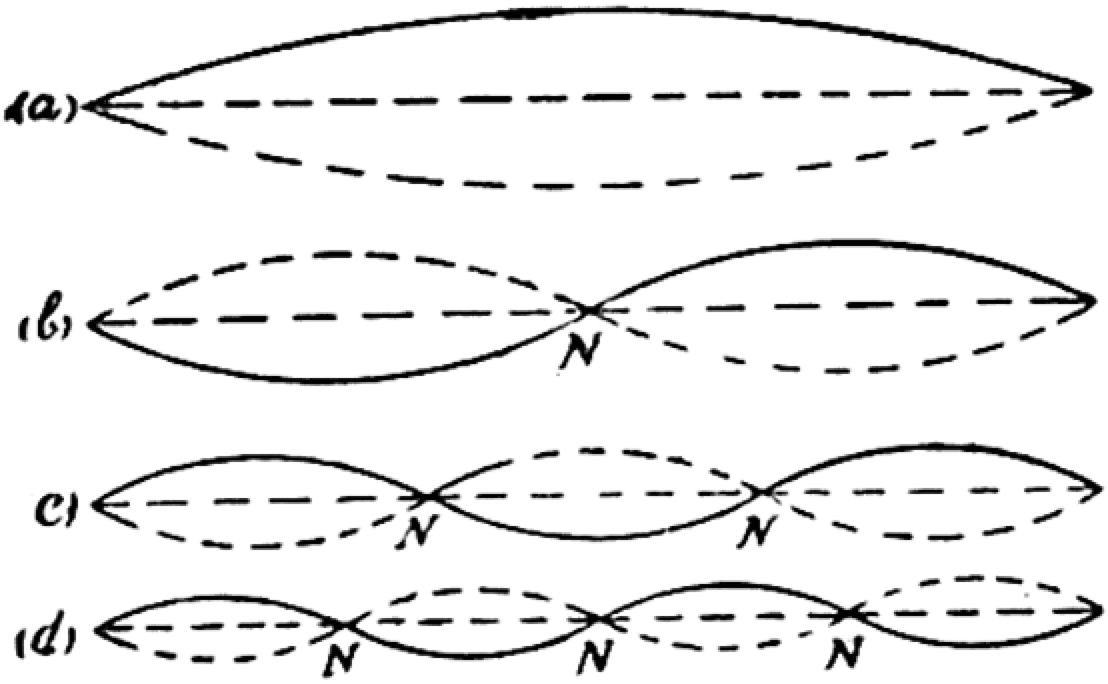

Fig. 41.

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*Though the nodes are described as at rest, this only means that their motion is very small compared with that of the loops. Really speaking, they must move slightly for the energy of vibration comes through them from the bowed segment. And the fixed ends of the string which are also nodes, must have some motion, for through them the energy passes to the sounding box or board on which the string is mounted.

string is touched at its middle point and bowed half way between one end and the point where it is touched, it will vibrate in two segments. The frequency, n', of this mode of vibration is obtained by putting $l/2$ for l in equation (3)

$$\therefore \quad n' = \frac{1}{l}\sqrt{\frac{T}{m}} = 2n, \quad \dots \quad \dots \tag{6}$$

i.e., the pitch is *double* that in the first case. Similarly, modes (*c*) and (*d*) give frequencies $3n$, $4n$, etc.

These tones emitted by a string besides its fundamental are called '*Overtones*'. If the frequencies of these overtones including the fundamental are as indicated above in the ratio 1, 2, 3, etc., they form a *harmonic series* and may be called *harmonic overtones* or simply Harmonics. A *Harmonic* may be defined as a note having a frequency an integral number of times that of the fundamental. Note that it is the presence of *harmonical overtones*, which is regarded as essential for the good musical *quality* of a vibrating string. Later we shall find that there are several cases when the overtones are *inharmonic*.

53. Compound Modes of Vibration of a String.—When a string is plucked or bowed, it does not vibrate in *any* one of the modes explained above but a number of these different modes of vibration are in general set up simultaneously and the vibration is the resultant of all the modes, of which the string is capable under the conditions of excitation. It is only necessary that the length of the string be a whole number of half wave-lengths. Hence when a string is apparently sounding its fundamental tone or a particular harmonic, this tone is, generally, only the predominant component of a compound note made up of all the possible modes under the conditions of excitation including the fundamental. The constituents can be picked out and recognized by suitable resonators or other means. The form of vibration assumed by a string in which such modes of vibration exist, is shown in Fig. 42. It will be easily understood if it is realised that in the compound mode, the motion of any point on the string is the resultant of all the motions proper to the component modes or partials present. The dotted lines show three positions

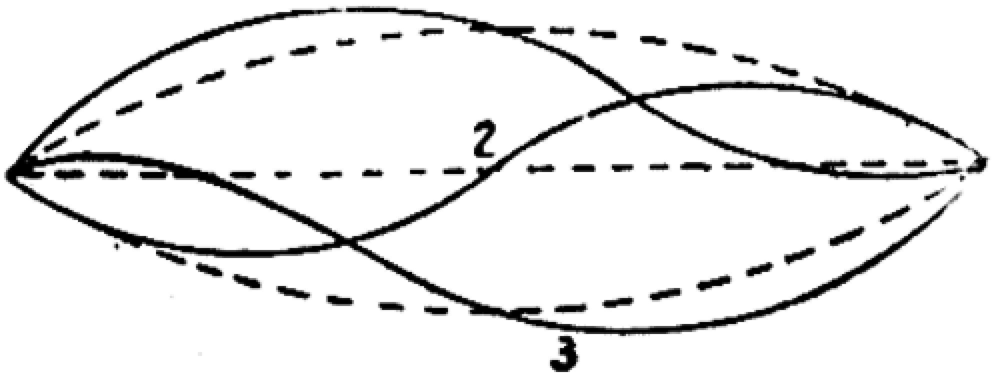

Fig. 42.

of the string simply vibrating as a whole in its fundamental mode. The full lines, however, show its form when it is at the same time vibrating in two segments, the two halves vibrating with reference to the dotted lines just as they would have done about the middle straight line if the fundamental mode of vibration had been absent.

The simultaneous existence of different modes of vibration in stretched strings may be demonstrated by a sonometer wire. Pluck the wire, say, at $\frac{1}{4}$ of its length from one end. Then touch the string lightly with a feather at the middle thus damping out its fundamental mode of vibration but it will be free to vibrate in two segments which will be plainly visible to the eye and easily recognizable by the ear to be an octave higher than the fundamental. Or if struck $\frac{1}{6}$ of its length from one end and then touched lightly at one-third of its length, the fundamental mode will be damped while it will still be heard sounding a tone having three times the frequency of the fundamental.

54. Demonstration of the Vibratory Motion of a String.—The peculiar motion of a vibrating string may be demonstrated to a class by projecting the image of a small portion of a monochord or a piano string upon a screen (or wall) with the help of a rotating mirror. The arrangement of the apparatus is shown in Fig. 43. A narrow slit, *S*, illuminated by an arc lamp, *C*, is placed on one side of the stretched string *AB* of the

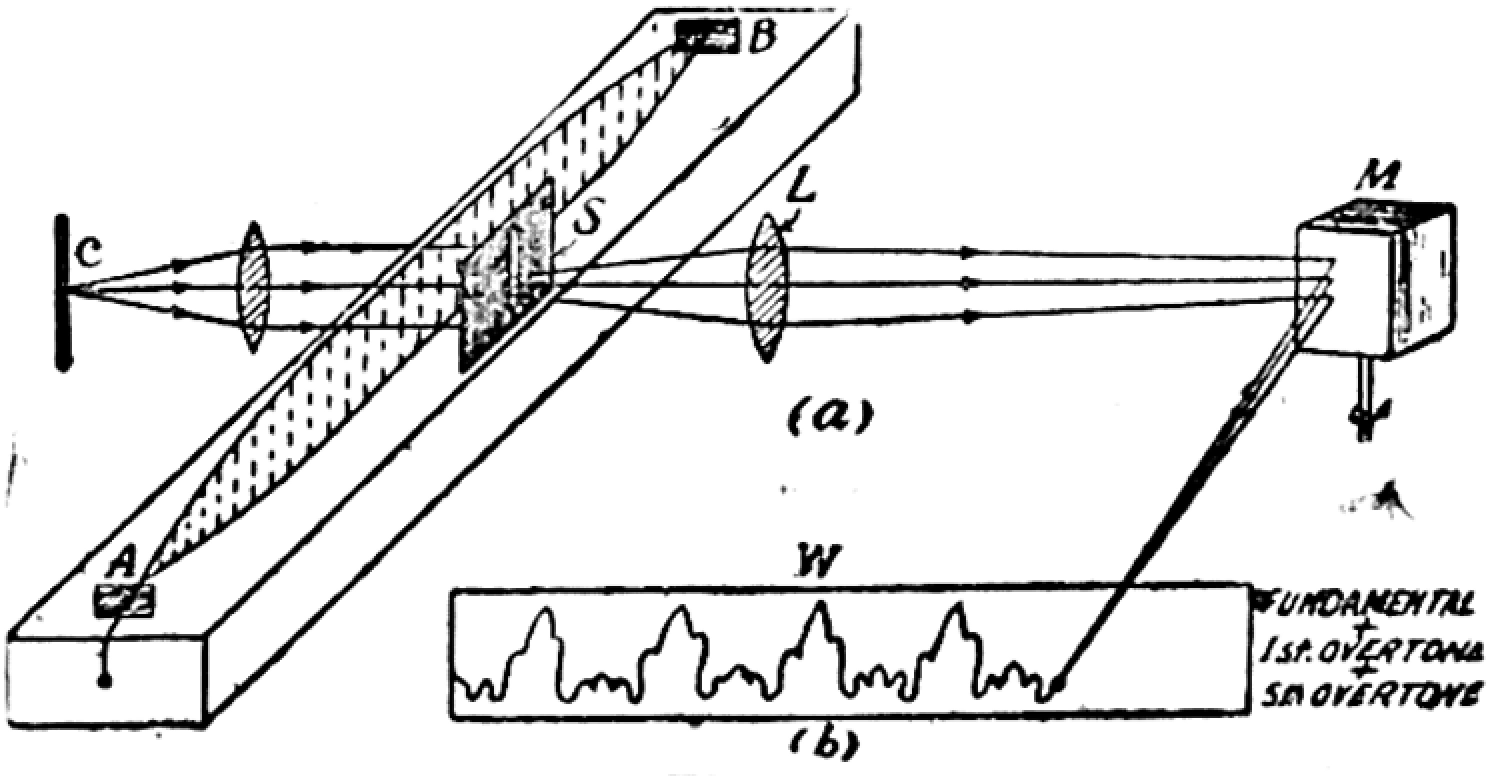

Fig. 43.

monochord. By means of a convex lens, *L*, an image of the slit and a small portion of the string is produced on the screen W. The light before reaching the screen falls upon the rotating mirror M. Without plucking the string, rotate the miror, and watch the image on the wall. On account of the *persistence* of vision the image of the string will appear to be drawn out into a horizontal straight line but when it is plucked or bowed vertically, the straight line becomes zig-zag and presents the appearance of a periodic curve such as is shown in Fig. 43 (*b*). The curve indicates that the vibration of the string is not simple harmonic for if it were so, the image on the screen would be a sine curve of the form shown in Fig 44 (*a*). The complex nature of the curve, however, shows that the vibration is the resultant of several components.

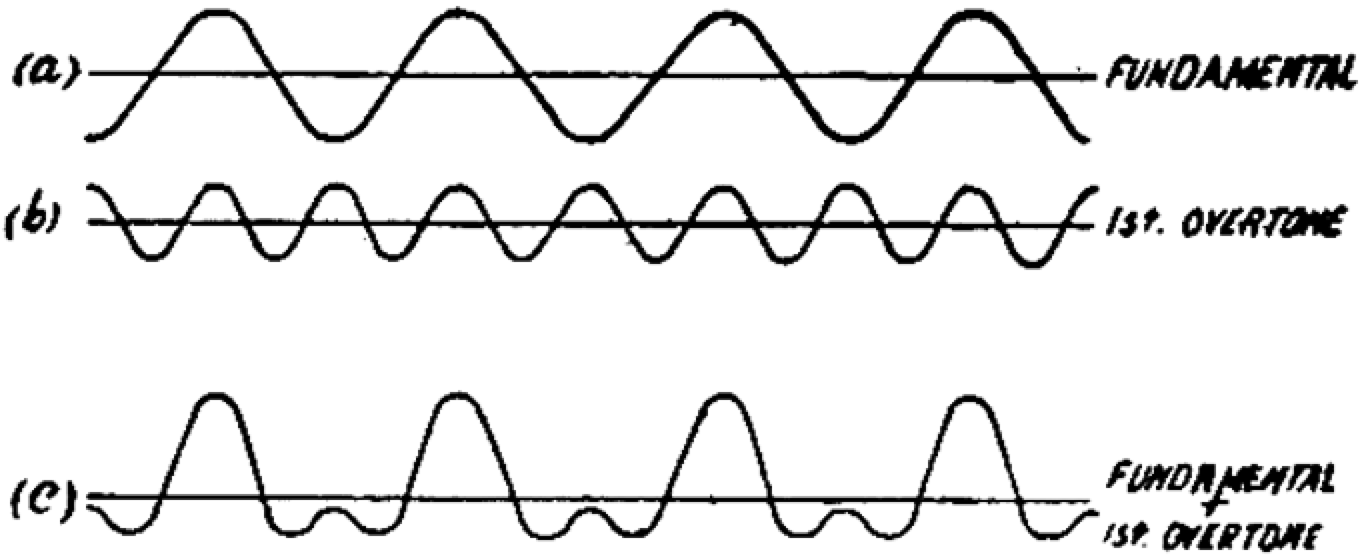

Fig. 44.

Various types of wave-form curves may be obtained depending on the conditions of excitation. The wave-form curve shown in Fig. 44 (*c*) will be produced when the string vibrates with the fundamental and the first overtone simultaneously. If, in addition to the fundamental, a string vibrates with the first and the 5th overtones the wave form will agree with Fig. 43 (*b*). Such curves can be obtained graphically by adding the displacements of the corresponding points in the component waves.

***54a. Motion of a point on a Vibrating String** :—The vibration Microscope described in Art. 21 (2) was used by Helmholtz for the experimental study of the motion of a point on the vibrating string of a violin under the action of the bow. The horizontal fork electrically maintained with the lens L attached to its prong is adjusted so that the vibrations of the lens and those of the string stretched vertically are in directions at right angles to each other. A bright point on the string is viewed by the microscope. If now the fork is made to vibrate and the string is at rest, the bright point will stretch out in a vertical line such as *AB* (Fig. 45). If the fork is at rest and the string is vibrating the bright point will be drawn out into a bright line *CD*. If both fork and string vibrate simultaneously, the two separate motions of the spot are compounded together and a Lissajous figure is produced whose shape depends upon the ratio of the two frequencies of vibration and on the phase difference.

The fork and the string are adjusted to unison but as the motion of the point on the string is not simple harmonic, though periodic, our curve will not resemble any of the curves in Fig. 11 (page 21). The general form of the curve obtained by Helmholtz is depicted in Fig. 45.

It gives us the details of the motion of the point on the string at times indicated by the motion of the fork which is simple harmonic

To investigate the displacement of the string against the tuning fork time scale draw a circle with centre on a horizontal line through *O* with its radius equal to the amplitude of the fork. Divide the circumference of the circle into any number of equal parts, say 16 at 0, 1, 2, 3 .. and through each of the points of division draw a horizontal line cutting the curve at 0, 1, 2, 3... zero, for convenience, being chosen where the vertical line through *O* cuts the horizontal line. The horizontal lines will mark a scale of equal time intervals on the vertical line through *O*. The numbers on the curve show the positions of the tracing point of light at the ends of successive equal intervals of time. On *AB* produced mark off 16 equally spaced points 0′, 1′, 2′, 3′......and from these points draw horizontals equal to the abscissae of the Lissajous figure at the point with a corresponding number and draw a smooth curve through the ends of these abscissae. The curve is the time displacement curve of the point on the string under observation, from which the speed of the point is easily deduced. It is obvious that any point on the string moves with uniform velocity forwards and backwards. This is true only when the point is in the centre of the string. If the point is nearer to one end, the forwards and backwards lines in the time displacement curve are unequally sloped showing that forward and backward speeds are unequal. Helmholtz concludes, "We learn therefore by these experiments "(1) The strings of a violin when struck by the bow, vibrate in one plane "(2)

That every point of a string moves to and fro with two constant velocities.

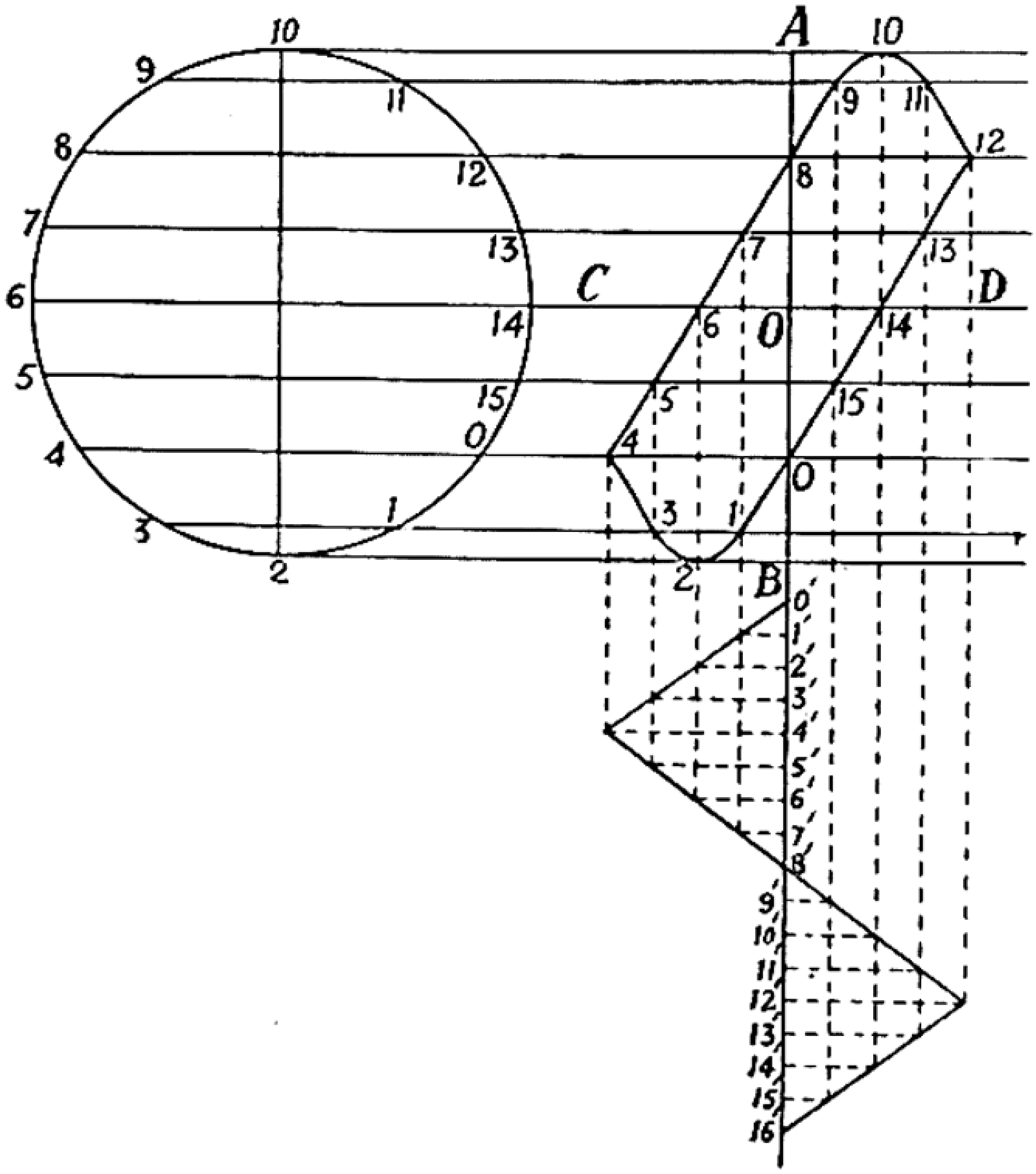

Fig. 45.

"These two data are sufficient for finding the complete equation of the motion of the whole string."

We shall have occasion to refer to these conclusions when discussing the action of the bow in a violin.

55. Stiffness of Strings.—The definition of the string given at the beginning of this Chapter, for which the laws have been established requires that the string should have no *stiffness i.e.*, offer no resistance to flexure, and therefore could be bent without any force being needed. Any actual string, however, departs somewhat from this behaviour and offers some slight resistance to bending. As a consequence, the restoring force acting on an element displaced from its normal position consists not only of the components of the applied-tension acting inwards, but an additional elastic force called into play by the bending of the wire. Thus the total restoring force is increased with a corresponding increase in the rate of vibration of the string. When the stiffness of the string is taken into account, theoretical investigation shows that not only is the fundamental raised in pitch but the overtones are also raised in pitch in an increasing proportion. The fundamental and the overtones do not fall in harmonic series but this departure in the case of lower partials of strings actually employed in musical instruments is of no serious importance.

56. **Melde's Experiment**.—A very convenient method of maintaining the vibration of a stretched wire and investigating the laws which govern such vibrations is due to Melde. A light string about 1 metre long is fixed at one end to the prong of a large tuning fork which is maintained in vibration electrically while the other end passes over an adjustable pulley and is fastened to a small scale pan. The tension in the string can be varied at will by placing weights in it. The vibrations of the fork may be either along the string as in Fig. 46(*a*), the **Longitudinal Mode** of Vibration, or at right angles to the string as shown in Fig. 46(*b*), the **Transverse Mode of Vibration**.

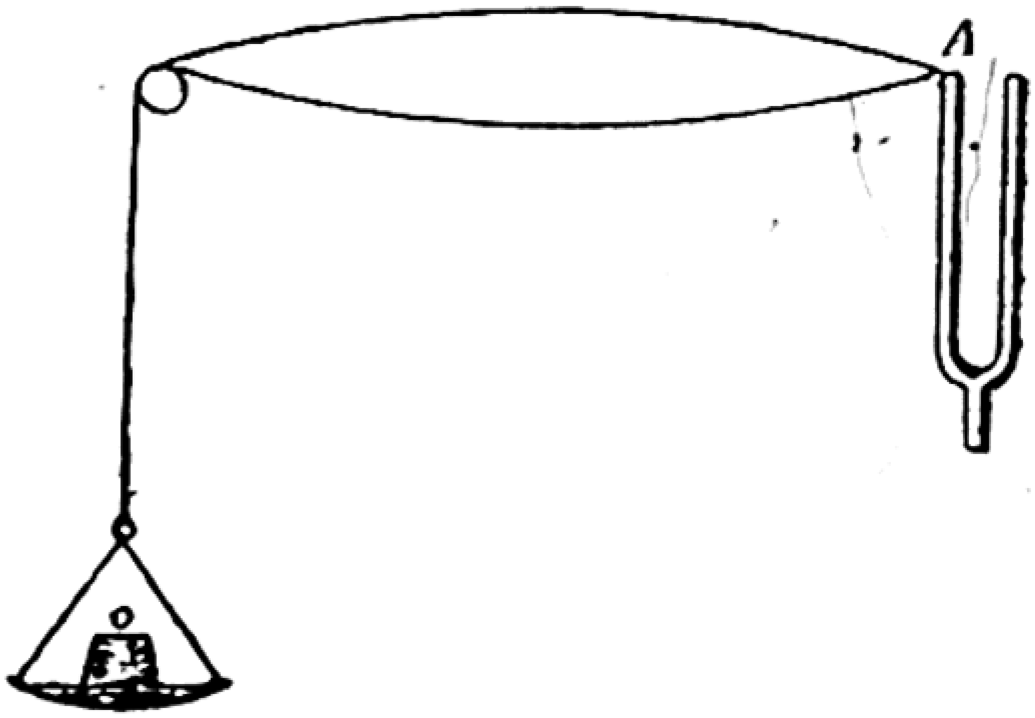

Fig. 46 (*a*).

Consider first the **Longitudinal Mode of Vibration**. With a given load adjust the length by moving the pulley near or farther from the fork so that it vibrates in a single segment. If the load is reduced to one-quarter the above value, two loops will be obtained, when it is reduced to one-ninth, three loops will be produced. If now in any one of these positions the fork is rotated through 90°, everything else being kept the same the wire will be found vibrating with twice the number of loops.

Let us now find the relation between the frequency of the wire and that of the fork in the *two* cases. In the first case when the end A of the prong is towards the extreme left, the amount of sag in the string is maximum. As the prong moves to the other end, the string is tightened becoming horizontal when it is at the extreme end of its vibration. When the prong returns, the wire does not sag again but travels up the horizontal on account of the inertia and the upward motion that it has acquired. It is in its extreme upward displacement of the wire that the prong completes one vibration. This shows that for every complete vibration of the fork, the string makes one-half of a complete vibration. Thus if the frequency of fork be N for a given length vibrating in a single segment, the vibration set in the string will be most *energetic* when the tension of the string T_1 is such that

$$\tfrac{1}{2}N=\frac{1}{2l}\cdot\sqrt{\frac{T_1}{m}} \quad \cdots \quad \cdots \quad (7)$$

If, however, we wish the same string to vibrate strongly in two segments with a node in the middle we must reduce the tension to T_2 such that

$$\tfrac{1}{2}N=\frac{1}{2l/2}\cdot\sqrt{\frac{T_2}{m}} \quad \cdots \quad \cdots \quad (8)$$

More generally if p segments are to be excited in the string each of length l' such that $pl'=l$, the tension should be so altered that

$$\tfrac{1}{2}N=\frac{1}{2l/p}\cdot\sqrt{\frac{T_p}{m}} \quad \cdots \quad \cdots \quad (9)$$

T_p being the altered tension.

Comparing (7) and (9) ; we have

$$\frac{1}{2l}\sqrt{\frac{T_1}{m}}=\frac{p}{2l}\cdot\sqrt{\frac{T_p}{m}}\text{, or squaring } T_p\times p^2=\text{constant.}$$

We conclude that the weights which are required to make the string vibrate in 1, 2, 3 loops are inversely proportional to the squares of the numbers. An experimental test of these relations affords a *verification* of the laws of vibrating strings.

Keeping the tension the same, the string can be set into vibrations strongly when its length is an integral multiple of the length of the string when vibrating with one ventral segment.

In the **Transverse Mode of Vibration** of the fork, when the prong moves towards the right the string also moves towards the right. When the prong passes through its mean position, the string will also be in its position of rest. It will move towards the left when the prong moves towards the left. The string will, in this case, vibrate in unison with the fork,

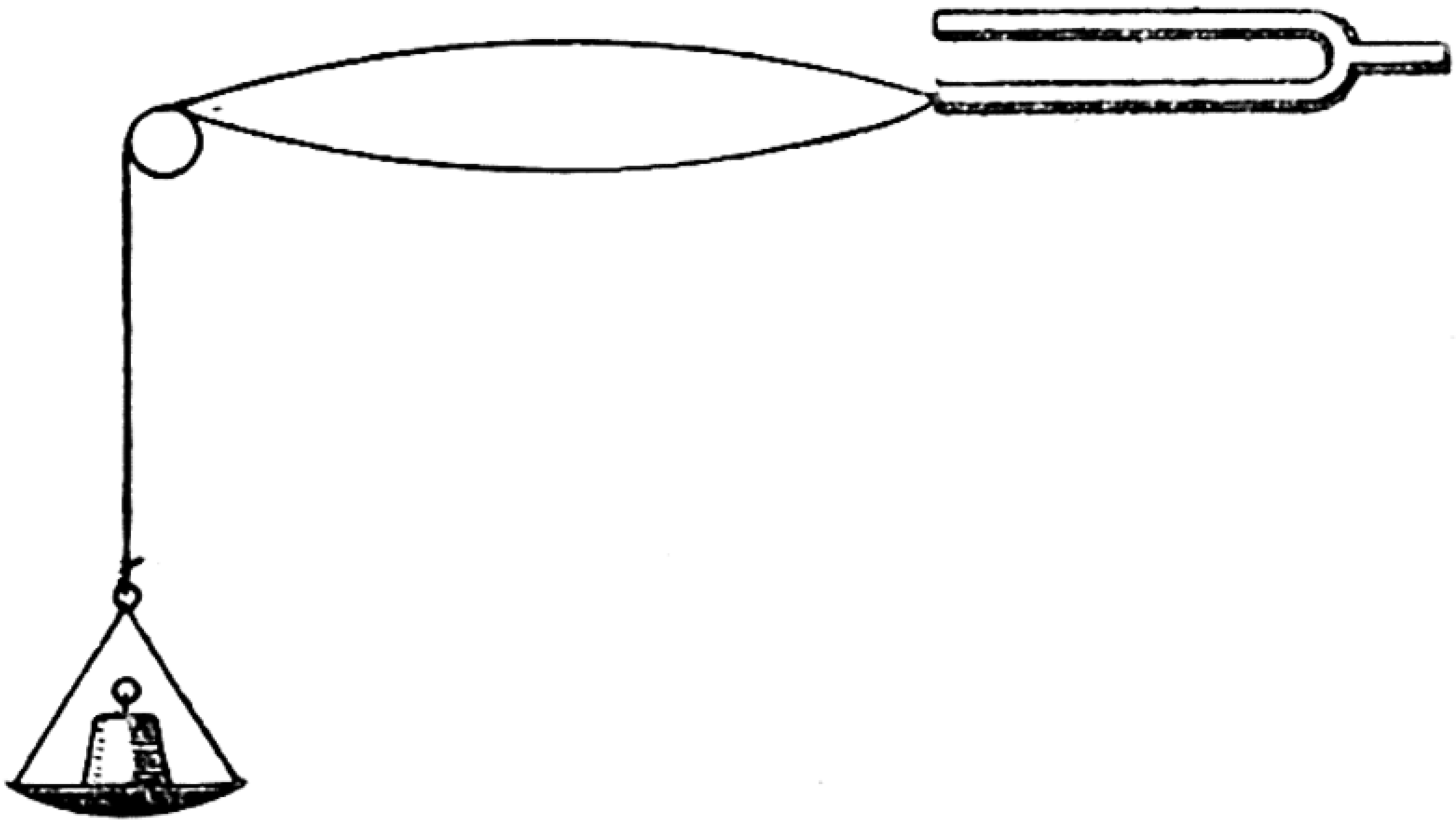

Fig. 46 (*b*).

i.e.,
$$N=\frac{1}{2l}\sqrt{\frac{T_1}{m}} \quad \ldots \quad \ldots \quad (10)$$

where l is the length of the fundamental mode of vibration. For the string to split up into p segments we have to alter the tension to T_p such that

$$N=\frac{p}{2l}\cdot\sqrt{\frac{T_p}{m}} \quad \ldots \quad \ldots \quad (11)$$

From 10 and (11) we have as before

$$T_p \times p^2=\text{constant.}$$

But a stretching force required to give a particular number of loops in this mode of vibration is four times that required in the first case, for in the first case

$$T_p = \frac{N^2 l^2 m}{p^2} \text{ from (9) above ;}$$

and in this case

$$T_p = 4 . \frac{N^2 l^2 m}{p^2} \text{ from (11) above.}$$

Example. *In a Melde's Experiment the string is stretched by a load of 10 grams. When the fork is bowed, the string vibrates in four loops. By what weights must the string be stretched for the number of loops to be 5, 3 and 1 respectively ?*

Using the relation,

$$T_p \times p^2 = \text{constant}$$

we have for 5 loops,

$$T_5 \times 5^2 = 10 \times 4^2$$

$$\therefore \; T_5 = \frac{160}{25} = 6{\cdot}40 \text{ gm.}$$

For 3 loops,

$$T_3 \times 3^2 = 10 \times 4^2$$

$$\therefore \; T_3 = \frac{160}{9} = 17{\cdot}77 \text{ gm.}$$

For one loop,

$$T_1 \times 1^2 = 1 \times 4^2$$

$$\therefore \; T_1 = 160 \text{ gm.}$$

The Melde's experiments, besides affording a practical method of verifying the laws of vibrating strings, illustrate in a very striking and convenient manner, the formation and maintenance of **Stationary Vibrations** in strings. The vibration of the prongs of the fork acts as a source of transverse vibrations which travel forward along the string to the pulley and are reflected back from there along the string. The superposition of the direct and the reflected train of waves under proper conditions produces stationary waves. The string will in

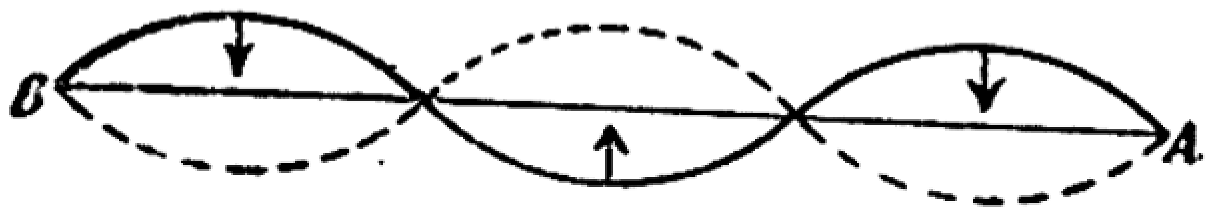

Fig. 47.

general split up into several equal segments depending upon the load. If the string vibrates in p segments, say, the alternate segments will be found to be moving in opposite directions with $(p-1)$ nodal points lying along the string equally spaced. In Fig. 47, the string AB is vibrating in three equal segments with two nodal points lying along the length between the end points, the motions of *alternate* segments are in opposite directions as indicated by the vertical arrows. For detailed study of these vibrations see Chapter VII.

CHAPTER V

PROPAGATION AND VELOCITY OF SOUND

57. Medium a Necessity for the Propagation of Sound.—We saw that sound waves originate in the vibratory motion of a body although the motion may not be visible. It is evident from the fact that sound is stopped when vibrations are stopped. If the sounding body is lying in air, the repeated vibratory motion sets up a wave motion which propagates in all directions. On falling upon the tympanic membrance of the ear it causes it to vibrate and thus produces the sensation of sound. It can be experimentally verified that sound waves cannot be propagated in vacuum by suspending a ringing electric bell in a partially evacuated vessel (Fig. 48). The sound becomes fainter and fainter, the better the vacuum becomes. It is easy to *infer* from this that an elastic medium is necessary in order that sound waves may be propagated and conveyed to the ear. Sound is transmitted not only by air but by liquids and solids *as well.* That water transmits sound, can be vouchsafed for by any one who has heard two stones struck together under water while his ears were submerged. As for solids, we all know that if we apply our ears to the rails, we can hear the sound made by the wheels of a distant train. A **toy telephone** with which every child is familiar, is another beautiful illustration of the transmission of sound by solids. Here the sound is transmitted along the cord. In some cases the transmission is so distinct that conversation can be carried on.

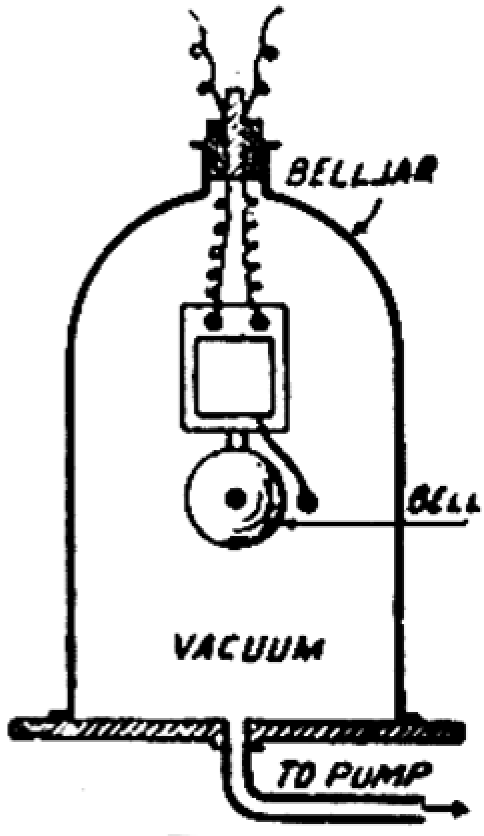

Fig. 48.

In the wooden form of the **stethoscope** used by medical men to hear the feeble sounds from the lungs and the heart, besides the column of air extending in the tube of the stethoscope wood plays an important role in conducting the sound.

It must be noted that sounds cannot pass from one medium to another of a very different density.

As already stated sound is a longitudinal wave motion in *material* media—solids, liquids and gases and the velocity with which it is propagated, depends upon the two characteristics of the medium, (*i*) its *elasticity*—a property which indicates that when the particles of a medium suffer relative displacements, a force is set up which tends to bring them back to their original configuration, (*ii*) its *inertia*, which depends upon its mass. The actual wave motion depends therefore on the kind of initial impulse given from outside, the elasticity and the mass contents of the medium.

Newton first showed that in a homogeneous medium the velocity of longitudinal waves is given by $v=\sqrt{\frac{E}{\rho}}$, where E is the modulus of Elasticity for the particular type of the strain set up and ρ, the density of the medium.

58. Velocity of Longitudinal Waves in Air.—Consider that a wave is travelling towards the right in the air contained in a long tube of unit cross-section as shown in Fig. 49. If the medium is caused to move towards the left with a velocity equal to that of the wave, then the wave, though moving with respect to the air, will be at rest relatively to the ground. In a small region surrounding any point, the pressure, velocity and density of the medium will be constant. These quantities will, of course, have different values at different points along the tube. Imagine two partitions *A* and *B* placed across the tube. Since there is no change in the average density of the medium, the mass of air entering the space between the two partitions in a time *t* must be equal to the mass leaving it in the same time. Since the particle velocities at *A* and *B* are, in general, not identical there will be a gain or loss of momentum accompanying the transfer of mass. For simplicity, suppose that *A* is a region of *normal* density and *B* a region of *condensation*. Then since the same mass of medium must flow out of second partition per second as enters the first in the same time, the velocity

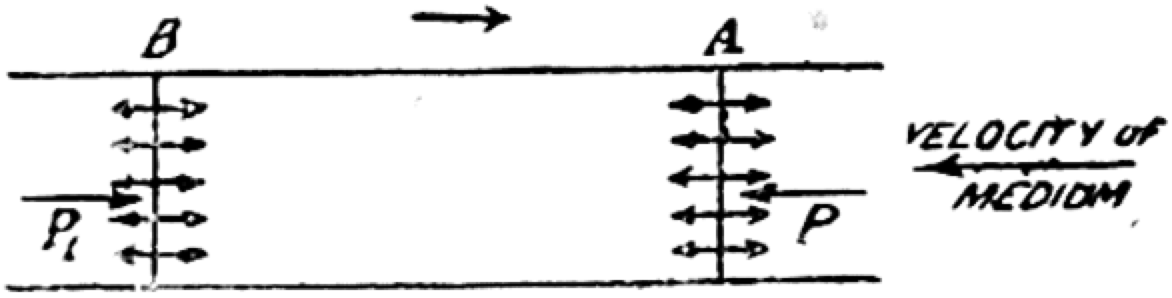

Fig. 49.

of outflow at the second will be less than the velocity of inflow at the first as the volume of a given mass of a fluid depends upon its pressure.

Let v, P and ρ denote the velocity, pressure and density respectively at A and v_1, P_1, and ρ_1, the corresponding quantities at B. Now

(*i*) Since the mass of air flowing in at A is equal to that flowing out at B, we have,

$$m = v\rho = v_1\rho_1 \qquad \ldots \qquad \ldots \qquad (1)$$

(*ii*) In one second a mass of gas m equal to that flowing through either partition has its velocity decreased from v to v_1 and therefore there is a gain of momentum/sec. $= m(v - v_1)$ and this, according to Newton's Second Law of motion, is due to some external force. Since the only force available is the slight difference of pressure $P_1 - P$, between the two partitions

$$\therefore \quad P_1 - P = m(v - v_1) \qquad \ldots \qquad \ldots \qquad (2)$$

or

$$P_1 - P = mv\left(1 - \frac{v_1}{v}\right) \qquad \ldots \qquad \ldots \qquad (2a)$$

From (1) $\qquad m = v\rho$, and $\dfrac{v_1}{v} = \dfrac{\rho}{\rho_1}$.

Substituting these values in (2*a*), we have

$$P_1 - P = v^2\rho\left(1 - \frac{\rho}{\rho_1}\right) = v^2\rho\,\frac{\rho_1 - \rho}{\rho}.$$

$$v^2\rho = \frac{\left(\dfrac{P_1 - P}{\rho_1 - \rho}\right)}{\rho_1} \qquad \ldots \qquad \ldots \qquad (3)$$

Now the volume elasticity of the medium is given by

$$E = \frac{P_1 - P}{(V - V_1)/V}$$

where V is the volume of unit mass $\left(= \dfrac{1}{\rho}\right)$.

Substituting $\dfrac{1}{\rho}$ for V we have

$$E = \frac{P_1 - P}{\dfrac{\rho_1 - \rho}{\rho_1}} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (4)$$

Putting this value in (3) above

$$v^2\rho = E, \quad \text{or} \quad v^2 = \frac{E}{\rho}.$$

Now v is the velocity of the air inflow at the first partition and is evidently equal to the velocity of the wave. Consequently the velocity of a longitudinal wave is $\sqrt{\dfrac{E}{\rho}}$ ... (5)

This equation holds good for a compressional wave in any elastic medium, *gaseous*, *liquid* or *solid*.

***59. Alternative proof for the velocity of a Longitudinal Wave.**—We shall apply the result obtained in Art. 43 to find the velocity of compressional waves in terms of the characteristics of the medium namely its 'elasticity' and 'inertia'.

Let us take the case of a longitudinal wave in air (or any elastic fluid). Suppose the waves are travelling from left to right along OX. Consider two planes of area a perpendicular to OX at e and f, and at a small distance δx apart. Let e' and f' be the displaced positions of the planes originally at e and f. If the displacement of the plane at e be y, then the rate of change of displacement is $\dfrac{dy}{dx}$. Therefore the displacement at $f = y + \left(\dfrac{dy}{dx}\right)\delta x$, the distance apart of the planes has been increased by $y + \left(\dfrac{dy}{dx}\right)\delta x - y = \dfrac{dy}{dx}.\delta x$.

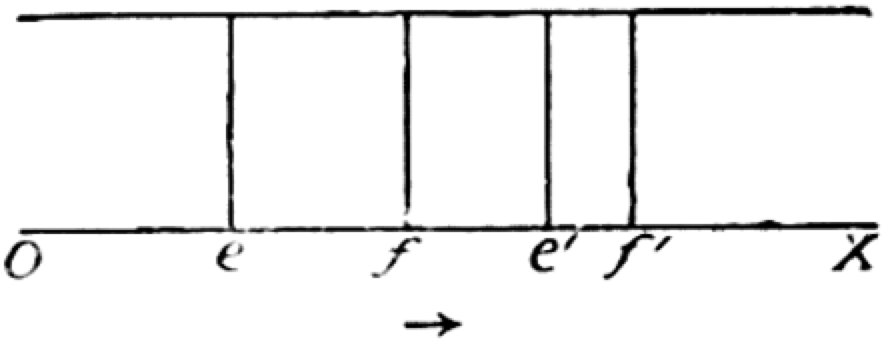

Fig. 50.

Change in volume of the layer $ef = a\dfrac{dy}{dx}.\delta x$.

Volume of the layer $ef = a\delta x$

$$\therefore \quad \text{Volumetric strain} = \frac{a\dfrac{dy}{dx}.\delta x}{a\delta x} = \frac{dy}{dx}.$$

If now F be the force per unit area on the plane e, then the total force on the plane $e = aF$

and ,, ,, ,, $f = aF + a\left(\dfrac{dF}{dx}\right)\delta x$

The resultant force on the element δx

$$=aF+a\left(\frac{dF}{dx}\right)\delta x-aF=a\left(\frac{dF}{dx}\right)\delta x.$$

Mass of air between e and f

$$=a.\delta x.\rho, \text{ where } \rho \text{ is the density of air.}$$

Acceleration of air particles $\frac{d^2y}{dt^2}$

$\therefore$ Mass × Acceleration$=a\delta x.\rho\frac{d^2y}{dt^2}$

By Newton's Second Law of motion, we have

$$a\ \delta x\ \rho.\frac{d^2y}{dt^2}=a\frac{dF}{dx}.\ \delta x.$$

or

$$\rho\ \frac{d^2y}{dt^2}=\frac{dF}{dx}\qquad \dots \qquad \dots \qquad \dots (6)$$

Now Elasticity $E=\frac{\text{stress}}{\text{strain}}=\frac{F}{\frac{dy}{dx}}$

$$\therefore\ F=E\ \left(\frac{dy}{dx}\right) \text{ and } \frac{dF}{dx}=\frac{d}{dx}\left(E\frac{dy}{dx}\right)$$

Putting this value in (6)

$$\rho\frac{d^2y}{dt^2}=\frac{d}{dx}\left(E\frac{dy}{dx}\right)=E\frac{d^2y}{dx^2} \text{ or } \frac{d^2y}{dt^2}=\frac{E}{\rho}\frac{d^2y}{dx^2}$$

Comparing it with equation (11) of Art. 43, we have

$$v^2=\frac{E}{\rho} \text{ or } v=\sqrt{\frac{E}{\rho}}$$

60. Calculation for Velocity in a Homogeneous Medium.— Let us apply Newton's formula to the case of air.

Now for a fluid $E=\frac{dP}{dV/V}$, where dP is the change in pressure P and dV the change in volume V. Newton assumed that the temperature during compressions and rarefactions in a sound wave remains constant and therefore Boyle's law ($PV=$ constant) holds good.

It can be easily shown that for a gas obeying Boyle's law $E=P$, where P is its pressure. For suppose that pressure is increased from P to $P+dP$ and the volume in consequence is changed from V to $V-dV$

$$\therefore\ PV=(P+dP)(V-dV)$$
$$=PV-PdV+VdP-dV\times dP.$$

Neglecting the product $dV\times dP$ being small, we have

$$PdV=VdP \text{ or } P=V\ \frac{dP}{dV}=E_\theta$$

where E_θ is the Isothermal Elasticity.

so $$v=\sqrt{\frac{P}{\rho}}$$

Now in the case of air at 0°C.,

$$P=13{\cdot}6\times76\times981 \text{ dynes/sq. cm.}$$

and $$\rho=0{\cdot}001293 \text{ gm./c.c.}$$

$$\therefore\ \text{Velocity of sound at } 0°\text{C}=\sqrt{\frac{76\times13{\cdot}6\times981}{{\cdot}001293}}$$
$$=28000 \text{ cm./sec. or } 918{\cdot}7 \text{ ft./sec.}$$

This result falls short of the experimental result according to which $v=33000$ cm./sec. by about 16%. Newton, however, by using imperfect data for P and ρ found the discrepancy to be about 12%. He accounted for it by assuming that the molecules of air occupied about $\frac{1}{8}$th of the space traversed by sound and that sound passed instantly through these, only taking time to go through the interspaces. He also supposed that the presence of water vapour had no share in conducting the sound. Both the assumptions, however, are unjustified and the true explanation was given by Laplace.

61. Laplace's Correction.—The discrepancy between the calculated and the observed values for the velocity of sound was first explained by Laplace in 1816. He said that compressions and rarefactions accompanying the passage of a sound wave occur so rapidly that the rise of temperature during compression and fall of temperature during rarefaction do not get enough time to be equalised by radiation or conduction to the surrounding air, as air is a medium of very low thermal conductivity. Hence the assumption that the changes in volume and pressure are isothermal and obey Boyle's law, is not valid. The change in this case is *adiabatic** for which the relation between the pressure and volume of a gas, is given by

---

*An *Adiabatic* change is one in which no heat flows in or out of any portion of the gas during any set of operations.

$$PV^{\gamma}=\text{constant},$$

where γ is the ratio between the specific heat at constant pressure and the specific heat at constant volume.

Let us find the value of Adiabatic Elasticity from this relation. Differentiating it, we get

$$V^{\gamma}dP+P.\gamma V^{\gamma-1}dV=0$$

or
$$\gamma PV^{\gamma-1}dV=-V^{\gamma}dP$$

or
$$\gamma P=-V\ \cdot\frac{dP}{dV}=E\phi,$$

where $E\phi$ is the Adiabatic Elasticity.

Substituting this value for E, we get

$$v=\sqrt{\frac{P}{\rho}}\quad\dots\quad\dots\quad\dots\quad(7)$$

For air, $\gamma=1{\cdot}41$. Using the data as before, velocity of sound in air

$$=\sqrt{\frac{1{\cdot}41\times76\times13{\cdot}6\times981}{{\cdot}001293}}$$
$$=33250\text{ cm./sec.}$$

This is in close agreement with the value obtained experimentally and therefore establishes the truth of *Laplace's Correction.*

It should be *remembered* that the most accurate method of determining γ is to measure the velocity of sound in a gas.

62. Discussion of the Formula.—We shall now use the formula, $v=\sqrt{\frac{\gamma P}{\rho}}$, to find how the velocity of sound in a gas depends upon the nature of the sound waves and on the pressure, density and temperature of the gas.

(i) Effect of amplitude and wave-length.—The expression for velocity is independent of the *amplitude* and the *wave-length* of the note and therefore we can safely conclude that velocity does not depend upon the amplitude and the wave-length, but this holds so long as the amplitude is *small* as is generally the case. Loud sounds such as those of a canon are found to travel *faster* than ordinary sounds in the immediate neighbourhood of the source. At a distance from it the sound velocity settles down to normal value corresponding to that for small amplitudes.

In 1905, in a tunnel 2 miles long, it was proved that difference in pitch has no effect, whatever, on velocity. Were it otherwise, indeed, a tune played by a distant band might become confused, and the characteristic quality of its instruments unrecognizable.

(*ii*) **Effect of Pressure.**—The change in the pressure of a given mass of gas would produce a corresponding change in volume. From Boyle's law, PV = constant. Since $V=\frac{m}{\rho}$

$$\therefore \quad \frac{Pm}{\rho}=\text{constant or } \frac{P}{\rho} \text{ remains unaltered.}$$

The velocity is thus independent of any change in pressure so long as any change in the atmospheric pressure is sufficiently slow and Boyle's law holds good.

The experiments have also shown that velocity of sound at high altitudes is the same as at sea-level.

(*iii*) **Effect of Density.**—The pressure remaining the same, velocities of sound in two gases are *inversely* proportional to the square root of their densities, provided γ has the same value for both of them. Oxygen is 16 times denser than hydrogen, therefore velocity of sound in hydrogen is 4 times that in oxygen,

$$\because \frac{\text{velocity in hydrogen}}{\text{velocity in oxygen}}=\sqrt{\frac{\gamma P}{\rho_h}} \div \sqrt{\frac{\gamma P}{\rho_0}}=\sqrt{\frac{\rho_0}{\rho_h}}=\sqrt{\frac{16}{1}}=4.$$

(*iv*) **Effect of Temperature.**—When the temperature increases, the density of the gas decreases without affecting the pressure. As the numerator in the expression $\sqrt{\frac{\gamma P}{\rho}}$ would remain unchanged, a rise of temperature would increase the velocity, and *vice versa*. We know from Charles's law that $\rho_t=\frac{\rho_0}{1+at}$, where ρ_t and ρ_0 are the densities at temperatures $t°$ and $0°$C., respectively and $a=\frac{1}{273}$. Substituting this in the equation for velocity, we get

$$v_t=\sqrt{\frac{\gamma p}{\rho_t}}=\sqrt{\frac{\gamma p}{\rho_0}\ (1+at)}$$

At 0°C. the velocity will be

$$v_0=\sqrt{\frac{\gamma P}{\rho_0}}.$$

$$\therefore\quad v_t=v_0\sqrt{1+at}.$$

or
$$\frac{v_t}{v_0}=\sqrt{(1+at)}=\sqrt{1+\frac{t}{273}}=\sqrt{\frac{273+t}{273}}$$

$$=\sqrt{\frac{T}{T_0}}$$

i.e., velocities are directly proportional to the square root of absolute temperatures.

If, however, the variation of t is not too large, we may, by the Binomial Theorem and neglecting higher powers of t, expand $(1+at)^{\frac{1}{2}}$ into

$$1+\frac{a}{2}t+\dots\dots\dots$$

so that, approximately

$$v_t=v_0\left(1+\frac{a}{2}t\right)$$

Using the value for $a=\frac{1}{273}$ per degree Centigrade, and 332 metres/sec. for the velocity of sound at 0°C., we have

$$v_t=332\left(1+\frac{t}{546}\right)$$

$$=332+0{\cdot}61\,t \text{ metres/sec.} \qquad \dots\quad (8)$$

and we see that the velocity increases by 0·61 m. or 2 ft./sec. for 1°C. rise of temperature. Note that this approximation holds good for air, and at ordinary temperatures.

This velocity of approximately 740 miles/hour at which sound is transmitted, is now being approached by modern transportation speeds and is exceeded by speeds of projectiles, wing tips of propellers, bullets, etc. Sound waves produced in the air by sources that have a greater speed than that of sound itself, in no way resemble ordinary sound waves. We shall deal with them later. (See Art. 133.)

63. Velocity of Sound in a Mixture of Gases.—To calculate the velocity of sound in a mixture of two or more gases or moist air, the actual density of the mixture is used in the formula. If the value of γ is different for the different components, the correct value is calculated from the relation

$$\frac{P}{\gamma-1}=\frac{P_1}{\gamma_1-1}+\frac{P_2}{\gamma_2-1}+\frac{P_3}{\gamma_3-1}.$$

where P is the pressure and γ is the ratio between the two specific heats of the mixture, P_1 and P_2 etc., being the partial pressures and γ_1 and γ_2 etc., the ratio of the two specific heats of the components of the mixture

To compare the velocity of sound in moist air with the velocity of sound in dry air at the same temperature, suppose ρ and ρ' are the densities, γ and γ' the ratio of the specific heats of the dry and moist air respectively. Then

velocity in dry air $=\sqrt{\frac{\gamma P}{\rho}}$, and velocity in moist air $=\sqrt{\frac{\gamma' P}{\rho'}}$.

$$\therefore \quad \frac{\text{Velocity in moist air}}{\text{Velocity in dry air}}=\sqrt{\frac{\gamma' P}{\rho'}}\div\sqrt{\frac{\gamma P}{\rho}}=\sqrt{\frac{\gamma'\rho}{\gamma\rho'}}$$

$$=\sqrt{\frac{\rho}{\rho'}} \text{ approximately} \qquad \ldots \quad (9)$$

Although the value of γ for air is 1·41 and that for water vapour about 1·3, but the amount of water vapour is so small that it would hardly affect the value γ' for the mixture which may be taken equal to the value of γ for air.

Now, applying the fundamental gas equation, $PV=RT$, and remembering that the density of water vapour at the same temperature and pressure as air is ·625 of the density of air, we can easily show* that $\frac{\rho}{\rho'}=\frac{P}{P_1+\cdot 625P_2}$.

$$\therefore \quad \frac{\text{velocity in moist air}}{\text{velocity in dry air}}=\sqrt{\frac{P}{P_1+\cdot 625P_2}}$$

Example. *To illustrate the use of this formula, let us compare the velocity of saturated air with dry air at 30°.*

Velocity of dry air at 30°.

$$=\text{Velocity at } 0°\text{C.}\times\sqrt{\frac{273+30}{273}}$$

---

*See a Book on Heat.

$$=332\times\sqrt{\frac{303}{273}}=349{\cdot}7\text{ m./sec.}$$

Now saturated vapour pressure at 30°C.$=31{\cdot}7$ mm. of mercury. If the barometric pressure be 752 mm. of mercury, then $P_1=752-31{\cdot}7=720{\cdot}3$ mm. and $P_2=31{\cdot}7$ mm.

$$\therefore\ \frac{\text{velocity in moist air at }30^\circ}{\text{velocity in dry air at }30^\circ}=\sqrt{\frac{752}{720{\cdot}3+{\cdot}625\times31{\cdot}7}}$$

$$=\sqrt{\frac{752}{740}}$$

$$\therefore\ \text{Velocity in moisture}=349{\cdot}7\times\sqrt{\frac{752}{740}}=752{\cdot}6\text{ m./sec.}$$

Thus the sound in saturated air travels *faster than* in dry air by 2·9 m./sec.

64. Relation between the Velocity of Sound and the Molecular Velocity in Gases.—Since a sound wave is superposed on the velocities of the molecules of a gas, it appears, *a priori*, that the velocity of sound (for small amplitudes) must depend upon the velocity of the molecular movements, particularly the *component* of the velocity of the molecules in the direction of the propagation of the wave.

We have seen above that the velocity of sound depends upon the pressure and the density of gas,

since
$$v=\sqrt{\frac{\gamma P}{\rho}}\,.$$

Also from the Kinetic Theory of gases, we know that the pressure arises from the to-and-fro motion of its molecules and is given by

$$P=\tfrac{1}{3}\rho\overline{C}^2 \quad \ldots \quad \ldots \quad (10)$$

where $\overline{C}$ is the *root mean square* velocity of the molecules.

From (10) $\dfrac{P}{\rho}=\tfrac{1}{3}\overline{C}^2$. Putting this value in the equation for v, we have
$$v=\sqrt{\frac{\gamma\overline{C}^2}{3}}=\overline{C}\sqrt{\frac{\gamma}{3}} \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

Also since $\overline{C}=C'\sqrt{\dfrac{3\pi}{8}}$, where C' is the *mean* velocity of the molecules, we have

$$v = C'\sqrt{\frac{\gamma\pi}{8}} \qquad \ldots \qquad \ldots \qquad (12)$$

It is clear that a knowledge of the velocity of sound in a gas at known pressure and density, gives us a method of determining the mean velocity of the molecules and the ratio of its specific heats.

The following table supports the conclusion arrived at above:

Gas	Observed Value of γ	Mean Molecular velocity m./sec. c'	Observed velocity of sound m./sec.	$c' \times \sqrt{\frac{\gamma\pi}{8}}$ m./sec.
Hydrogen	1·41	$17{\cdot}4 \times 10^2$	$12{\cdot}86 \times 10^2$	$12{\cdot}9 \times 10^2$
Air	1·40	4·47 ,,	3·31 ,,	3·31 ,,
CO_2	1·30	3·6 ,,	2·57 ,,	2·56 ,,

65. **Velocity of Sound in Liquids.**—The velocity of longitudinal waves in an extended mass of liquid depends upon the bulk modulus of elasticity, K, and the density, ρ, so that Newton's formula becomes

$$v = \sqrt{K/\rho} \qquad \ldots \qquad \ldots \qquad (13)$$

For water at ordinary pressure

$$K = 2{\cdot}2 \times 10^{10} \text{ dynes per sq. cm.}$$

$$\rho = 1 \text{ gram per c.c.}$$

$$\therefore \quad v = \sqrt{\frac{2{\cdot}2 \times 10^{10}}{1}} \text{ cm./sec.}$$

$$= 1480 \text{ m./sec.}$$

Since liquids are highly incompressible, they have very large values for bulk modulus of elasticity. This more than compensates for their great density and therefore velocity of sound in them is relatively very high.

For water, values lying between 1400 m. and 1800 m./sec. have been observed. This great divergence in the result is due to the fact that experiments have been performed at different depths and therefore at different temperature and pressure conditions. The variation in density is not so large as is the variation of elastic modulus K* with pressure and temperature. Various impurities also effect the result.

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*K for water increases six fold with pressure increase from 1 to 25,000 atmospheres.

Colladon and Sturm in 1826 found the velocity of sound in water at mean temperature of 8·1°C. to be 1435 m./sec. Recent experiments by Ionescu in 1924 give its value for distilled water at 20°C. and atmospheric pressure to be 1470 m./sec.[*]

Explosive waves in water also show an increase of velocity with intensity but the phenomenon is much less marked than in air.

66. Velocity of Sound in Isotropic Solids.—Solids are capable of experiencing various types of strains and therefore several kinds of waves are possible in them. If the substance is in the form of a thin rod or wire, we have longitudinal waves accompanied by lateral shrinkage at regions of rarefaction and expansion at regions of condensation involving only Young's Modulus (Y), therefore

$$v=\sqrt{\frac{Y}{\rho}} \qquad \dots \qquad \dots \qquad (14)$$

For steel Young's Modulus,

$$Y=2{\cdot}14\times10^{12} \text{ dynes per sq. cm.}$$
$$\rho=7{\cdot}6 \text{ grams per c.c.}$$
$$\therefore \quad v=\sqrt{\frac{2{\cdot}14\times10^{12}}{7{\cdot}6}} \text{ cm./sec.}$$
$$=5300 \text{ m./sec.}$$

In *extended solids*, where lateral shrinkage and expansion are not possible, the longitudinal waves are propagated with a velocity given by

$$v=\sqrt{\frac{K+\frac{4}{3}n}{\rho}} \qquad \dots \qquad \dots \qquad (15)$$

where K is the bulk and n is the rigidity modulus, $(K+\frac{4}{3}n)$ denotes what we call *Elongational Elasticity*[†]. For torsional vibrations the Coefficient of Elasticity concerned is the modulus of rigidity and the velocity therefore is given by

$$v=\sqrt{\frac{n}{\rho}} \qquad \dots \qquad \dots \qquad (16)$$

67. The following table gives the velocity of sound in various substances. The values in the case of solids are only approximate, as different samples vary greatly in their composition, properties etc.

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[*] Taking this value for velocity, $K=v^2\rho=2{\cdot}16\times10^{10}$ dynes per sq. cm.

[†] See a Book on Properties of Matter.

Table of Velocity of Sound

<table>
<tr><th>Substance</th><th>m./sec.</th><th colspan="2">Substance</th><th>m./sec.</th></tr>
<tr><td>Iron (cast)</td><td>4700</td><td colspan="2">Water at 20°C.</td><td>1470</td></tr>
<tr><td>Steel</td><td>5100</td><td colspan="2">Mercury at 25°C.</td><td>1469</td></tr>
<tr><td>Brass</td><td>3600</td><td colspan="2">Turpentine at 11°C.</td><td>1326</td></tr>
<tr><td>Oak</td><td>4200</td><td>Air</td><td rowspan="5">at 0°C</td><td>331·8</td></tr>
<tr><td>Glass</td><td>4000 to 5300</td><td>Oxygen</td><td>317</td></tr>
<tr><td>Paraffin wax</td><td>1400</td><td>Coal-gas</td><td>490</td></tr>
<tr><td>Rubber</td><td>30</td><td>CO_2</td><td>262</td></tr>
<tr><td></td><td></td><td>Hydrogen</td><td>1286</td></tr>
</table>

CHAPTER VI

INTERFERENCE, BEATS, AND COMBINATION TONES

68. **Superposition of Waves : (Interference) :**—Let us now consider the phenomena which result from the superposition of two or more wave trains travelling in the same medium at the same time. Each particle of the medium is acted on at any instant by two forces, one due to each wave train. The motion of the particle will be the vector sum of the separate displacements and as a consequence the *distribution* of energy in the medium will be very different from that due to the component wave trains. When the forces act in the same direction, there will be a higher or deeper trough or condensation than for either wave train. When they oppose, the particles may remain undisturbed. Such phenomena are generally grouped under the name '**interference**'.

There are several important cases of interference, some of which are illustrated below.

Case I. Two waves of the same frequency and amplitude travelling in the same direction (Coincident Wave Trains).

(*a*) When they are in the same phase, crest coinciding with crest and trough with trough and are represented by continuous and dotted curves as in [Fig. 51 (*a*)], they reinforce each other and the resultant wave shown thick has twice the amplitude of either separate wave.

(*b*) Fig. 51 (*b*) represents the two wave trains when they are in opposite phase, such that the crest of one coincides with the trough of the other and *vice versa*. Here the difference in phase is one-half wavelength. The displacements cancel each other and the medium is motionless.

(*c*) Two waves of the same frequency but with arbitrary amplitudes and difference of phase result in a wave train which is itself Simple Harmonic and has the same shape and frequency as the components. The amplitude of the resultant vibration is less than the sum and greater than the difference of amplitudes of the component vibrations.

All the above cases may be treated analytically as under :—

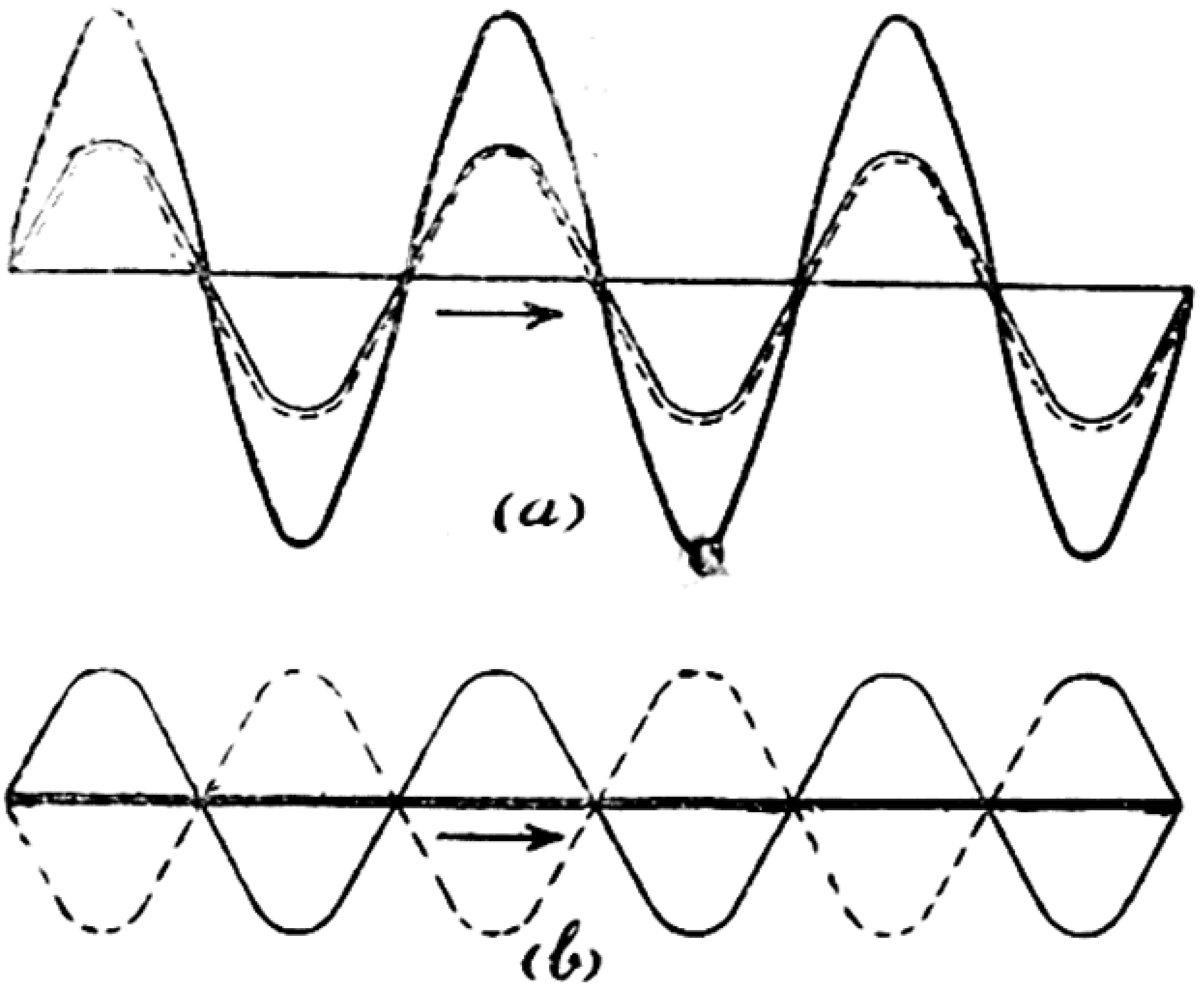

Fig. 51.

Analytical Treatment.—

Let
$$y_1 = a_1 \sin \frac{2\pi}{\lambda}(vt-x) \quad \ldots \quad \ldots \quad (1)$$

and
$$y_2 = a_2 \sin \left\{ \frac{2\pi}{\lambda}(vt-x)+\phi \right\} \quad \ldots \quad (2)$$

represent the displacement equations of two wave trains of the same frequency but with different amplitudes and phases, ϕ being the phase difference. The equation of the resultant wave train is given by

$$y = y_1 + y_2 = a_1 \sin \frac{2\pi}{\lambda}(vt-x) + a_2 \sin\left\{ \frac{2\pi}{\lambda}(vt-x)+\phi \right\}$$

$$= a_1 \sin \frac{2\pi}{\lambda}(vt-x) + a_2 \sin \frac{2\pi}{\lambda}(vt-x) \cos \phi + a_2 \cos \frac{2\pi}{\lambda}(vt-x) \sin \phi$$

$$= \sin \frac{2\pi}{\lambda}(vt-x)[a_1 + a_2 \cos \phi] + \cos \frac{2\pi}{\lambda}(vt-x)\,[a_2 \sin \phi]$$

Let
and
$$\left.\begin{aligned} a_1 + a_2 \cos \phi &= r \cos \theta \\ a_2 \sin \phi &= r \sin \theta \end{aligned}\right\} \quad \ldots \quad \ldots \quad (3)$$

Then $y = r \sin \frac{2\pi}{\lambda}(vt-x) \cos \theta + r \cos \frac{2\pi}{\lambda}(vt-x) \sin \theta$

$$=r\left[\sin\frac{2\pi}{\lambda}(vt-x)\cos\theta+\cos\frac{2\pi}{\lambda}(vt-x)\sin\theta\right]$$

$$=r\sin\left[\frac{2\pi}{\lambda}(vt-x)+\theta\right]$$

$$\left.\begin{aligned}\text{where } r&=\sqrt{a_1^2+a_2^2+2a_1a_2\cos\phi}\\ \text{and}\quad \tan\theta&=\frac{a_2\sin\phi}{a_1+a_2\cos\phi}.\end{aligned}\right\}\text{ from (3)}$$

So we have the resultant wave train of the same frequency but with a different phase and amplitude.

Special Cases :—

If the amplitudes are equal ($a_1=a_2$) and the phases are the same, $\phi=0$, superposition gives a vibration of double amplitude $2a$ [*Case I*, (*a*) above] but if phases are opposed *i.e.*, $\phi=180°$, the resultant amplitude is zero, [*Case I*, (*b*) above]. In the more general *Case I*, (c) above, the amplitude may vary between (a_1+a_2) and (a_1-a_2) according to the phase difference ϕ.

These conclusions are arrived at on the supposition that the interfering wave trains are plane and do not spread out laterally. For in the event of the lateral spreading out of the waves, their amplitudes gradually diminish, and the intensity of the wave will fall off inversely as the square of the distance from the source.

Case II. Two wave trains of the same frequency and amplitude travelling in opposite directions (*Opposite Wave Trains*).

Such wave trains give rise to a very important group of interference phenomenon and are known as standing or stationary waves. They are separately dealt with in Chapter VII and briefly referred to in Arts. 56 and 69 (*ii*).

Case III. Two wave trains of the same frequency but crossing at a point (*Oblique Wave Trains*).

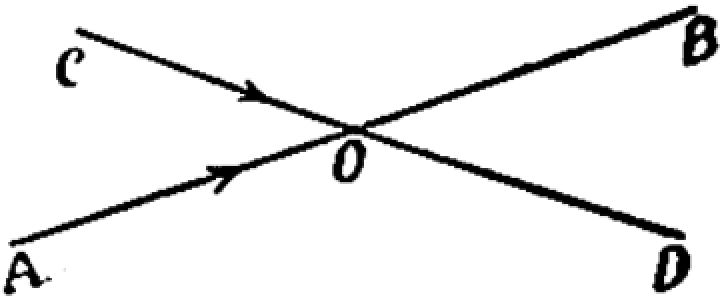

Fig. 52.

Let the wave trains proceed along *AB* and *CD* crossing at *O*. Here we are concerned with the velocity, displacement, and the pressure variations at the point of confluence *O* and not along the entire path of the wave trains. As the wave trains are of the same frequency, their phase difference at *O* will be throughout the same, but, whatever its magnitude, there will never be complete annulment in all respects. The following cases are worth considering :—

(*a*) When the amplitudes are equal and phases are also alike *i.e.*, the wave trains make the particle *O* pass through the mean position in the direction of the propagation of the waves simultaneously. By the parallelogram law of addition we find that the resultant will be a linear vibration along the direction bisecting the $\angle AOC$.

(*b*) When the amplitudes are equal and phase is 180° apart, we again find that the resultant will be a linear vibration bisecting the $\angle COB$

(*c*) With any other magnitudes of the phase difference the vibrations of *O* may be reduced to two rectangular vibrations of different amplitudes and varying phases The particle will then describe any of the Lissajous' figures of vibration periods in the ratio of 1 : 1 with *O* as centre. Such figures have already been discussed in Chapter II.

We learn that the point of confluence of the wave trains will always have some velocity and displacement, whatever the phase difference. Since pressure variations have no direction but magnitude only, it is possible that condensation due to one wave train may neutralize the rarefaction due to the other. This would happen when the amplitudes are equal and phases opposite. Suppose, at any instant, *O* is passing through its equilibrium position in the direction *OB* under the influence of one wave train, and in the direction *OC* under that of the other, it would therefore be at the centre of the region of condensation due to wave train *AB* and at the centre of the region of rarefaction due to the wave train *CD*. The result will be no variation of pressure at *O* and as regards the effect on the ear, *O* would be a point of silence. If however, the amplitudes are unequal and phases opposite, the intensity of sound at *O* would be minimum.

69. Superposition of Spherical Waves—Suppose *A* and *B* are the centres of two series of waves of the same wavelength and amplitude. In order to obtain two such centres of disturbance, attach wires to both the prongs of an electrically maintained tuning-fork at right angles to them so that they dip lightly below the surface of mercury or any other mobile liquid. When the tuning-fork vibrates, each wire sends out concentric circular waves. These waves consist of alternate crests and troughs. In Fig. 53, crests are indicated by continuous lines and troughs by dotted ones. There are certain points marked × where the crest of one wave is superposed on the crest of the other or the troughs of the two waves come together ; a half period later the crests and troughs interchange places. The result is that at these points the waves from the two sources reinforce each other. Such points lie on curves *aa'*, *cc'*, etc., denoting a series of waves of double the amplitude of the original waves. The differences of the distances of the points on any one of these hyperbolic curves from the sources must be either zero or a whole number of wavelengths.

Let us call the distances from *A* and *B*, d_A and d_B. So in general the waves denoting reinforcement are defined by

$$d_A - d_B = n\lambda, \text{ where } n \text{ is an integer } 0, 1, 2, \text{etc.}$$

At the points marked by dots, however, the crest of a wave from one centre reaches at the same instant as a trough from the other and *vice versa*. If the waves are of equal amplitude, the two together neutralize each other so that the resultant displacement at these points is zero. Such points lie on curves

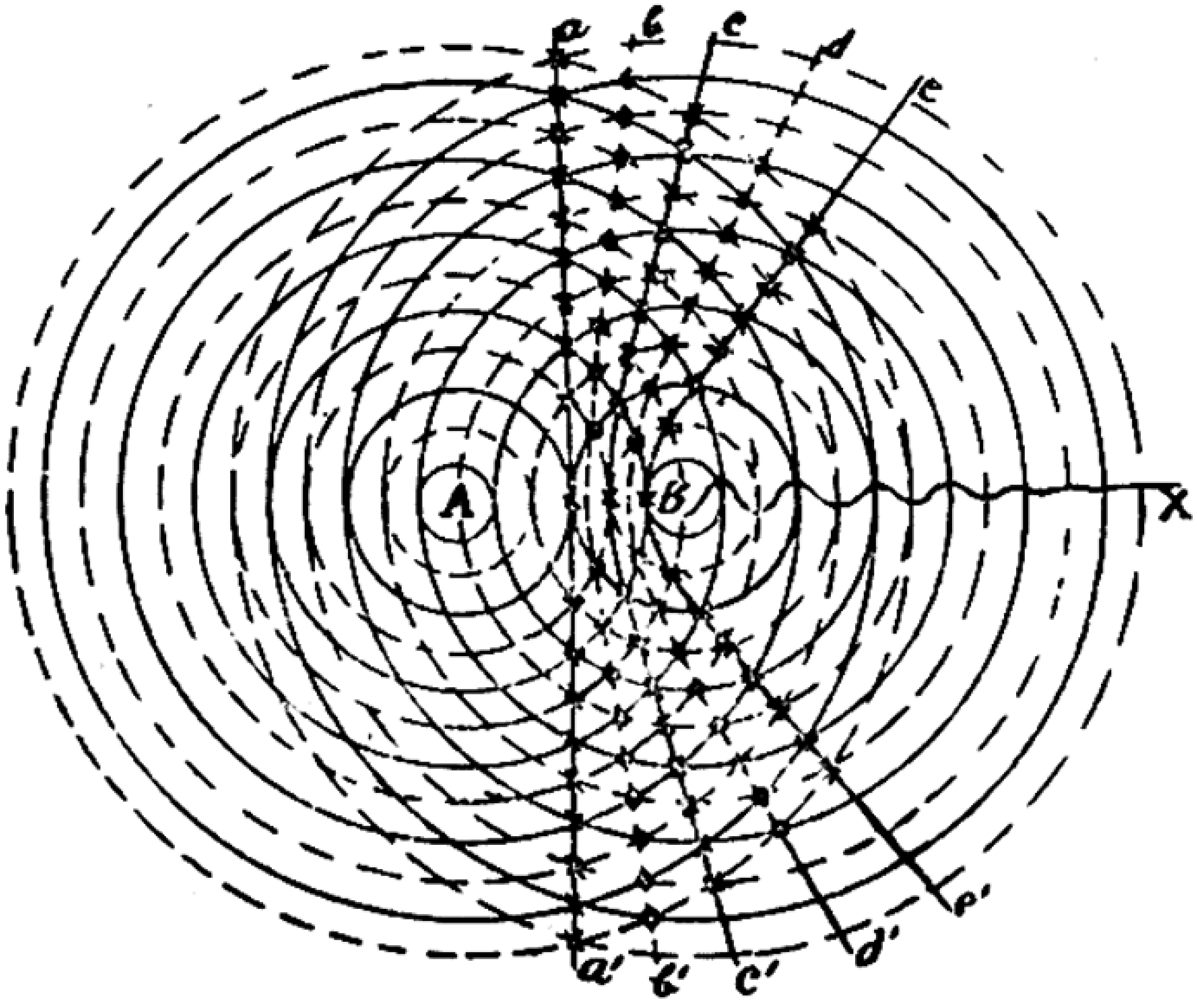

Fig. 53.

bb', *dd'* shown dotted in the figure. The difference of the distances, of any such point from the two sources is an odd multiple of half a wavelength and the curves on which they lie are defined by

$$d_A - d_B = (2n-1)\lambda/2, \text{ where } n=1, 2, 3, \text{ etc.}$$

The two important cases of the phenomenon of interference may be illustrated with reference to Fig. 53.

(*i*) Along the line *BX* we have two wave trains travelling along the same direction and, therefore, giving rise to a single progressive harmonic wave. If, for the sake of simplicity, we assume that the distance *AB* is an exact number of wave lengths, the two trains will be in the same phase and the amplitude of the resultant wave will be the sum of the separate constituents. If, however, the distance *AB* is an odd number of half wavelengths, the constituent wave trains will be in opposite phase and the amplitude at each point will be the difference of those due to the two constituents.

On account of the lateral spreading of the wave trains, the amplitudes of the component wave trains will diminish as we go away from the sources and so does that of the resultant. The intensity of sound will, therefore, fall off as we pass on outwards along *BX* or any of the hyperbolic curves.

(*ii*) **Standing Waves.**—In the region between *A* and *B* we have two wave trains going in opposite directions. Let us confine ourselves to points *a*, *b*, *c*, where the hyperbolic curves referred to above, cut the line *AB* (Fig. 54). Starting at *A*, which is always in vibration we come to the point, *d*, on a

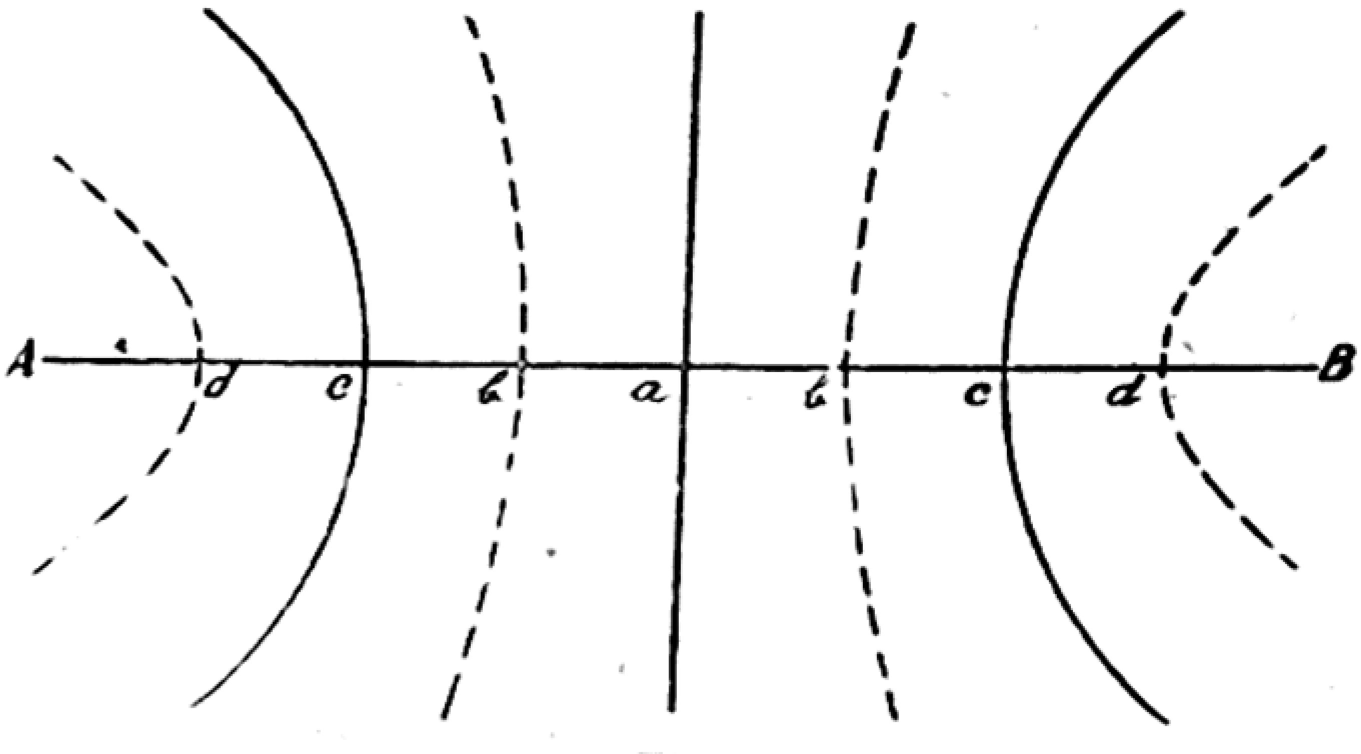

Fig. 54.

curve denoting interference where the disturbance is minimum. At *c*, the vibration is once more a maximum, at *b* a minimum, and at *a* another maximum. Moreover, if two troughs meet at *a*, two crests must be meeting at *c* and so on; so that *a* is always in opposite phase to either point *c*. These are the stationary or standing waves (see Chapter VII). They do not advance but oscillate up and down with alternate segments in opposite phase. Between these, there are points like *b* and *d* which vibrate very little or not at all, but where the pressure changes are maximum. These points are called *Nodes* while the segments between them are called loops and their centres *a*, and *c*, *Antinodes* where there is maximum motion but no change of pressure.

70. Energy is not lost in Interference.—The interference of two sets of waves does not involve loss of energy as might be expected. When there is complete interference at any point, there is no motion of the medium and no energy at that point. The energy of the two interfering wave trains is neither lost nor destroyed, but is transferred to neighbouring points where they meet in the same phase, and the amplitude becomes twice that due to a single wave. At these points the energy of the resultant vibration is four times the energy of the either wave acting independently at such points. There results from the interaction of the two waves systems, a different *distribution of energy* but the total energy remains unchanged.

71. Interference near a Fork.—If we strike a tuning-fork and hold it close to the ear, on turning it round its own stem between the fingers we hear alternations of loudness and softness four times in each rotation. The same phenomenon is observed, if the ear is moved round the fork. This is due to the interference of the sound waves originating at the prongs. Let us see how? When the prongs of the tuning-fork move outwards, they transmit condensations in the direction *AE* and *BF* and as the distance between *A* and *B* increases, simultaneously

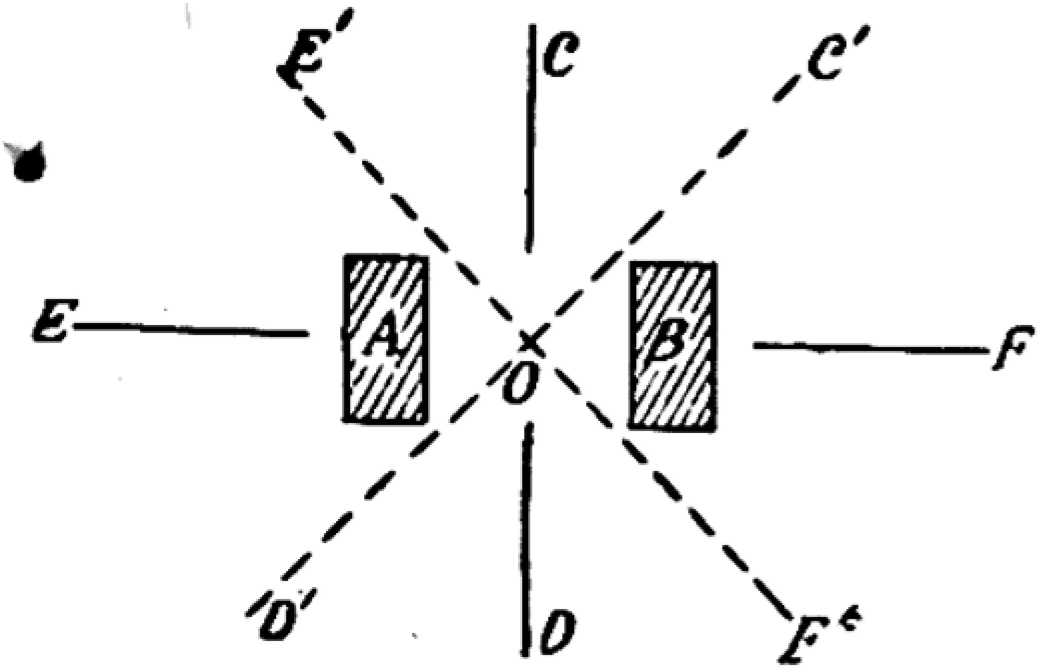

Fig. 55.

a rarefaction is sent in the directions OC and OD. When the prongs move inwards, the reverse happens, consequently we have two sets of waves in opposite phases. In the four directions OE, OF, OC and OD we have maximum amplitude and, therefore, maximum intensity of sound. Along the directions shown by dotted lines the two sets of waves interefere and produce silence, for in these directions condensations from A and B arrive at the same time as rarefactions, therefore they neutralize each other producing zero amplitude and therefore no sound.

72. Quincke's Interference Tube.—In a more direct method of demonstrating interference between two trains of sound waves we make use of the branched tube shown in Fig. 56. A tube, A, divides into two branches S and T which reunite at L. The part T of the tube is made to slide telescopically over the other part so that the length of the part ATL can be varied. A tuning-fork, F, of high pitch say about 1000 p.p.s. is placed over the end A and a rubber tube attached to a side tube at L, is used to convey to the ear the sound which arrives at L *via* the paths ASL and ATL. If the sliding tube is adjusted so that the two paths are of the same length, the waves take the same time to travel by the either path from A to L and therefore the two sets always arrive at L in the same phase. They will, therefore, reinforce each other at the ears of the listener and produce a loud sound. If now T is drawn out so that the path ATL is made longer than the path ASL by $\lambda/2$, the compression of one path arrives at L at the same time as a rarefaction of the other; there will be complete neutralization of the two wave trains and no sound will be heard at L. Drawing T still further out, the sound is again heard till when the path difference amounts to one wavelength, it reaches a maximum. Thus, according as difference of both branches amounts to an odd or even number of half wavelengths, we have destruction or reinforcement of the two wave trains. The experiment besides illustrating interference,

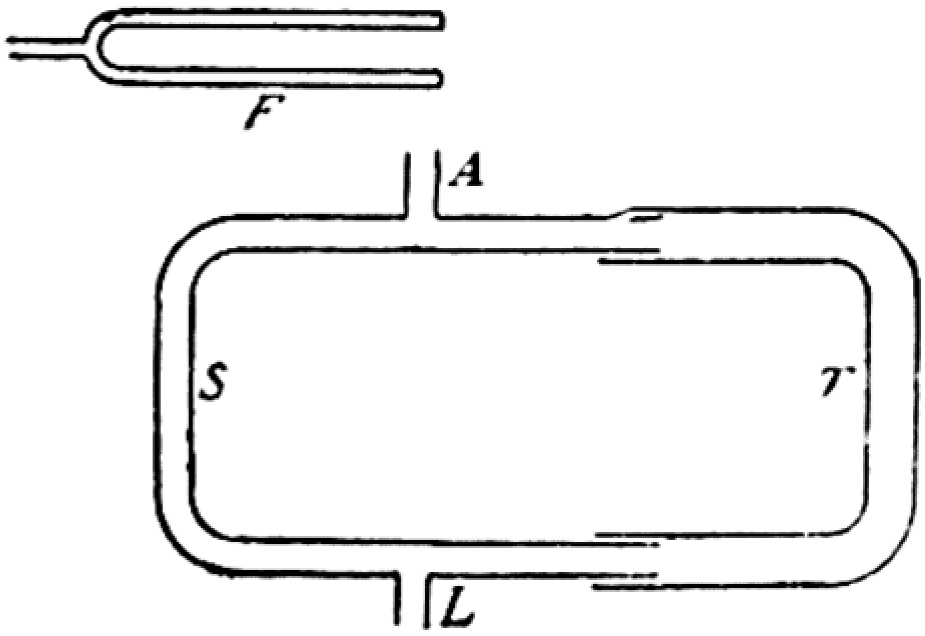

Fig. 56.

provides a good laboratory method of measuring the velocity of sound in a tube. Adjust the apparatus so that the two paths *ASL* and *ATL* are equal. Next draw the sliding tube out till the sound is reduced to a minimum. Measure the difference in the two paths. This is $\lambda/2$, hence λ is known. If n be the frequency of the fork, $v=n\lambda$.

The student might ask what has become of the energy of the sound waves in this case. The answer is that it has been reflected from *L* back to the end *A*. As the two wave trains are in opposite phase at *L*, there are no variations of pressure at this point. We shall find later in the case of open organ pipes that it is on account of this condition of no variation of pressure at the open end that reflection takes place with a reversal of phase. Here *L* is acting similar to the open end and reflects the sound energy back to *A*.

73. Interference by Seebeck's Tube.—The phenomenon of interference may be easily demonstrated by means of a tube *AB* stoppered at the end *B* by a movable piston and with a small side tube, *L*, near the end *A*. A rubber tube attached to *L* conveys the sound to the ear. A vibrating tuning-fork is placed close to the end *A*. On properly adjusting the piston at the end *B*, when $2BC=\frac{\lambda}{2}$ or any odd multiple of $\lambda/2$, we hear no sound at *A*. This is due to the super-position of the two wave trains, one going direct from *F* to *C* and the other which goes to the end *B*, gets reflected there and again comes to *C*. If $BC=\lambda/4$, the two wave trains will be in opposite phase and will neutralize each other. The method may be used to find the wavelength and consequently the velocity of sound if the frequency of *F* is known. The method has the advantage that the volume of the air involved being smaller, its temperature and humidity can be kept constant. The arrangement can also be adapted to the determination of the velocities of sound in gases other than the air.

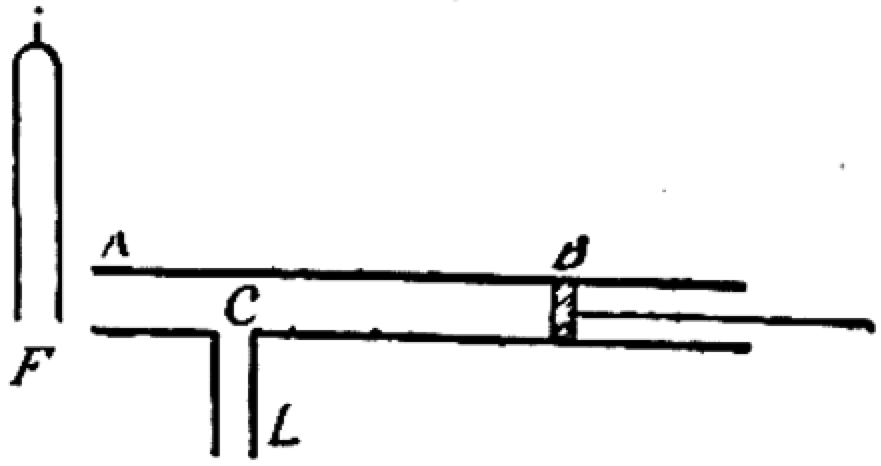

Fig. 57.

74. Fog Signalling and Zones of Silence.—During recent years large power-sirens have been used in foggy weather for signalling from light houses or light ships for the safety of ships at sea. It has been noticed that during foggy weather light cannot penetrate to long distances but if a powerful siren is blown steadily, the sound can be heard at much longer distances. Tyndall and others have demonstrated that the homogeneous medium of the fog instead of being an obstruction is more transparent to sound waves and particularly beneficial The fading of sound, as we shall see later, is due to the discontinuities in the composition in the atmosphere arising from temperature differences, convection currents or humidity variations etc. Tyndall while experimenting on fog signals noticed that sometimes a ship finds itself in a zone of silence, a signal being audible, for example from, 0 to 2 or 4 to 6 miles but quite inaudible from 2 to 4 miles. Such *Zones of Silence* are due to the interference of sound waves.

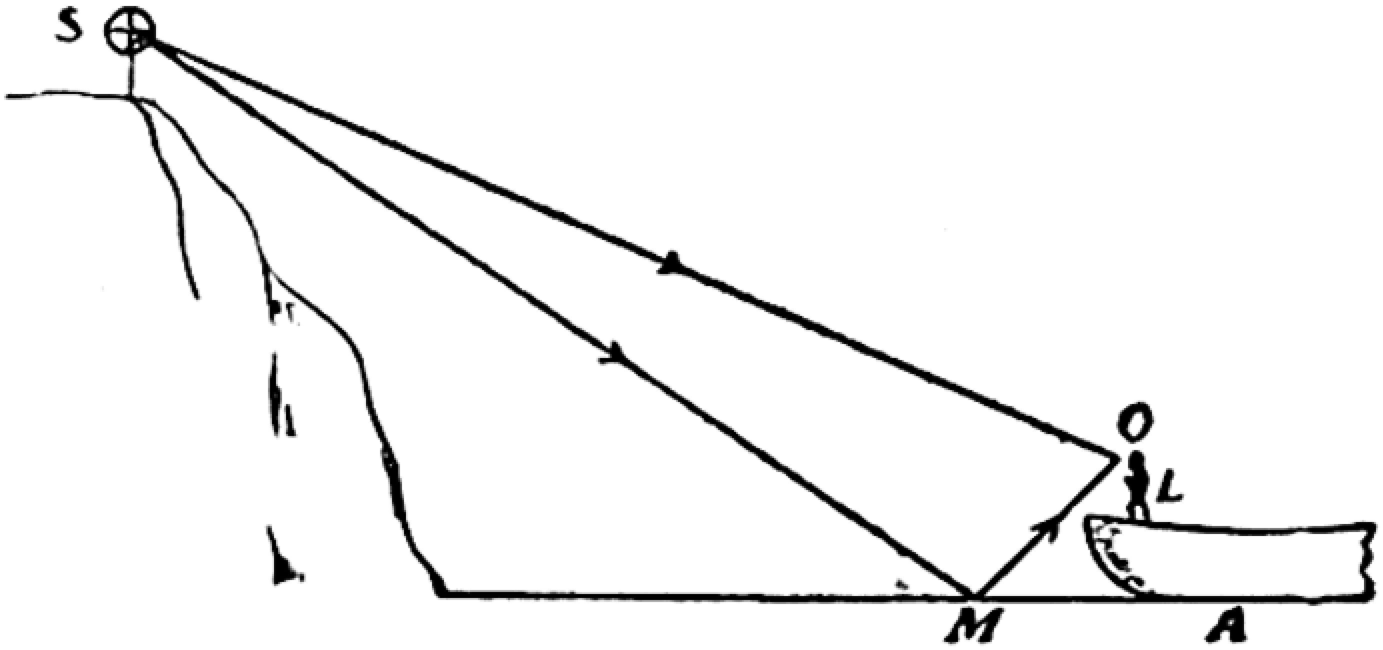

Fig. 58.

Suppose a siren *S* is installed on a light-house and a ship *A* is approaching towards it. Two wave trains reach the listener *L*, one along the direct path *SO* and the other *via* the path *SMO* after reflection from the surface of water. If these paths differ by $\lambda/2$ or an odd multiple of $\lambda/2$, the two wave trains annul each other causing silence. If the ship comes nearer, the difference between the two paths alters and the sound becomes audible again, reaching a maximum when the path difference between the two wave trains becomes an even number of half wavelengths.

75. Superposition of Harmonic Waves of nearly the same Frequency (Beats).—If two wave trains of nearly the same frequency are travelling along the same line in the same direction, the condition at any fixed point is continually changing and the resultant displacement alternately waxes and wanes in amplitude on account of the wave trains getting in and out of step at regular intervals. If at a given point they are in the same phase, the condensation or rarefaction of one falls on the condensation or rarefaction of the other, the result is that there is maximum disturbance. Now, as the wavelength of one train is shorter than that of the other, they gradually get out of step, the wave train with shorter, wavelength lagging behind the other until after a certain number of vibrations depending upon their relative frequencies it has fallen behind by half a wavelength so that the compression of one arrives at the ear at the same time as the rarefaction of the other producing a minimum of sound. Some time later, the shorter has lost one entire vibration and is again in phase with the other, and the two sounds reinforce as in the beginning. Thus we hear a single note which rises and falls in intensity. These periodic *alternations* of sound between a maximum and minimum loudness are called '**Beats**'. They may be graphically depicted as in Fig. 59, where the thin continuous curve and the dotted curve represent the two wave trains whose frequencies are in the ratio of 5 : 4, *i.e.*, every five waves of one train exactly superpose over four waves of the other. In the neighbourhood of *A* they are in step and assist each other. At *B* they are half a wave length apart, *i.e.*, completely out of step and thus annul each other. At *C* they again agree in phase and, therefore, reinforce each other. The resultant wave train is indicated by the thick continuous curve showing clearly the variations in amplitude. From this we see that during the interval between *A* and *C* when one wave train gains one vibration over the other, one beat is produced. If the frequencies of the wave trains are n_1 and n_2, then one gains $n_1 - n_2$ vibrations over the other in one second. Each time it gains one complete vibration, the two sounds are in the

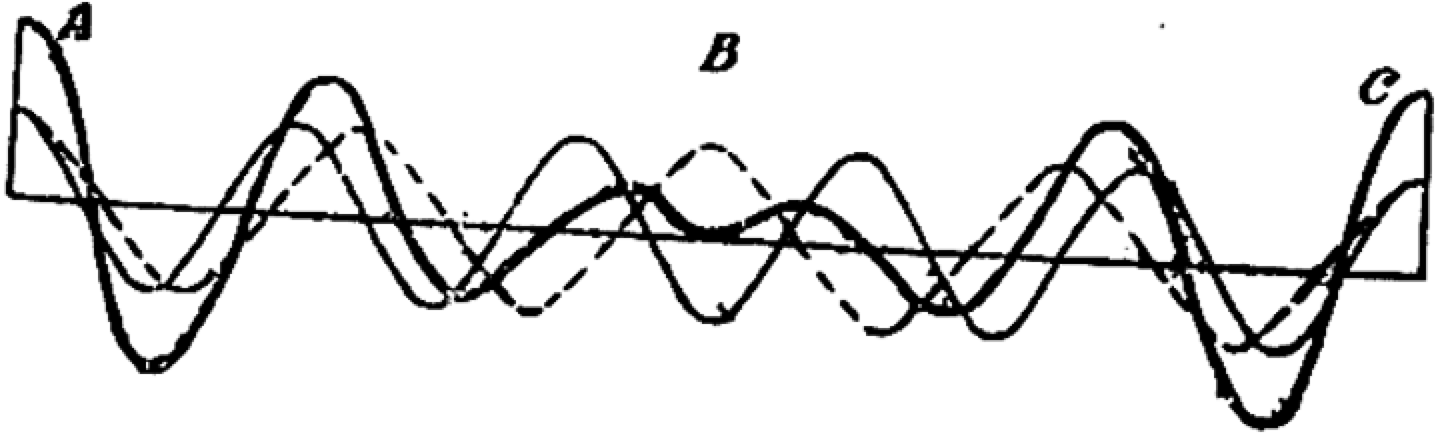

Fig. 59.

same phase and reinforce each other again ; there are n_1-n_2 maxima/sec. and a similar number of minima/sec. *i.e*, the frequency of the beats is equal to the difference in the frequencies of the two notes. These beats play a very important part in acoustical determinations and in the theory of *Music*.

Example. *The points of the prongs of a tuning fork A originally in unison with a fork B of frequency 512 are filed, and the forks produce 5 beats per sec. when sounded together. What is the pitch of A after filing ?*

Originally A and B are in unison, therefore the frequency of A is also 512. The filing of prongs results in a reduction of the mass and hence a rise in the pitch of the fork. The number of beats per second denotes the difference in the frequencies of the fork. Hence the pitch of A after filing $=512+5=$**517**.

76. Mathematical Treatment of Beats.—Let n_1 and n_2 be the frequencies of the two sources. For simplicity, let us suppose that the two sources are in phase when $t=0$, and their respective amplitudes are equal. The corresponding vibrations are represented by

$$y_1=a \sin 2\pi\, n_1 t \qquad \ldots \qquad \ldots \qquad \ldots \qquad (1)$$

and

$$y_2=a \sin 2\pi\, n_2 t \qquad \ldots \qquad \ldots \qquad \ldots \qquad (2)$$

The resultant displacement is given by

$$y=y_1+y_2=a \sin 2\pi n_1 t+a \sin 2\pi n_2 t.$$

Applying the well-known theorem in Trigonometry

$$\sin A+\sin B=2 \sin \frac{A+B}{2} \cos \frac{A-B}{2},$$

we get

$$y=2a \sin 2\pi \left(\frac{n_1+n_2}{2}\right)t \cos 2\pi \left(\frac{n_1-n_2}{2}\right)t$$

$$=2a \cos 2\pi \left(\frac{n_1-n_2}{2}\right)t \sin 2\pi \left(\frac{n_1+n_2}{2}\right)t.$$

This represents a periodic vibration of amplitude

$$2a \cos 2\pi \left(\frac{n_1-n_2}{2}\right)t \text{ with a frequency } \frac{n_1+n_2}{2}.$$

Let $2a \cos 2\pi \left(\frac{n_1-n_2}{2}\right) t$, be A, then

$$y=A \sin 2\pi \left(\frac{n_1+n_2}{2}\right) t.$$

The amplitude is not constant but slowly changes with time, the maximum value being nearly $2a$ and the minimum value 0. Let us calculate the interval between two successive maxima or minima. For a maximum

$$\cos 2\pi \left(\frac{n_1-n_2}{2}\right)t=\pm 1$$

$$\therefore \ \pi\,(n_1-n_2)\,t=k\,\pi, \text{ where } k=0,\ 1,\ 2,\ 3, \text{ etc.}$$

or

$$t=\frac{k}{n_1-n_2}=0,\ \frac{1}{n_1-n_2},\ \frac{2}{n_1-n_2},\ \frac{3}{n_1-n_2} \text{ and so on.}$$

$\therefore$ Time interval between two maxima $=\dfrac{1}{n_1-n_2}$.

or the frequency of maxima $=n_1-n_2$

For a minimum value

$$\pi\,(n_1-n_2)t=k\pi+\pi/2, \text{ where } k=0,\ 1,\ 2,\ 3, \text{ etc.}$$

$$\therefore \ t=\frac{k}{n_1-n_2}+\frac{1}{2\,(n_1-n_2)}$$

$$=\frac{1}{2(n_1-n_2)},\ \frac{3}{2(n_1-n_2)},\ \frac{5}{2(n_1-n_2)} \text{ and so on.}$$

so that the minima are regularly timed in between the maxima and the interval between two minima is also $\dfrac{1}{n_1-n_2}$ or their frequency $=n_1 \quad n_2$.

Since the interval between successive maxima and that between successive minima is $\dfrac{1}{n_1-n_2}$ sec., the frequency of the beats $=(n_1-n_2)$/sec.

Hence we may say that the resultant of two simple Harmonic vibrations of nearly equal frequencies may be regarded approximately a simple harmonic vibration whose frequency is the arithmetical mean of the component frequencies, and whose amplitude is alternately the sum and the difference of the component amplitudes, the variation having a frequency equal to the difference of the component frequencies.

77. Stationary Interference and Beats.—Earlier in this chapter we saw that two sources of exactly the same pitch produced interference pattern which remained stationary, that is the positions of maximum and minimum disturbances remained at the same place while the sounds were maintained. There was no shifting of the positions of maxima and minima with time. At a given place we had either no sound or maximum sound whereas in the case of beats where we have two sources of slightly different frequencies, a periodic waxing and waning of sound is perceived. These maxima and minima travel onwards and can be heard at all places at different times. In other words, we have with sounds of equal pitches throughout time, places of maximum and minimum effects; with sounds of slightly different pitches we have throughout space, times of maximum and minimum effects. The former is a case of '**permanent interference**' and the latter being always meant by the term '**beats**'. Referring to Fig. 54, we have noticed that the points lying on the '*hyperbolas*' are either permanently in a region of silence or in a region of maximum sound. Such stationary interference effects are easily observable in the case of light waves.

To sum up, for permanent or stationary interference pattern between two sources it is necessary and sufficient that the two sources must be of exactly the same pitch and that they must have a constant phase relationship at all times, whereas for beats or which is the same thing that the interference pattern should be moving, the two sources must be of slightly different frequencies. In this case a procession of 'nodes' and 'loops' past the ear produces a sound of rising and falling amplitude.

78. Combination Tones.—So far we have been discussing composition of wave trains with a small amplitude and in which the restoring force is proportional to the displacement so that the ordinary vector law of addition holds good. But when two *primaries* or *pure tones* of frequencies m and n are loud and sustained, tones of frequencies other than the primaries and their harmonics are produced, they are called **Combinational Tones**, for their existence depends upon both. These tones arise from the failure of the 'Principle of Superposition'. The most important and the easily recognizable of these tones is the *First Difference* tone of frequency $n—m$. It may be easily heard when two organ pipes or flageolets are strongly blown together close to the listener, especially if the ear is previously prepared for the pitch it is to hear. This difference tone may produce with the first primary a second difference tone of frequency $n—2m$ and a third difference tone of frequency $n—3m$ and so on. There are also vibrations of frequencies equal to the sum of the original vibrations called the *Summation Tones*, the first summation tone of frequency $m+n$, second summation tone of frequency $2m+n$ or $m+2n$ and so on. Besides there are self-combination tones of frequencies $2m$ and $2n$. Theoretically, a large number of these tones are possible but the First Difference tone is easy to hear, as it is pretty loud, while the First summation tone is weak, and difficult to hear. The rest are too faint to be audible. These tones were first observed by an Italian violinist Tartini and therefore, are sometimes called **'Tartini Tones'.**

79. **Origin of Combination Tones.**—Much controversy has been going on regarding the *origin* of these tones. The problem was to decide whether the tones have objective existence, *i. e.*, there is present in the air a vibration of the frequency equal to that of the combination tone or it is merely a subjective effect, *i.e.*, an interpretation of the brain only. According to Koenig and several others, the first difference tone is merely the **'Beat Tone'** The fact that the frequency of this tone is equal to the frequency of the beats, led them to the supposition that the beats themselves merge into a musical note. But this view in the first place leaves the summational tones entirely unexplained and secondly it cannot be reconciled with the Ohm's experimental law, according to which the only tones, which the ear can hear, correspond to the pendular vibrations of the ear. To understand it clearly let us see how the ear perceives the sensation of a tone A tone is ordinarily due to a succession of waves, such that the air in the first half of each wave is displaced forwards and the pressure is in excess of the normal, while in the second half the air is displaced backwards and the pressure is in defect. These excesses and defects produce the sensation of a tone. But in each half of a beat the mean displacement of the air is zero and the mean pressure normal. There is no reason to suppose that mere loudness produces a pressure on the drum of the ear inwards while a faintness allows a relaxation and return outwards, suppositions which would appear necessary if beats are to produce in the ear a tone of their own frequency. But as stated, such suppositions compel us to abandon Ohm's law.

An entirely different explanation of this and other combination tones requiring no such supposition with regard to beats has been given by Helmholtz. Mathematical analysis shows that with an unsymmetrical membrane such as the drum of the ear or a limited air cavity subjected to vibrations, the restoring force is not proportional to the displacement. When two harmonic forces of different frequencies act on the membrane or the cavity, besides producing self-combination tones they produce vibrations having frequencies equal to the difference or the sum of the components.

Helmholtz experimentally proved that under certain similar conditions the combinational tones existed objectively, independently of the ear. He showed that the condition for the generation of the combinational tones was that the same mass of air should be agitated violently by two simple tones simultaneously This occurs in the case of harmonium or his 'double siren' if the tones are produced by two series of holes blown upon from the same wind chest. He showed their objective existence by the aid of his resonators and sympathetic resonance of membranes tuned in unison with them. On experimental basis Helmholtz asserts that when the places in which the two tones are struck, are entirely separated and have no mechanical connection as, for example from two separate wind instruments, the reinforcement of combinational tones by resonators is small and dubious. Here then there does not exist in the air any clearly sensible pendular vibration corresponding to the combinational tone and we must conclude that such tones which are often powerfully audible are really produced in the ear itself.

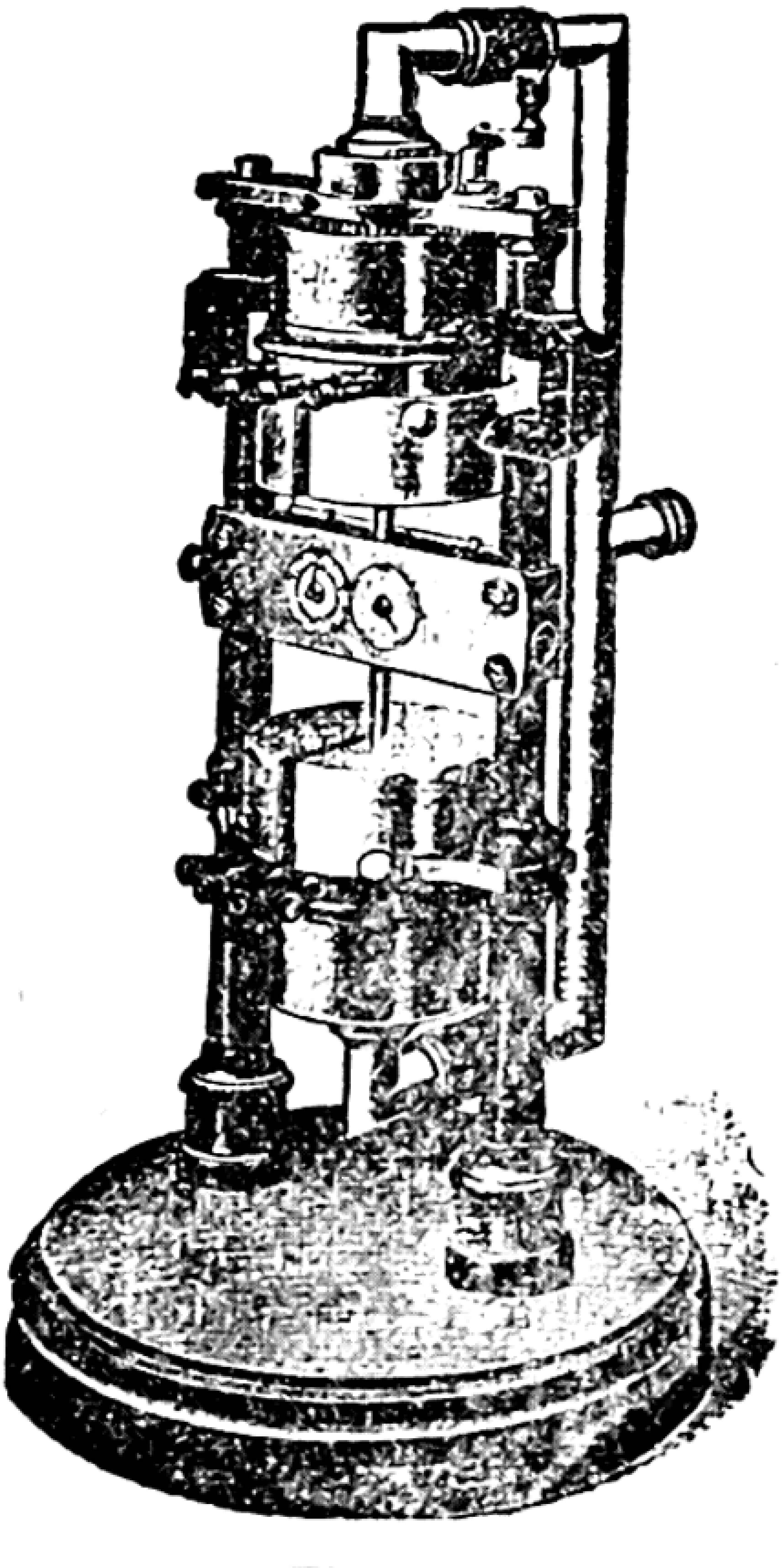

Fig. 60.

80. Rucker and Edser Expt.—Profs. A. W. Rucker and E. Edser performed experiments showing conclusively the objective reality of the difference and summation tones supporting the conclusions of Helmholtz.

To produce the generating

tones Helmholtz' double siren (Fig. 60), was employed. It consists of two ordinary sirens mounted facing each other on the same axis. Their revolving discs are carried at the ends of a common spindle which operates the counter. Each disc and the corresponding wind chest are pierced with 4 sets of holes arranged in circles 10, 12, 15, and 18 in the lower and 8, 9, 10 and 12, in the upper. One or more of these circles can be put in use by means of stops fitting into the side of the instrument and the frequencies of the tones produced by them can be adjusted at will. Both sirens are blown from the same air chest.

The resonator to respond to the combinational tone was a tuning-fork of frequency 64. The siren was adjusted to give a different tone of frequency 64 and placed in front of a Koenig resonator *R*. The sound waves were picked up by a large cone *C* and concentrated on a prong of the tuning fork *via* the narrow apex. This prong carried a small strip of wood, *W*, to compensate for the mirror, *m*, of an equal mass attached to the other prong. This formed one of the mirrors of the Michelson and Morley's interferometer* which was used to detect the motion of the prong. Michelson's interference bands were produced in the telescope (*see* any textbook on Physical Optics). A movement of the prong of the fork *F* by quarter of a wavelength *i.e.*, $\frac{1}{6500}$ mm. would alter the path of one of the interfering beams by half wavelength the bright bands of the interference pattern would occupy the positions of the dark bands and *vice versa*. So even a vibratory motion of this amplitude would continually interchange bright and dark bands causing them to disappear. It may be noted that none except the

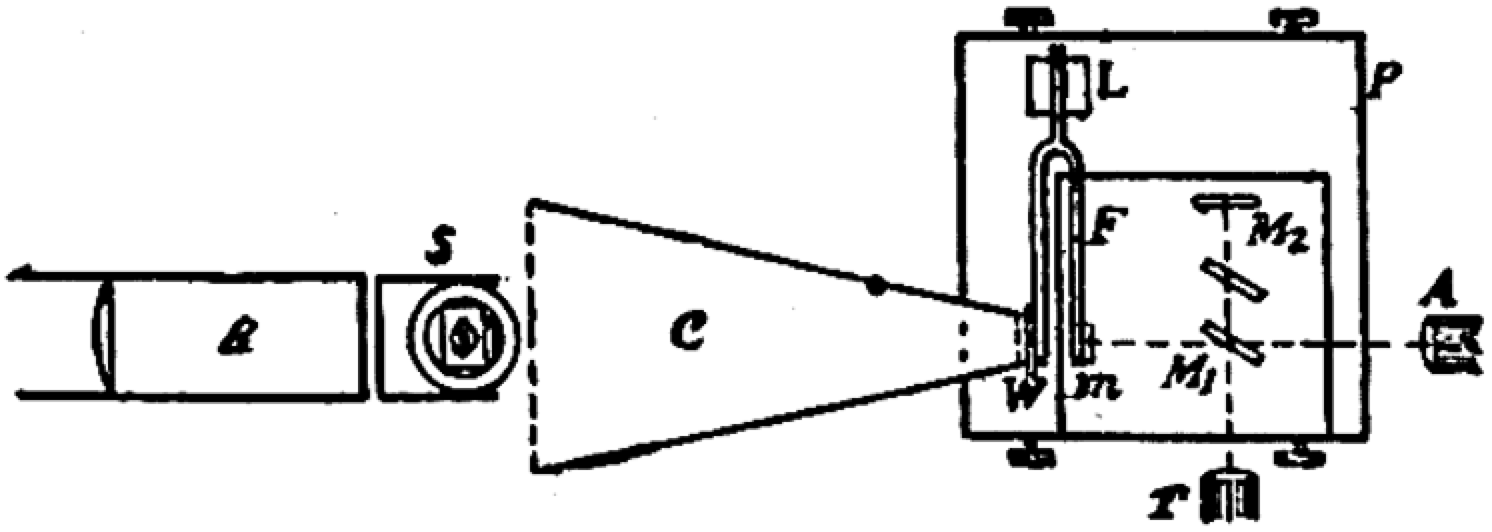

Fig. 61.

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*As the tuning-fork is relatively difficult to excite by resonance, it was necessary to use a delicate method of detecting its motion, if present.

pendular vibrations of the same frequency as the fork, would excite it.

To make the whole arrangement steady and to provide against accidental disturbances, the stem of the tuning fork was embedded in a lead block, *L*, and both the block and the interferometer were mounted on a heavy stone slab, *P*.

For small intervals between the two primary tones the difference tone and the Koenig's beat tone would agree in pitch, but for large intervals the difference tone would still be equal to the difference between the pitches of the two primaries while the beat tone would be the difference between the pitch of the upper note and the octave of the lower. In all the experiments the pitch of the different tone was maintained at 64, while that of the beat tone was changed from 64 to 12·8. In all cases the objectivity of the difference tone and not that of the beat tone was proved. The siren was next adjusted to give a summation tone of 64 and in each case the interference bands were disturbed. The results of the experiments are given in the table below.

Table

No. of holes in siren	Pitches of tones produced	Pitches of combination tones detected by the fork		Pitch of Koenig's beat note
10 and 8	35·5 and 28·4	64	Summation Tones	
12 and 9	36·57 and 27·43	64		
12 and 10	384 and 320	64	Differential Tones	64
15 and 12	320 and 256	64		64
15 and 9	160 and 96	64		32
18 and 8	115·2 and 51·2	64		12·8

These results were later confirmed by Prof. C. V. Boys using a specially sensitive mirror resonator. Helmholtz was therefore correct in stating that the siren produces two objective tones of frequencies equal to the difference and the sum of the frequencies of the original tones and Koenig's explanation is therefore untenable.

80a. Sir W.H. Bragg* has recently given attention to the conditions which must be satisfied in order that the combination tones may have objective existence. The underlying principle is very simple. "When vibrations of two different frequencies are imposed upon a medium as, for example, when two tuning forks are sounded, the chord consists only of the imposed frequencies and there are no combination tones in the air. But if during the production of sound the amplitude of one source depends at every instant upon the amplitude of the other, the combination tones appear. In the first case the disturbance in the medium may be represented by

$$a \sin 2\pi\, mt + b \sin 2\pi\, nt.$$

In the second case, there is sure to be a term of the form :

$$2c. \sin 2\pi\, mt \times \sin 2\pi\, nt,$$

which is equal to

$$c. \cos 2\pi\,(m-n)t - c. \cos 2\pi\,(m+n)t,$$

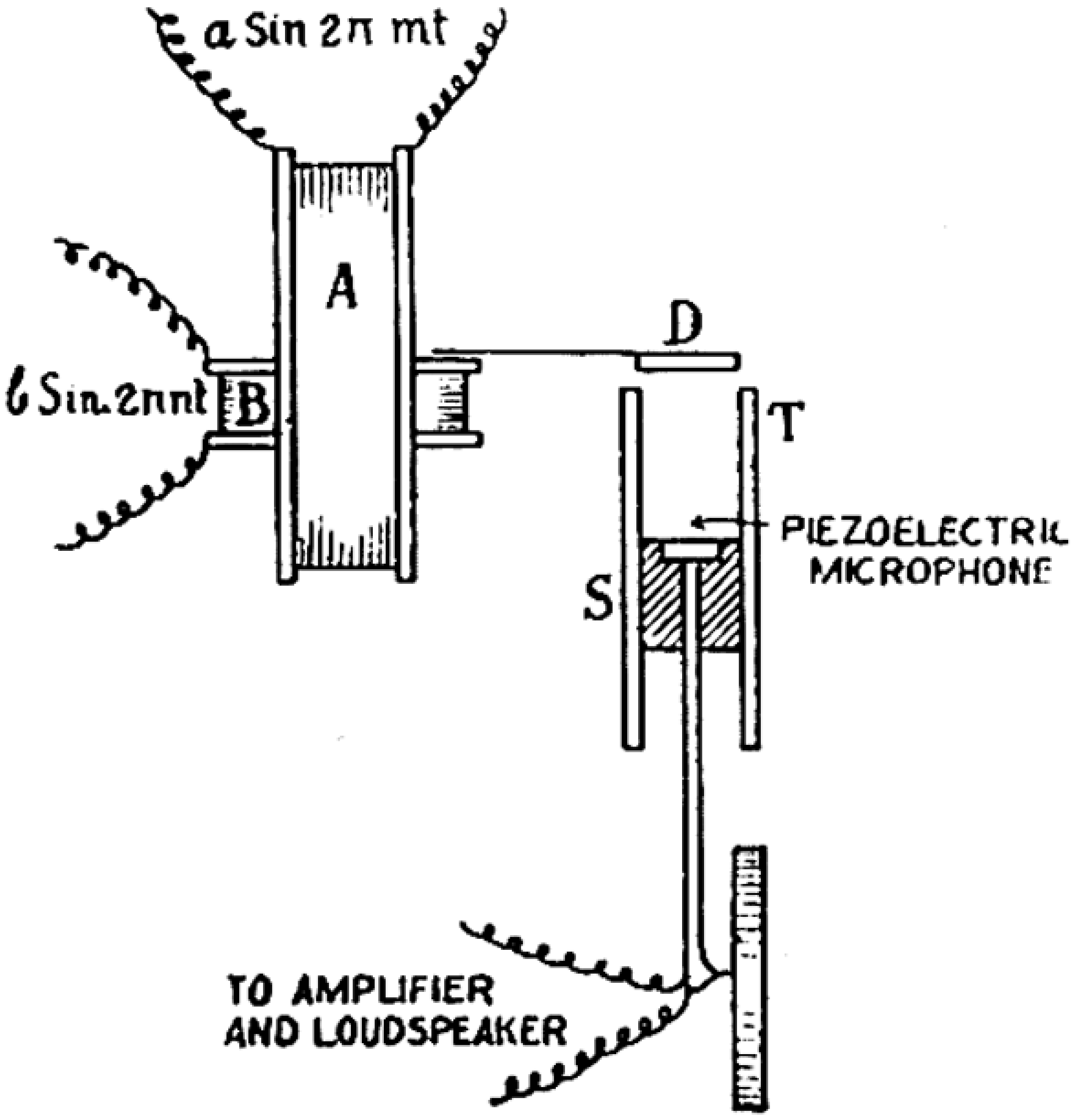

Fig. 61 (*a*)

and the combinational tones have an objective existence.

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*Discourse delivered at the Royal Institution on Nov. 11—Nature. Vol. 143, p. 542 (1939).

This conclusion is readily illustrated by experiment. In Fig. 61 (*a*), *A* represents a coil seen edgeways, through which either a steady current *a*, or an alternating current $a \sin 2\pi\, mt$ can be made to run. B is a smaller coil at right angles to A; the centres of the two coils coincide. Through the second coil, either a steady current *b* or an alternating current $b \sin 2\pi\, nt$ may be made to run. The second coil can turn about a vertical axis which is perpendicular to the plane of the diagram. *T* is a cylindrical tube in which a slider *S* is mounted; the slider contains a piezo-electric microphone connected to a loud speaker. When *B* vibrates about its axis, which it must do if there are currents in the coils and one or both are alternating, the disc *D* flutters before the mouth of the tube. The tube, as a resonator, can be tuned by moving the slider.

In an actual experiment, the current of frequency $m=250$ was taken from a high-frequency generator, that of frequency $n=50$ was drawn from the lighting circuit. When the current in *A* was alternating and that in *B* was steady, the resonator responded to the 250 note. When the current in *B* was alternating, the response to the 250 note became much weaker but the resonator could be made to respond strongly to either 200 or 300. The torque on the *B* coil was proportional to

$$\sin 2\pi .\ 250 .\ t \times \sin 2\pi .\ 50 .\ t,$$

i.e.

$$\text{to } \cos 2\pi .\ 200t - \cos 2\pi .\ 300 . t.$$

The way in which a detecting apparatus can generate accessory tones may be illustrated by the following example [Fig. 61 (*b*)].

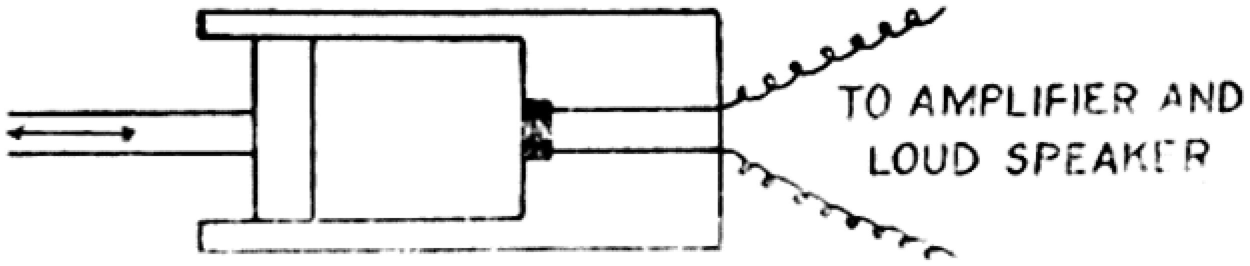

Fig. 61 (*b*).

Suppose that a piezo-electric cell is inserted at the end of a short cylinder, and that a piston executes a harmonic motion, so that the volume of the air may be put equal to $v=v_0+v^1 \sin 2\pi\, mt$.

Then the pressure *p*, supposing the motion to be isothermal (though it makes no difference to the general result if it be taken as adiabatic) is given by $pv=c$, a constant.

$$\therefore\ p=\frac{c}{v_0+v^1\ \sin 2\pi\ mt}$$
$$=\frac{c}{v_0}\left\{1+\frac{v^1\sin 2\pi\ mt}{v_0}\right\}^{-1};$$

and this when expanded shows overtones, although the piston itself moves in a simple harmonic motion

If there are two pistons operating on the one chamber,

$$p=\frac{c}{v_0}\left\{1+\frac{v^1\sin 2\pi\ mt}{v_0}+\frac{v^1\sin 2\pi\ nt}{v_0}\right\}^{-1};$$

and this when expanded shows a system of combination tones.

The variations of pressure can be converted into a complex sound in which the tones appear."

81. Beats caused by Difference Tones.—Sometimes we find that even two notes which differ too much in pitch to beat directly, when sounded together, give rise to beats. This can be easily demonstrated by taking two tuning-forks nearly an octave apart mounted on resonance boxes. If they are an exact octave apart, the frequency of one of them may be slightly altered by attaching a small quantity of wax to its prongs On sounding the two together beats will be heard. Let us see how the beats are explained ? If m and n be their pitches such that $n=2m$ nearly, $n-m=m$ nearly and beats can be heard between this difference tone and the original tone m. Suppose one of the forks has frequency 256 and the other 510, the first difference tone will have a pitch $510-256=254$, and this will give two beats per sec. with the note of the lower fork. *These beats can be made use of in tuning the two forks to an exact octave.* If the forks are adjusted until the beats disappear, the pitches must be exactly in the ratio of 2 : 1. Also if $m=256$, $n=764$, the second difference tone $n-2m=764-512=252$. This gives 4 beats per sec. with the lower fork.

The difference tones play a very important part in the concord of musical intervals and we shall deal with this point in Chapter XIII.

CHAPTER VII

STATIONARY WAVES

82. Stationary Waves.—In the preceding Chapter we have mainly dealt with the composition of two wave trains travelling in the same direction. There is another important type of wave phenomenon briefly referred to in Art. 69 (*ii*), in which two similar and equal wave trains travelling in opposite directions in a straight line with equal velocities superpose over each other and give rise to what are called **stationary** or **standing waves**.

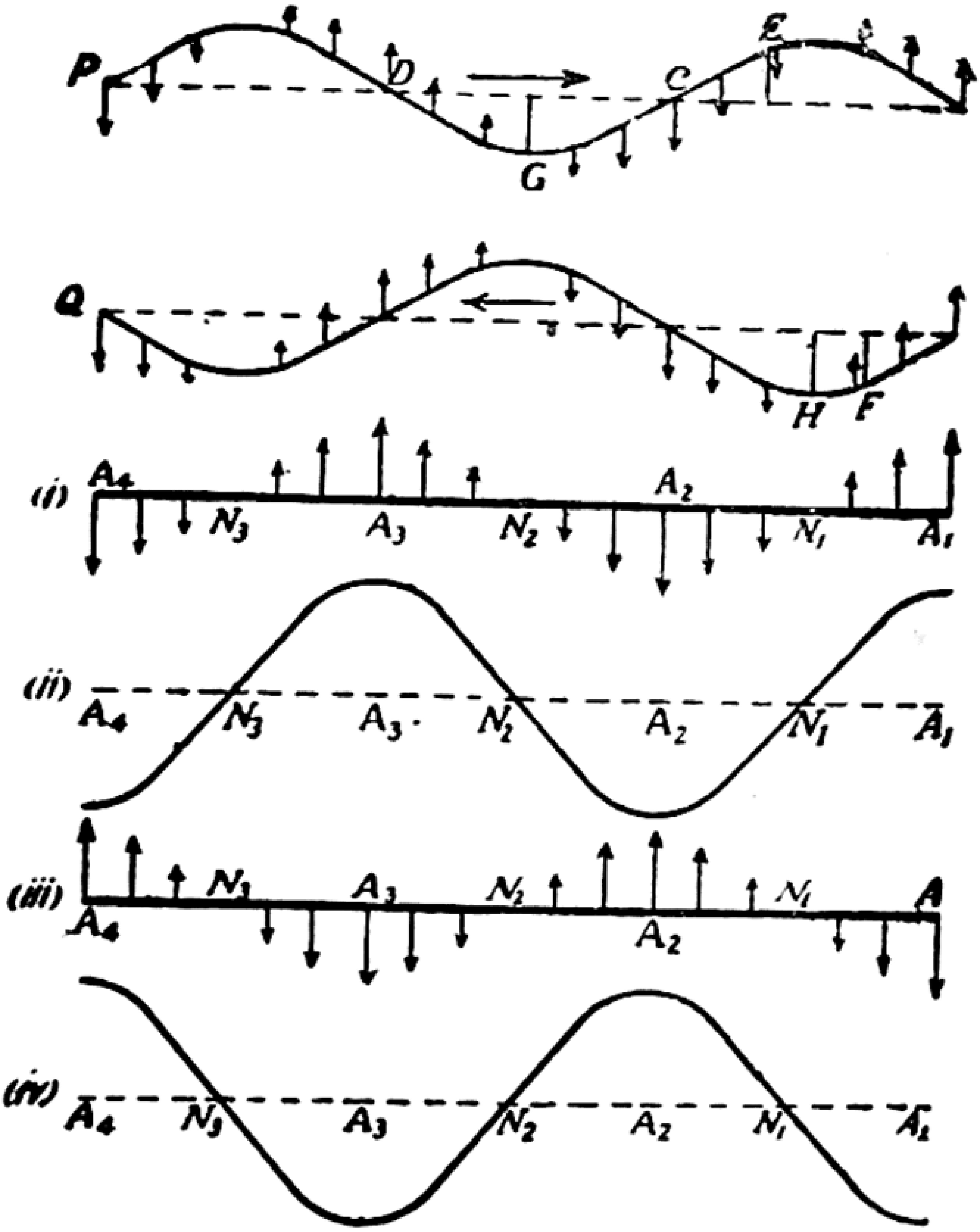

Fig. 62.

The method of production of these standing waves can be understood by the aid of Fig. 62. *P* and *Q* are the two wave trains travelling in the directions indicated by the horizontal arrows, the transverse arrows denoting the velocities of the particles in magnitude and direction. Such two waves trains, as you will learn later, represent at a certain instant the incident and the reflected waves respectively in an open organ pipe or in a string free at the other end and by superposition give rise to stationary vibrations.

(*i*) At the instant when the two component waves have the positions represented in the figure, the resultant wave will have no displacement at any point along the line of transmission [Fig. 62 (*ii*)]. The crest of one wave coincides with the trough of the other at every point. The displacements due to the component waves being equal and opposite, the resultant curve is a horizontal straight line. Although all the points are passing through their equilibrium position, yet the points marked N_1, N_2, N_3 etc., which are simultaneously at the crests and troughs of the component waves have no resultant velocity; whereas the points like *C* and *D* in the component wave train which are also passing through their equilibrium position have velocities in the same direction. At such points marked A_1, A_2, A_3. etc., the velocity of resultant wave train is equal to twice the velocity of the points *C* and *D*.

As the component wave trains advance in their respective directions†, the point *E* in the wave train *P* will arrive at N_1 simultaneously with the point *F* of the wave train *Q*. The resultant displacement is zero as the component displacements are equal and opposite. Since the transverse particle velocities are also in the opposite direction the velocity at N_1 will also be zero. In other words the velocities and displacements of the point N_1, due to the wave trains *P* and *Q* will always *differ in phase by* π. Thus it appears that points N_1, N_2, N_3, will remain permanently stationary. Those points are called **Nodes**.

(*ii*) After quarter time period each wave has travelled through a distance equal to quarter wave length* the troughs

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†It is to be assumed in this case that the wave train *Q* extends further towards the right and is being drawn towards the left with the wave velocity.

*The corresponding stages of the component wave trains are not shown in the figure for this and the following cases. The student is expected to draw them for himself for a clear understanding of the formation of the resultant wave.

G and H will simultaneously arrive above A_2, so the resultant displacement at this point will be at its maximum value being equal to twice the value of either of the component waves. All the particles are at the extreme ends of their swings and momentarily at rest. The resultant curve takes the form shown in Fig. 62 (*ii*).

(*iii*) After half time period from the beginning, when each wave has travelled through half a wavelength, the conditions with respect to displacements and velocities of the particles in the resultant wave are represented in Fig. 62 (*ii*). All the particles are again passing through their normal positions. The velocities at the point A_1, A_2, A_3, etc., are again maximum but in the reversed directions.

(*iv*) After another quarter period the resultant curve takes the shape shown in Fig. 62 (*iv*). As a result of the superposition of the component waves all the particles are at rest again and the displacement of each is maximum but in a direction opposite to the corresponding segment in Fig. 62 (*ii*).

After moving through another quarter wavelength, the component wave trains will give rise to a train identical with that in the beginning so that the resultant displacement and velocity will be as indicated in Fig. 62 (*i*). It has been noticed that the points A_1, A_2, A_3, etc. suffer maximum displacement. Such points are called **Antinodes.**

83. Characteristics of Stationary Waves.—(1) Every particle excepting nodes oscillates in S. H. motion with the same period as the component waves but the amplitude varies continuously along the line assuming +ve, zero and −ve value.

(2) The medium splits up into segments, all the particles in the same segment being simultaneously in the same phase but in opposite phase with the particles in the consecutive segment.

(3) Condensations (crests) or rarefactions (troughs) do not advance as they do in a progressive wave and consequently there is no advance of wave energy in either direction *i.e.*, the transmission of energy across any plane is zero. These condensations and rarefactions appear and disappear at the same place to be succeeded by opposite conditions every half period.

(4) **Nodes** are the points where displacement and velocity

are zero, but strain *i. e.*, change in density or pressure, is maximum. Midway between them are **antinodes** which are alternately points of maximum displacement and maximum velocity but zero strain.

(5) The general appearance of the wave can be represented by a sine curve but twice in each period all the particles shrink to a straight line.

(6) All the particles are having their maximum and minimum velocities at the same time, the velocity being maximum when they are passing through their mean positions and zero when they are at the extremes of their swings. On either side of an antinode, the velocity diminishes proportionately being reduced to zero at the nodes.

(7) The wavelength and the periodic time in which the stationary wave goes through all its changes are the same as those of the component waves.

(8) In the case of longitudinal stationary waves the *linear* density of the particles repeats itself after every wavelength *i.e.*, regions of condensations and rarefactions are situated alternately. Further this change of linear density does not advance forward in the medium. There is next a proportional change in its magnitude at each point resulting in a state of normal density everywhere. This occurs twice in each period, and the air has at each point maximum velocity. It continues to move in the same direction but with diminishing velocity and the nodes towards which it moves which were previously rarefied ones, now become condensed and *vice versa*. This goes on till the particles come to rest again. Then all the movements begin again but in a reverse direction and so on.

84. We have described above the formation of stationary waves in general without making a reference to the two types (*a*) when the component waves are **Transverse** ; (*b*) when they are **Longitudinal** The former may be called *Transverse Stationary Vibrations* and are formed by sending properly timed impulses along a rope or a string one end of which is fixed, and the latter are termed *Longitudinal Stationary Vibrations*, as are set up in a vibrating column of air in an organ pipe.

Both types, however, are due to the interaction of similar wave trains running in opposite directions and are usually due to reflected waves meeting those which are advancing. We shall first consider the formation of transverse stationary vibrations in strings.

85. Reflection of Waves in Strings.—Suppose a crest is sent along a string of which one end is held in the hand and the other end is tied to a hook in a wall (Fig. 63). When

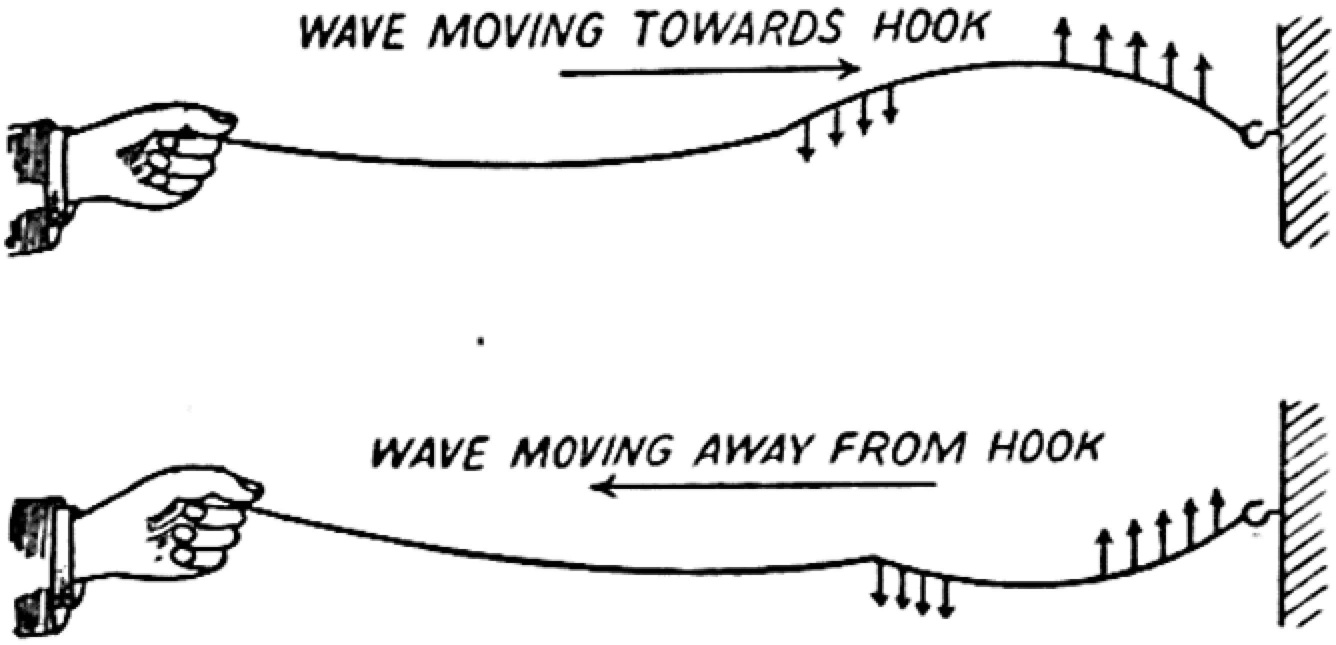

Fig. 63.

the hand is jerked so as to send a crest along the string we find that the particles in front of a crest pull up those ahead of them and thus the crest advances. As the crest approaches the hook, the particles in front of it are rising and in so doing, the string exerts an upward pull on the hook. By Newton's Third Law of motion the hook exerts an equal downward pull on the string. This is equal and opposite to the force communicated by the hand in sending the crest along the string. As the hook is fixed this force is greater than the normal reaction exerted by one position of the string upon another. Consequently the force gives rise to a trough which moves away from the hook. Similarly, it may be shown that a trough is reflected as a crest. *Whenever a wave is reflected from the fixed end of a string, crests are reflected as troughs and troughs as crests.* That is, there occurs a complete reversal of phase as indicated. If $y=a\sin\frac{2\pi}{\lambda}(vt-x)$ is the equation of the incident, wave then the reflected wave will be represented by $y=-a\sin\frac{2\pi}{\lambda}(vt+x)$, —sign of a indicates that the amplitude is reversed, a crest returns as a trough. The change in the sign of x is due to the fact that the reflected wave is now travelling in the direction opposite to that of the incident wave taken as +ve.

If the string is free at the other end a crest returns as a crest and a trough as a trough. Suppose the end *B* is free and a crest is first reaching it. *B* will be jerked towards the right by the arriving wave and as it is free to do so it will acquire a velocity towards the right as if a pulse had been communicated to it by the hand and so it sends back a crest towards the right.

Hence the equation of the reflected wave is given by

$$y=a\sin\frac{2\pi}{\lambda}(vt+x).$$

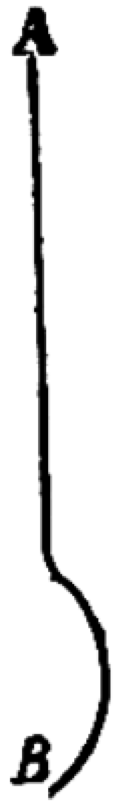

Fig. 64.

86. Stationary Vibrations in Strings.—Consider a string fixed at the end *B*. Suppose waves are sent along it and at any instant the shape of the incident wave is shown by the thick lined curve Fig. 65 (*a*). To get the position of the reflected wave in the string at the instant considered, imagine the direct wave to be optically reflected twice, first in *AB* when it will take the shape represented in Fig. 65 (*b*) and then in *CD*. After these two imaginary reflections the shape of the curve is shown in Fig. 65 (*c*). The continuation of this wave to the left of *CD* is the required reflected wave. The superposition of the direct and the reflected waves gives rise to stationary undulations.

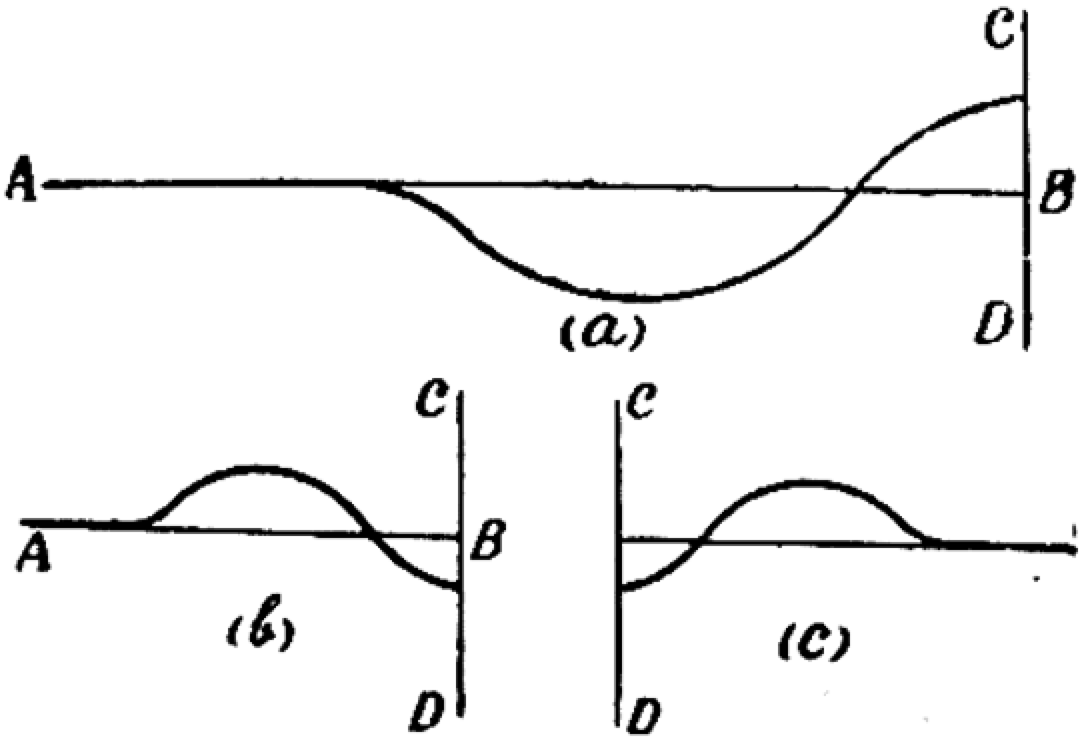

Fig. 65.

In Fig. 66 four stages at intervals of a quarter period have been depicted. The thin continuous curve represents the direct wave incident at the interface *CD* and the dotted curve the reflected wave obtained by the method given in the preceding para. To the right of the interface is shown the imaginary prolongation of the reflected wave. In stage (i) the resultant shape of the curve shown in thick line is obtained by compounding the direct and reflected wave trains and represents the steady vibration of the string with double the amplitude of the either constituent. It will be noticed that the phase of the reflected wave at the fixed end is always such that the resultant displacement at the interface due to the direct and the reflected waves is zero. This is a *nodal* point. At this a point a crest due to one wave is meeting a trough due to the other. Such points are indicated by N_1, N_2, N_3 etc., in the diagram.

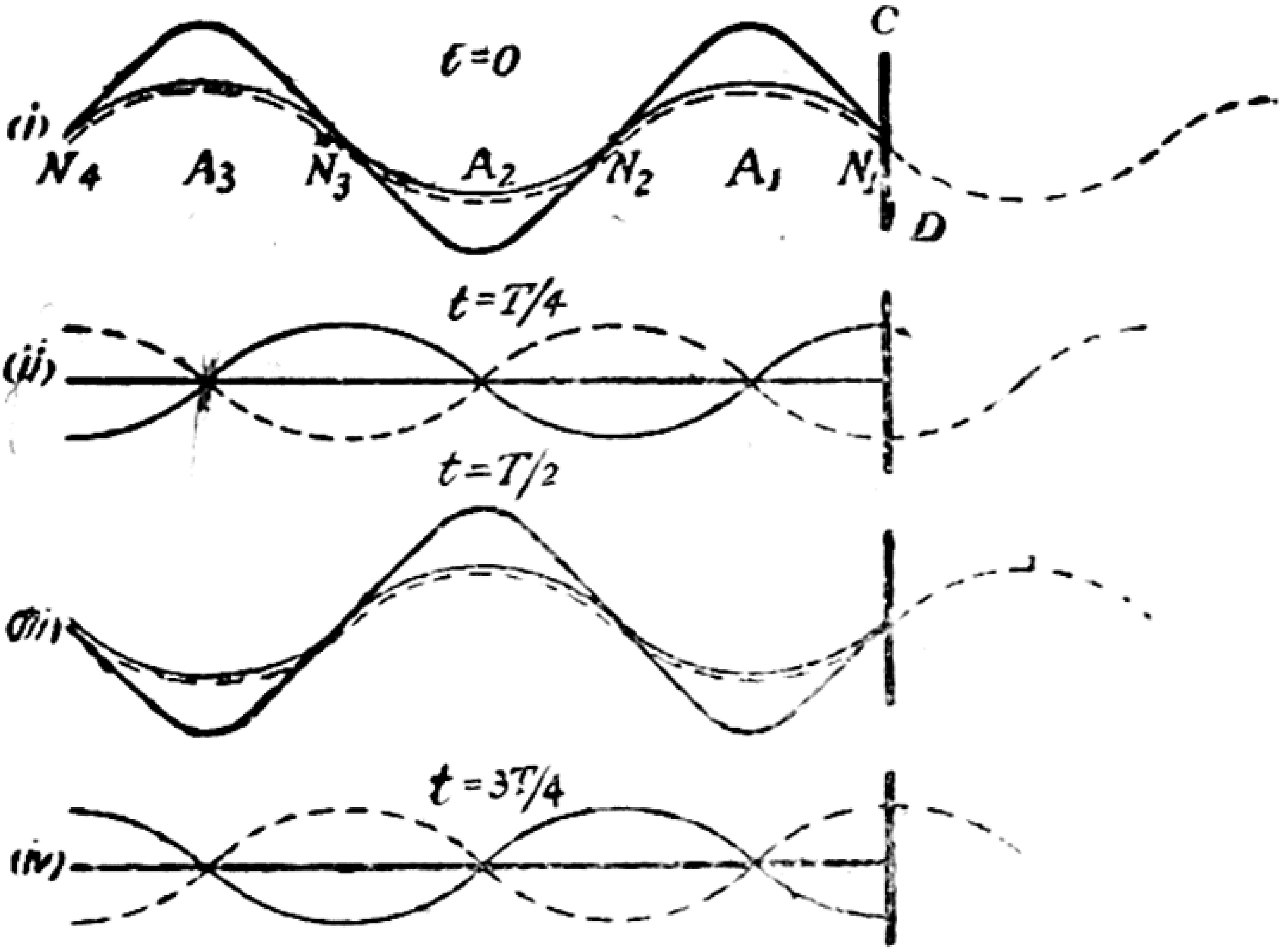

Fig. 66.

At points A_1, A_2, A_3, etc., either two crests or two troughs are meeting. These points are *antinodes*.

A quarter period later, stage (*ii*), the crest of one wave train coincides with the trough of the other *i.e.*, the two constituents are in opposite phase and their ordinates will, everywhere, be equal and opposite. Hence the resultant curve is simply a straight line. The string will be in the equilibrium position everywhere at the moment.

In stage (*iii*) again a trough falls upon trough and a crest upon crest but they have interchanged positions with respect to stage (*i*).

In stage (*iv*) there is a coincidence of a trough with a crest. Proceeding further we find that there will be a repetition of these four stages once in each time period and so on. The nodal points N_1, N_2, N_3 are permanently at rest, whereas the antinodes A_1, A_2, A_3 have maximum displacement and velocity. The wavelength is equal to the distance between alternate nodes or antinodes.

The segments of the string vibrate from side to side, all the points in the same segment being in the same phase but in opposite phase with the neighbouring segment.

87 Reflection at the end of a pipe.—Before we consider the formation of longitudinal stationary vibrations in air, let us see how reflection takes place at the closed and the open end of a pipe :—

(i) Closed End.—Let *PQ* be a pipe closed at the end *Q* upon which a train of waves is incident normally. Since the end is rigid, none of the energy incident upon it can be transmitted forward so that the layer of air in contact with the end must remain permanently at rest, for if it moved away there would be a vacuum towards the right and almost normal atmospheric pressure on the other side. Hence when a compression reaches the closed end, the only way whereby this layer can free itself from the compressional strain is by sending a wave of compression towards the left. Thus we see that a pulse of compression is reflected from the fixed end at *Q* as a pulse of compression and a rarefaction returns as a rarefaction. In each case the displacements of particles due to the reflected wave are opposite to those due to incident wave. The waves are said to have been reflected with reversal *of phase* since the motions of the particles are reversed by reflection. If, therefore, $y=a \sin \frac{2\pi}{\lambda}(vt-x)$ is the equation of the incident wave that of the reflected one is given by $y=-a \sin\frac{2\pi}{\lambda}(vt+x)$.

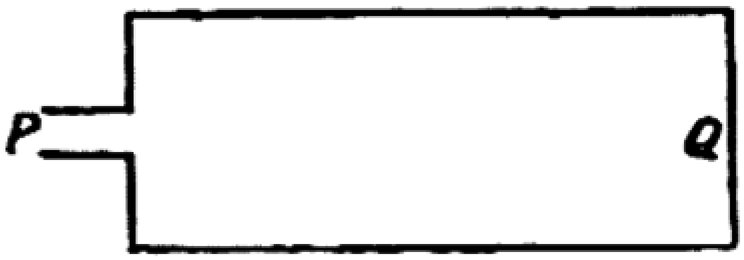

Fig 67

(ii) Open End.—When a pulse of compression is travelling in an open pipe the energy transference as in the above case is taking place in the direction of propagation till the pulse arrives at the open end where it meets much smaller resistance than inside. It expands into the medium outside. In doing so it more than releases its strain and continues to move beyond its normal amplitude. This causes the air behind to become rarefied. Thus a wave of rarefaction is sent back in the pipe. At first thought this reflection of a compression from an open end as a rarefaction might seem to represent a change of phase π. But as compared with a reflection from a fixed end there is *a delay of half a period* in the reflection of this wave of opposite phase. Hence in the reflection of a compressional wave from the fixed end of a tube, or a denser medium, there is a reversal of phase or change of phase π. In the reflection from the open end or a less dense medium, however, there is no change of phase, since the particles continue to move in the same direction as the incident wave. Similarly a wave of rarefaction is sent back as a wave of compression. The equation of the reflected wave is given by $y=a\sin\frac{2\pi}{\lambda}(vt+x)$.

It may be noted that at the open end the reflection is partial as part of the energy of the direct wave will pass out into the open air. The amplitude of the reflected wave will be less than that of the incident wave so that interference of the waves within the tube is not complete. For this reason the velocity and the displacement at the nodes is never exactly zero. The equation of the reflected wave may therefore be taken as $y=a'\sin\frac{2\pi}{\lambda}(vt+x)$ where $a' < a$.

88. Longitudinal Stationary Vibrations.—Consider a closed end pipe. Let as before the thin curves of Fig. 66 (*i*) to (*iv*) represent the *associated* displacement curves of the direct and reflected waves and the thick continuous curve their resultant. In this case also let us divide the whole period into four equal parts and discover the state of the air subjected to such vibrations.

At time $t=0$, the displacements are shown in stage (*i*), those due to one wave towards the right are superposed by those due to the other wave also towards the right so that each particle is having twice the displacement due to either wave and is in the extreme end of its swing.

Also the maximum compressions and maximum rarefactions due to the two waves are coincident and the resultant compressions and rarefactions will be double of either but as regards velocity there is instantaneous rest.

After one quarter period the displacements are shown in Fig. 66 (*ii*) and the maximum compression of one coincides with the maximum rarefaction of the other and they annul each other. So the particles along the entire wave have linear density. But since the velocities will be in the same direction, the resultant velocity will be double of either.

Another quarter period later [stage (*iii*)] we shall again have the two compressions or two rarefactions arriving simultaneously at a point but as during the interval the waves have advanced by half a wavelength they have interchanged places *i. e.*, where in stage (*i*) two compressions were arriving simultaneously, now two rarefactions are arriving together and *vice versa.*

In the last periodic stage shown in (*iv*), the maximum rarefaction of one again coincides with the maximum compression of the other and they annul each other so that the medium has once more normal density every where. The resultant velocity of a particle will be double the velocity due to either component but in a direction opposite to that shown in stage (*ii*). Hence we see that there are certain points at regular intervals of $\lambda/2$, which are alternately points of maximum compression and maximum rarefaction. At such points velocities are also zero. These points have already been termed as *nodes.* Note that the nodes which were previously rarefied ones now become condensed and *vice versa.* We also see that conditions depicted in stages (*i*) and (*iii*) recur twice in each period and alternate with those depicted in stages (*ii*) and (*iv*). It is also observed that midway between them are *antinodes*, where the maximum compressions of one set coincide with the maximum rarefactions of the other and the velocity is double of either component.

The various stages of stationary vibrations in open end pipes have been discussed in Art. 82. Note that the open end forms an antinode.

89. Analytical Treatment of Standing Waves.—The results arrived at in the preceding Arts. can be obtained mathematically by the equations of superposing wave trains. Let

$$y_1 = a \sin \frac{2\pi}{\lambda}(vt - x) \qquad \text{... (1) [incident]}$$

and

$$y_2 = a \sin \frac{2\pi}{\lambda}(vt + x) \qquad \text{... (2) [reflected]}$$

represent the two wave trains of the same wavelength, λ, and of the same amplitude, a, travelling with the same velocity, v, in opposite directions. Note, here the *interface* where the reflection takes place, is denoted by $x=0$.

The displacement equations refer to the incident and reflected waves in an open end pipe or a string free at the other end.

The displacement equation of the resultant wave is

$$y=y_1+y_2=a \sin \frac{2\pi}{\lambda}(vt-x)+a \sin \frac{2\pi}{\lambda}(vt+x)$$

$$=a\left[\sin\frac{2\pi}{\lambda}(vt-x)+\sin\frac{2\pi}{\lambda}(vt+x)\right]$$

$$=2a \sin \frac{2\pi vt}{\lambda} \cos \frac{2\pi x}{\lambda}.$$

$$=2a \cos \frac{2\pi x}{\lambda} \sin \frac{2\pi vt}{\lambda} \quad \ldots \quad (3)$$

This represents a Simple Harmonic vibration of amplitude $A=2a \cos \frac{2\pi x}{\lambda}$ and of the same period as the originals.

The strain or the compression at any point of the resultant vibration is obtained by differentiating (3) with respect to x

$$\therefore \text{ strain }\left(\frac{dy}{dx}\right)=-\frac{4\pi a}{\lambda} \sin\frac{2\pi x}{\lambda} \sin\frac{2\pi vt}{\lambda} \quad \ldots \quad (4)$$

Differentiating (3) with respect to t we get the velocity of the particles.

$$\therefore \text{ Velocity }\left(\frac{dy}{dt}\right)=\frac{4a\pi v}{\lambda} \cos\frac{2\pi x}{\lambda} \cos\frac{2\pi vt}{\lambda} \quad \ldots \quad (5)$$

Now let us discuss the changes in the condition of the vibrating column as regards displacement, velocity, and strain (change of density or pressure) at different positions and at different times.

Changes with respect to Position: —

(i) At positions where $\sin \frac{2\pi x}{\lambda}=0$ and therefore $\cos \frac{2\pi x}{\lambda} =\pm 1$, we find from equations (3) and (5) that at any given instant the displacement and velocity are greater than at other points but strain [equation (4)] is zero, *i.e.*, there is neither any compression nor rarefaction. These points are *antinodes.* The position of such points is obtained by putting

$$\frac{2\pi x}{\lambda}=m\pi, \text{ where } m \text{ is an integer } 0,\ 1,\ 2,\ 3, \text{ etc.}$$

Then
$$x=m\frac{\lambda}{2} \text{ or } 0,\ \frac{\lambda}{2},\ \lambda,\ \frac{3\lambda}{2},\ \ldots\ldots$$

These antinodal points are separated from each other by $\lambda/2$; the interface being an antinodal point.

(*ii*) At positions where $\cos\frac{2\pi x}{\lambda}=0$ and so $\sin\frac{2\pi x}{\lambda}=\pm 1$, the displacement and velocity are throughout zero but compressions and rarefactions are greater than at other places but occurring alternately. These are *nodes.* They can be obtained by putting

$$\frac{2\pi x}{\lambda}=\frac{(2m+1)\pi}{2}=m\pi+\frac{\pi}{2}.$$

$$\therefore\ x=m\frac{\lambda}{2}+\frac{\lambda}{4} \text{ or } \frac{\lambda}{4},\ \frac{3\lambda}{4},\ \frac{5\lambda}{4},\ \frac{7\lambda}{4},\ \ldots\ldots$$

for $m=0$, 1, 2, 3 etc.

Such points are also separated by $\frac{\lambda}{2}$, the distance of the first node from the interface is $\frac{\lambda}{4}$.

At points other than nodes and antinodes the displacements and the velocities lie between these extreme values and decrease gradually from their maximum values at an antinode to zero at the nodes on either side of it. Similarly, the change of density lies between the extreme values deduced above and decreases on either side of a node, the alternate nodes are centres of condensation and rarefaction and each node is alternately a centre of condensation and rarefaction.

Changes with respect to Time :—

(*iii*) At times when $\sin\frac{2\pi vt}{\lambda}$ or $\sin\frac{2\pi t}{T}=0$ and so $\cos\frac{2\pi t}{T}=\pm 1$; there is neither any displacement nor any compression or rarefaction [equations (3) and (4)] but velocity [equation (5)] is greater than at other times.

The particles are passing through their normal positions everywhere.

To obtain such times put $\frac{2\pi t}{T}=m\pi \therefore\ t=m\frac{T}{2}$ or $0, \frac{T}{2}, T, \ldots$

Such instants occur twice in each time period.

(*iv*) At times for which $\cos\frac{2\pi vt}{\lambda}$ or $\cos\frac{2\pi t}{T}=0$, and so $\sin\frac{2\pi t}{T}=\pm 1$, the velocity is everywhere zero and displacements and compressions or rarefactions are greater than at other times.

Such times are found by putting $\frac{2\pi t}{T}=(2m+1)\frac{\pi}{2}=m\pi+\frac{\pi}{2}$

$$\therefore\ t=m\frac{T}{2}+\frac{T}{4} \text{ or } \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}, \ldots$$

These conditions will also be repeated twice in each time-period but $\frac{T}{4}$ later than those reached in (*iii*).

A similar analytical treatment may also be applied to the case of a closed end pipe or a string fixed at the other end provided care is taken that the proper sign of the amplitude (negative in these cases) is substituted before the displacement equation of the reflected wave.

90. Energy of Stationary Waves.*—The *distribution* of energy in a stationary wave is not the same as in a progressive wave. The total energy per wavelength in a stationary wave is *double* that in a progressive wave. This should be the case, seeing that the waves now under consideration are compounded of two sets, except that one is reversed in direction of propagation. Moreover, in the case of stationary waves there is *no flow of energy*. This is obvious for the resultant energy current is the algebraic sum of the energy currents in the two oppositely travelling progressive waves of which the stationary waves may be built up. But these two energy currents are equal and opposite, hence the *resultant energy transfer in any direction is zero.*

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*Read Art. 44 and contrast the statements made in that section with those made in this.

While in any progressive wave half the energy is at any moment kinetic and the other half potential, in a stationary wave the ratio of the kinetic to the potential energy is the same at every point at any instant that is, it is independent of place but the ratio varies from instant to instant. The energy is all kinetic everywhere when the particles are moving through their equilibrium positions. At this moment most of the energy is near the *antinodes*, where the air or the element of the string is moving fastest. When the particles are at the extreme end of a swing—which occurs at the same instant everywhere—there is no kinetic energy, as all the air is at rest ; it is all potential everywhere depending upon differences of pressure and none is at the antinodes. In the intermediate stages it is partly in one form and partly in the other.

***90a. Energy is not transferred in a Stationary Wave :—** We have shown in the preceding Art. that in a stationary wave there is no transfer of energy across any plane. This can be mathematically proved as follows :—

When waves are set up in a fluid the excess of pressure is given by

$$p=-\mathrm{E}\frac{dy}{dx},$$

where $\mathrm{E}(=v^2\rho)$ is the volume elasticity and $\frac{dy}{dx}$ denotes the volumetric strain

For a stationary wave from equation (4) page 128

we have $$\frac{dy}{dx}=-\frac{4\pi a}{\lambda}\sin\frac{2\pi x}{\lambda}\sin\frac{2\pi vt}{\lambda}$$

whence $$p=v^2\rho.\frac{4\pi a}{\lambda}\sin\frac{2\pi x}{\lambda}\sin\frac{2\pi vt}{\lambda}$$

$$=p_{max}\sin\frac{2\pi x}{\lambda}\sin\frac{2\pi vt}{\lambda}.$$

$$=p_x\sin\frac{2\pi vt}{\lambda},$$

where $$p_x=p_{max}\sin\frac{2\pi x}{\lambda}.$$

The velocity of the particle from equation (5) on the same page is given by

$$U=\frac{dy}{dt}=\frac{4\pi\ av}{\lambda}\cos\frac{2\pi\ x}{\lambda}\cos\frac{2\pi\ vt}{\lambda}$$

$$=U_x\cos\frac{2\pi\ vt}{\lambda},\ \text{where } U_x=\frac{4\pi\ av}{\lambda}\cos\frac{2\pi\ x}{\lambda}$$

Therefore work done or energy transferred per unit area in a small time dt

$$=p\ U\ dt$$

The energy transfer in the periodic time T

$$=\int_0^T p\,U dt$$

∴ Rate of transfer of energy

$$e=\frac{1}{T}\int_0^T p\ \ Udt$$

$$=\frac{1}{T}\int_0^T p_x\ \sin\frac{2\pi\ vt}{\lambda}\times U_x\cos\frac{2\pi\ vt}{\lambda}\,dt$$

Since p_x and U_x are not the functions of time, they can be taken out of the limits of integration and we have

$$c=\frac{p_x U_x}{T}\int_0^T \sin\frac{2\pi\ vt}{\lambda}\cos\frac{2\pi\ vt}{\lambda}dt$$

$$=\frac{p_x\ U_x}{2T}\int_0^T \sin\frac{4\pi\ vt}{\lambda}dt$$

$$=0 \qquad \left[\because \int_0^T \sin\frac{4\pi\ vt}{\lambda}dt=0.\right.$$

Thus in the case of stationary vibrations no energy is transferred across any section.

91. Demonstration of Longitudinal Stationary Waves in open Air.—For this purpose we need a high frequency sound, such as is produced when a Galton's whistle is blown. Galton's whistle (Fig. 68) is a miniature organ pipe fitted with an adjustable screw piston. It is blown by means of air collected in a rubber bag attached above its mouth. By turning the screw around, the volume of the air in the whistle can be altered any the pitch of the note emitted, changed. The frequency for and given position is read from graduations on the scale. The positions of nodes and antinodes are located by a sensitive flame (see Art. 100).

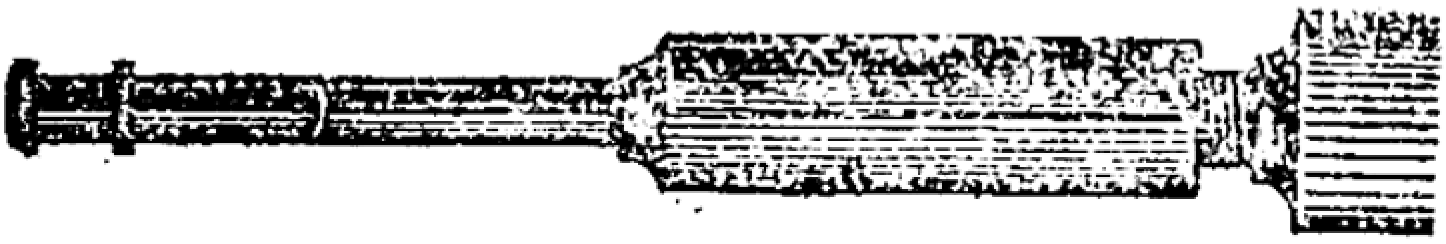


Fig. 68.

Place the Galton's whistle at a distance of a few feet from a vertical board or a wall of the room, and blow it steadily by means of the air supplied by a gas bag. The sound waves emitted by the whistle are reflected by the wall and travel backwards. They interfere with the direct waves and give rise to stationary vibrations. Bring the sensitive flame between the board and the whistle. When the flame is at a node, it does not flicker since the air is at rest and when at an antinode the flame will flare up, since at these points the disturbance is most pronounced. The distance between two successive nodes or antinodes is $\lambda/2$.

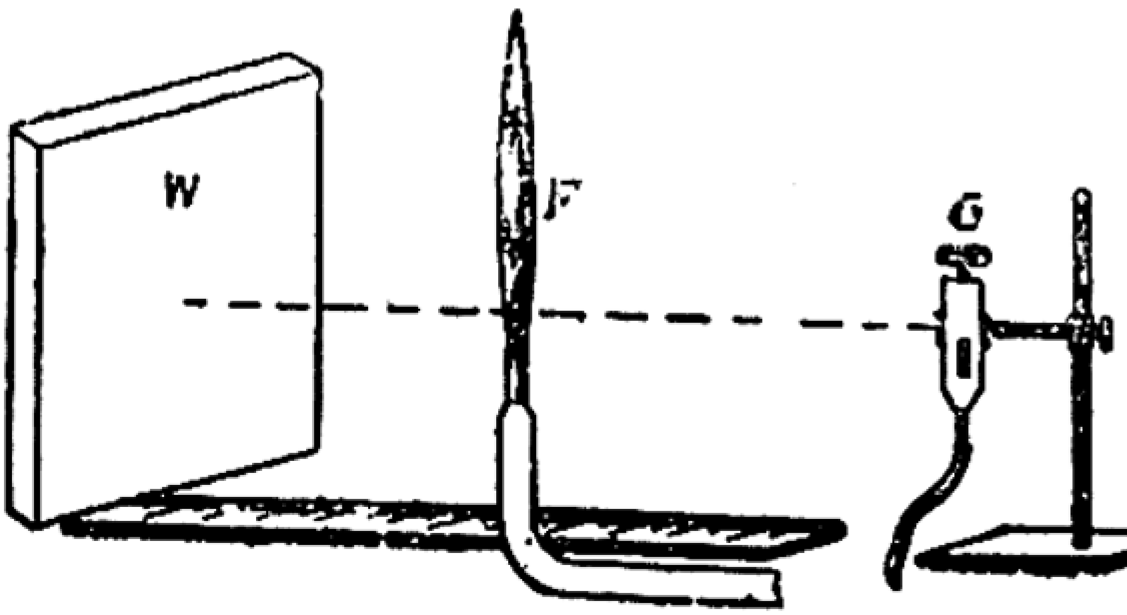


Fig. 69.

The experiment may be used to find the pitch of a high frequency note, if the velocity of sound is known. For that purpose a metre scale is placed alongside the flame. The positions of the nodes are located and the mean distance between two consecutive nodes is determined. This distance is $\lambda/2$, hence λ is known. If v be the velocity of sound at the room temperature, we can calculate the frequency from the relation, $v = n\lambda$.

In this manner the frequency of a note which is so high that it is beyond the upper limit of audibility, may be found and since the frequency of the note emitted by a Galton's whistle is continuously variable, the upper limit of audibility may be fixed.

92. Longitudinal stationary waves in a Resonating Air Column.—The phenomenon of standing waves in a resonating air column may be demonstrated to a big audience by a thin walled hollow tube made of iron, 2″ in diameter and about 6 ft. long. It is closed at one end by a thin rubber membrane or a tissue paper and at the other end with an adjustable piston. Arranged in a row at the top are a number of tiny holes spaced at regular intervals. Coal gas is conducted into the tube by a small side tube, L, and can be ignited at the holes so as to give a number of small flames of *equal* height. An organ pipe is next sounded in front of the membrane which is thereby set into vibrations and produces waves inside the tube. When resonance is obtained by adjusting the piston, the gas flames burn with *different* heights and produce the result shown in Fig. 70. Where the nodes occur in the gas column, the air is at rest; at these points the pressure is high and the flames are tallest. Midway between them are the antinodes where the amplitude of the gas particles is maximum and the flames are low. The pressure variations are in accordance with the famous *Bernoulli's Principle*, according to which at places where the velocity of the gas particles is low the pressure is high, and where the velocity of the particles is high the pressure is low.

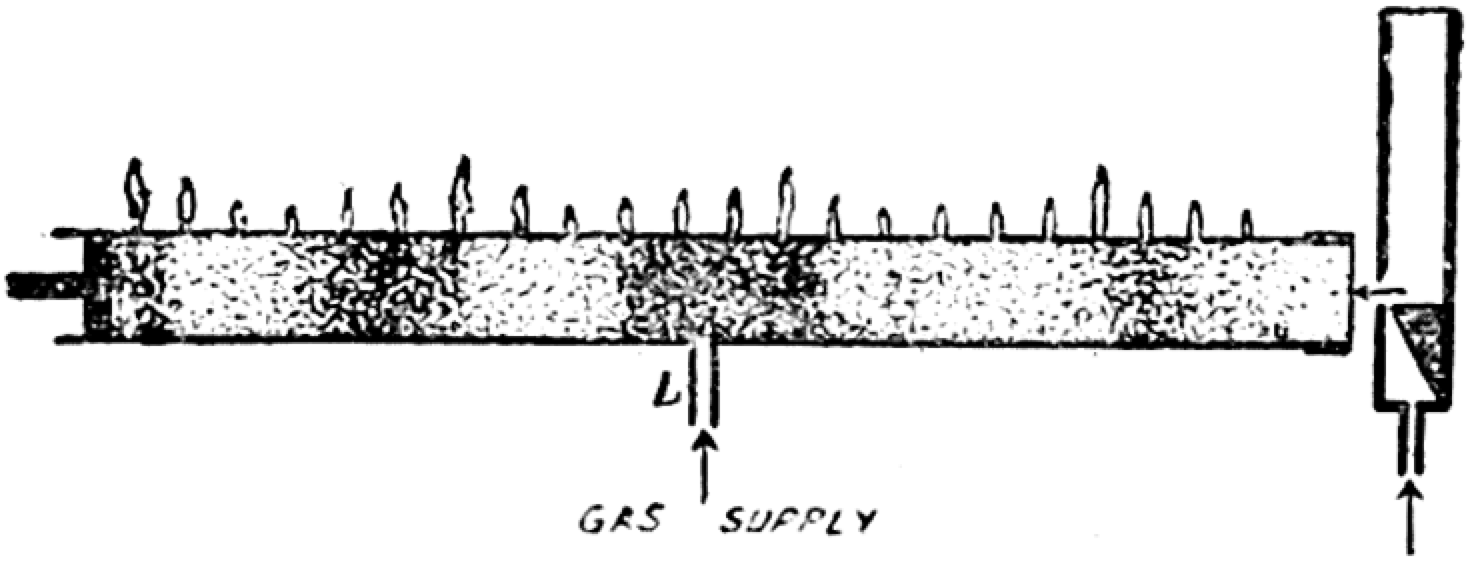
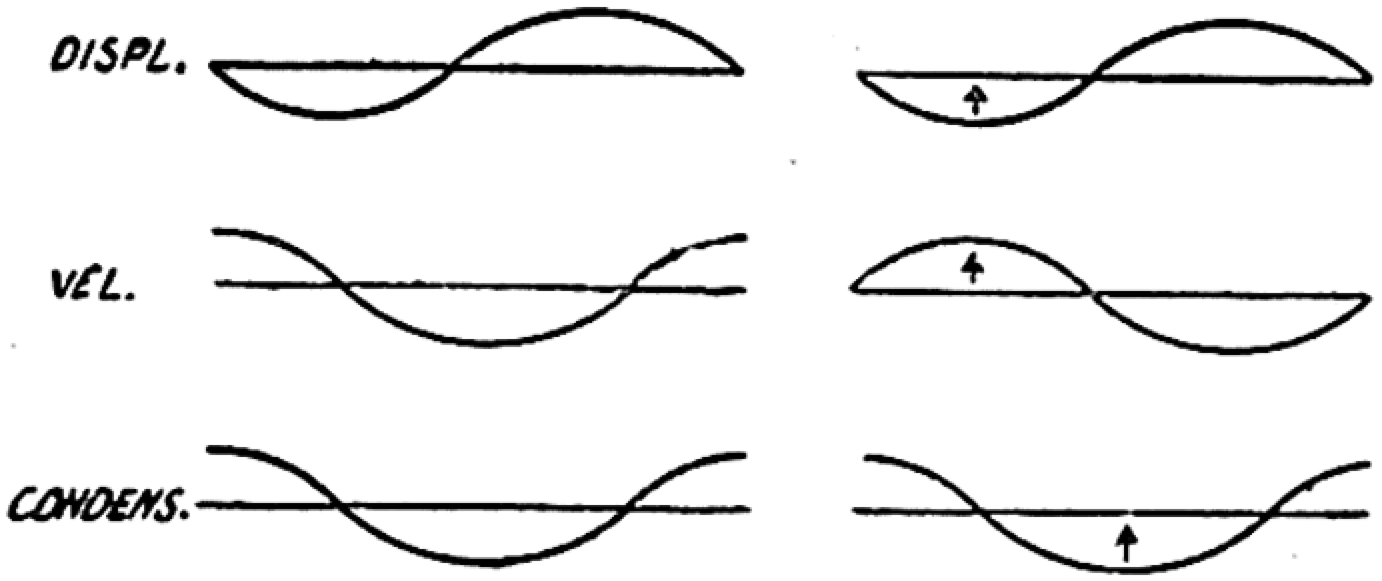


Fig. 70.

93. Comparison between Stationary and Progressive Waves.—

Progressive Wave	Stationary Wave
1. The disturbance moves onward, being handed on from one particle to next after some time. In other words here all the characteristics of a point at any instant will be *transferred* to the next point in the next instant.	1. The disturbance is fixed as it were. No particle transfers its motion to the neighbouring one. In other words vibration characteristics of a point are peculiar to itself and are *not* transferred to its neighbouring point at *any* time.
2. Every particle has a constant amplitude but it reaches any stage of its displacement at a time quite different from that of its neighbouring particles *i.e.*, the phase *varies* along the wave.	2. The amplitude is not the same but its decrease from antinode to node follows a *cosine law*. All particles reach their maximum or minimum displacements at the *same* time Particles between consecutive nodes are in same phase, but they are in opposite phase with those in the preceding and succeeding segments.
3. No particle is permanently at rest but each particle comes to a state of *momentary* rest at the end of its swing. This condition may be achieved by different particles at different times.	3 Particles at the nodes are always at rest All other particles have maximum displacement and rest simultaneously and they repeat this condition after every half period.
4. Maximum velocity that a particle develops in course of a single period is independent of the value of x, *i.e.*, all particles have the *same* maximum velocity but they obtain it when they pass through their mean position one after another.	4. Here at nodes, the velocity is always zero, and at antinodes it has its greatest value. All particles pass through the mean position at the same time, but each has a velocity which is its *own maximum*.
5. In no region, conditions of compression, normal pressure or no change of density, and rarefaction are constant, but as these conditions travel onward, each region is affected turn by turn.	5. As stated above there is no outward motion Condensation at antinodes is least or zero and for nodes it is greatest.
6. Velocity curve agrees with condensation curve, and is $T/4$ ahead of displacement curve. These travel onwards with the wave.	6. Condensation curve is $T/4$ ahead of displacement curve, while the velocity curve is $T/2$ ahead of it. These retain the same position and merely change in amplitude. In Fig. 71 (*b*) it is assumed that the air particles are neither at the the end of the swing nor passing through the equilibrium position but in some intermediate position and moving towards their equilibrium position.

Progressive Wave (*contd.*)	**Stationary Wave** (*contd.*)
$[y=a\sin\frac{2\pi}{\lambda}(vt-x)]$	$[y=2a\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi vt}{\lambda}]$
$\frac{dy}{dt}=\frac{2\pi av}{\lambda}\cos\frac{2\pi}{\lambda}(vt-x).$	$\frac{dy}{dt}=\frac{4\pi av}{\lambda}\cos\frac{2\pi x}{\lambda}\cos\frac{2\pi vt}{\lambda}.$
$\frac{dy}{dx}=-\frac{2\pi a}{\lambda}\cos\frac{2\pi}{\lambda}(vt-x).$	$\frac{dy}{dx}=-\frac{4\pi a}{\lambda}\sin\frac{2\pi x}{\lambda}\sin\frac{2\pi vt}{\lambda}.$



(*a*) Fig. 71 (*b*)

Progressive Wave	**Stationary Wave**
7. There is transmission of energy across every plane along a progressive wave. Condensation curve and velocity curve *agree* together and so during a condensation the elastic bond between the particles will be pushing the particles in front forwards and work will be done on the particles ahead *i.e.*, there is *a flow of energy* in the direction that a wave travels. The energy is not uniformly distributed over the wave for we saw that at the points of no velocity there is no compression and consequently at these points there is no energy. Further the energy in a whole wave is at every instant half kinetic and half potential.	7. Here there is *no flow* of energy across any plane. As stated above in No. 6, condensation and velocity curves differ in phase $T/4$ *i.e.*, they are said to be in quadrature, and hence no energy is transmitted.* This is why the waves are said to be standing. Here the distribution of energy is quite different from that of a progressive wave. The ratio of the K. E. to P. E. is the same at every point at any moment but the ratio varies from moment to moment. When all particles are in their undisplaced positions, the energy is wholly kinetic, $T/4$ later all particles are in their extreme positions, they being stationary for a moment, the energy is wholly potential.
8. There is no place where there is no change in density.	8. Antinodes are places of *no* change in density, whereas nodes are places of maximum change of density.

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*See Art. 90.

CHAPTER VIII

REFLECTION, REFRACTION, AND DIFFRACTION

94. **Reflection of Sound Waves.**—It is a matter of common observation that reflection of sound plays a very important part in our everyday life. For instance, who is not familiar with its consequences, namely, reverberation in buildings, echoes and rolling of thunder : and with its applications in speaking tubes, sounding boards, ear trumpet, etc. Before we consider their explanation, let us study the **mechanism of reflection**.

When a sound wave consisting of condensations and rarefactions meets the bounding surface between two different media, it is partially reflected and a wave travels in the opposite direction in the first medium with the same velocity as that of the incident wave. In order to understand how this reflection is caused, let us consider a mechanical analogy. Suppose a heavy railway truck is standing in a station siding and a number of light trucks are being shunted towards it. As a light truck strikes the heavy stationary truck, it rebounds and starts to move off in opposite direction to that before impact, while the heavy truck moves slowly in the direction in which shunting occurs. Here the heavy truck corresponds to a denser medium and lighter ones to a rarer medium. We find that when a compression wave meets the surface of separation of light and heavy media, a small part of its energy is transmitted in the heavy medium and a large portion of it is sent back as a wave of compression but a forward movement of the particles is converted into a backward movement. In other words, the reflected waves suffer a *change in phase* by an amount π. This is reflection with a *change of sign but no change of type*, *i.e.*, condensation is reflected as a condensation and rarefaction as a rarefaction just as at the closed end of a pipe.

Next consider that a light truck is stationary and a number of heavy trucks impinge against it. After the impact both the trucks will be seen to move in the same direction. So here again if we compare the light truck to a rarer medium and the heavy one to a denser medium as before, we find that a wave of compression proceeds in the second medium as a wave of compression and is reflected back in the first medium as a rarefied wave. As the displacement of the heavy truck continues to be in the same direction after impact as before it, we find that here the condensation has changed into rarefaction but no change in sign has occurred. The intensity of the reflected wave will depend upon the relative densities of the two media *i.e.*, the greater the difference in their densities the greater the intensity of the reflected wave.

To sum up, we learn that at the boundary of a rarer to a denser medium a wave of compression is reflected with a *change of type but no change of sign.*

95. Reflection of Sound Waves at a Rigid Wall.—Consider a sonorous body *A* sounding in front of a rigid wall *LM* perpendicular to the plane of the paper. Spherical waves of compression and rarefaction spread out in all directions. A section of the wave front* of such a wave is shown in the figure. Let this wave front marked 1 at any instant meet the wall at *O*, a point obtained by dropping a perpendicular from *A* on *LM*. An instant later it occupies the position 2, meeting the wall between *B* and *C* from *O* successively outwards. Every point between *B* and *C* will be the centre of the secondary wavelets whose envelope gives the reflected wave *BDC* just the *position BEC* (which the wave would have occupied if the wall had not been there) placed oppositely.

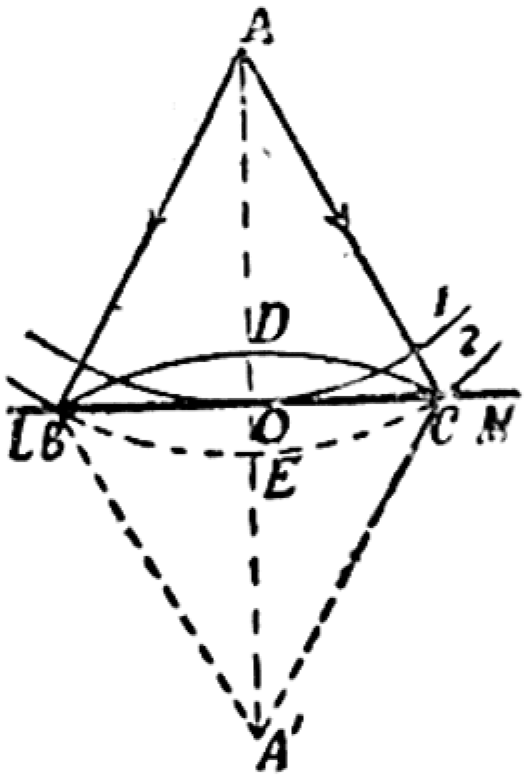

Fig. 72.

It is still spherical in shape but is travelling outwards from *A′* as centre. The point *A′* is the **acoustical image** of *A*.

96. Acoustical Image.—To conveniently obtain the position of the image of *A* we have to remember that the geometrical laws of reflection of sound waves are the same as apply to light rays, namely :—

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*It is the *locus* at any instant of all the adjoining points in the same phase of vibration. According to Huygen's Principle of wave propagation the wave-front at any instant may be derived as the *envelope* of the wavelets whose centres are all points constituting the wave-front which existed *t* seconds previously, the radii of such secondary wavelets being equal to *vt*, where *v* is the velocity of propagation of the wave.

(1) The angle of incidence is equal to the angle of reflection.

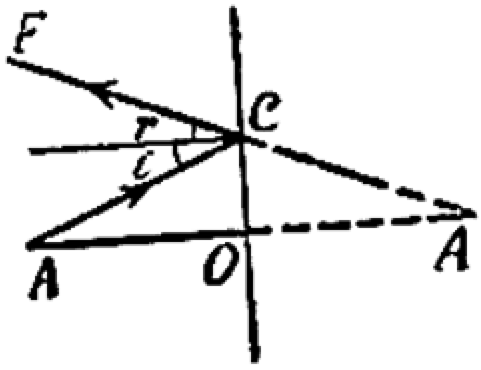

Fig. 73.

(2) The incident ray, the normal to the reflecting surface and the reflected ray lie in one plane. With the help of these laws we can easily show that the sound image *A* lies as far behind the surface as *A* is in front of it. (Fig. 73.)

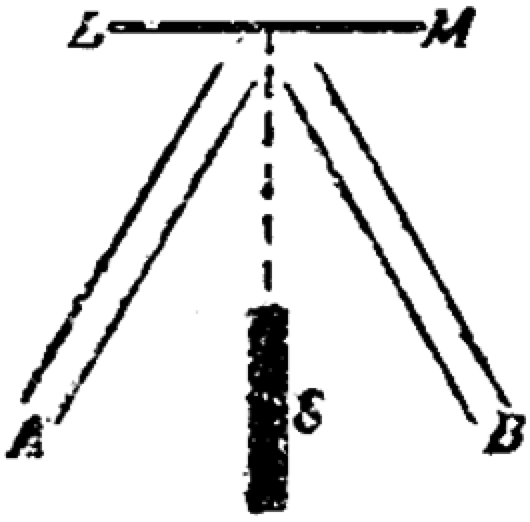

Fig. 74.

To verify the laws of reflection of sound waves arrange two tubes *A* and *B** horizontally at an angle as shown in Fig. 74. Place a reflecting surface such as a polished wooden block *LM* vertically behind them. At the end, *A*, place a watch or preferably blow a Galton's whistle. Next adjust the block or the tube *B* by rotating it, so that on placing the ear near the end *B*, ticking of the watch or the whistle is distinctly heard. The sound of the high-pitched Galton's whistle may also be detected by a sensitive flame. To cut off the direct sound from *A*, a felt screen or a wet cloth should be placed between *A* and *B*. In the adjusted position it will be found that the axes of the tubes are equally inclined to the normal to the surface, *i.e.*, the angles of incidence and reflection are equal and also the waves lie in the same plane with the normal.

It will be interesting to see as referred to in Art. 95. how the reflected wave may be regarded as the resultant of secondary wavelets coming from each point of the reflecting surface. Let *AB* be a plane wave-front meeting the reflecing surface *AC* at *A*. By the time the portion *B* of the wave-front *AB* reaches *C*, *A* has already sent back a secondary wavelet of radius *AC* equal to *BC*, since the velocity of the reflected wave is the same as that of the inci-

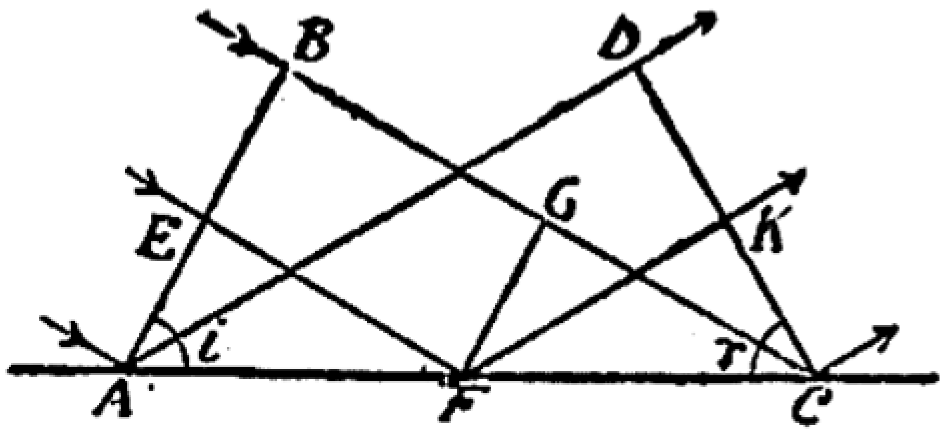

Fig. 75.

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*Glass tubes each about 3 ft. long and 2 inches in diameter may be used.

dent one. So also the circular wavelet starting backward from F, when the incident wave has reached the position FG, will have a radius FK equal to GC, when the wave-front at B reaches C.

From each point of the reflecting surface between A and C, in succession, these elementary waves spread out just as from A and F in arcs of circles whose centres lie on AC. The envelope of the secondary wavelets is the tangent plane CD. So CD is the reflected wave-front. Since AD is equal to BC, and is at right angles to the tangent DC, just as BC is at right angles to AB, the two $\triangle$s ADC and ABC are equal and the angle of incidence BAC is equal to the angle of reflection DCA.

97. Echoes.—The reflection of sound waves from the surface of a large obstacle, as for example a cliff, a hill-side or the walls of a building is described as an echo. In order that an echo may be heard the dimensions of the obstacle must be very large as compared to the wavelength of the incident sound, otherwise sound energy is scattered in all directions and soon becomes inaudible. Thus a sound of short duration and high pitch such as a pistol shot or a sharp crack gives a loud and clear echo, and with a flat or a low pitch sound the echo may not be heard at all. Since the sensation of sound persists for $\frac{1}{10}$th of a second after the source has ceased, so in order that an echo may be distinguished as separate, it must reach the ear $\frac{1}{10}$th of a second after the original sound. Taking the velocity of sound as 1120 ft./sec. it means that sound must come after traversing a distance of 112 ft. *i.e.*, the minimum distance of the obstacle from the source must be half of this *i.e.* 56 ft. If, however, the distance of the reflecting surface is less than this, the sound will appear to be drawn out. It is obvious, therefore, that for sounds of longer duration the interval between the direct and the reflected waves must be longer to hear the echo.

If, for instance, we want to hear the echo of articulate sounds, the minimum distance of the obstacle should be 112 ft. It is because a person can, on an average, utter 5 syllables per sec. Thus to hear the echo of a monosyllabic word the one in which the last syllable of a word is distinctly heard, the time interval between the direct and the reflected sounds should be $\frac{1}{5}$th second *i.e.* the sound should cover 224 ft. in all before it falls on the ear. Therefore the distance of the reflecting surface should be 112 ft. from the observer for the reception of a monosyllabic echo. For a disyllabic echo the minimum distance should be 224 ft. and for a poly-syllabic echo consisting of p syllables the minimum distance should be p times 112 ft.

98. Multiple Echoes.—Multiple echoes may be heard when a sound is reflected from a number of reflecting surfaces suitably placed such as two parallel cliffs. The rumbling and rolling of thunder is due to the successive reflection of a peel of thunder from a number of reflecting surfaces such as clouds, mountains, rocks and surfaces of separation between atmospheric currents and various strata of air.

It has been generally noticed that in public halls and auditoriums the sound persists even after the source of sound has ceased. This persistence of sound is called *Reverberation*. It is due to the multiple reflection of the speaker's or the singer's voice from various surfaces such as the floor, the walls and the ceiling, etc. These reflected sounds interfere and cause a jumbling or prolongation of sound leading to great indistinctness and render the hall acoustically bad for speaking. We shall discuss the subject of Architectural Acoustics in Chapter XV.

99. Musical Echo.—Sometimes when a person walks on a hard footpath, or produces a sharp short sound by clapping in front of a flight of stairs or a number of regularly spaced rails or palings, he hears that each footstep or clap is followed by an echo which produces the sensation of a **musical note.** It is due to the successive reflections of the incident sound wave from each step or pale at regular intervals—the interval being given by the time taken by the sound to traverse twice the distance between each step or pale. These successive impulses, if sufficiently rapid, blend into a musical note. The musical ring heard, sometimes, when an observer walks in a bare hall or a corridor is explained in a similar way. The frequency of the musical echo may be calculated as follows. Let d be the distance between the pales, and θ the angle which the line joining the pales makes with the line joining the observer with the pales. The distances between the palings in the direction of the observer is $d \cos \theta$ and therefore the time interval between successive impulses is $2d \cos \theta / v$ or the frequency of the echo **is equal to $v/2d \cos \theta$. When θ is small it reduces to $v/2d$.**

Example. *A boy claps his hands in front of a staircase and hears a musical echo. If the velocity of sound is 1120 ft. per sec. and if each step is 10 inches wide, what will be the pitch of the sound heard?*

Interval between reflections from two successive steps is equal to the time taken by the sound to traverse twice the width of the step *i.e.* $\frac{2d}{V}$, where d is the width and V the velocity of sound.

Now

$$d = 10''$$

$$V = 1120 \text{ ft./sec.}$$

$$\therefore \text{Time interval } (T) = \frac{2 \times 10}{12 \times 1120}$$

$\therefore$ The pitch of the echo,

$$n = \frac{1}{T} = \frac{12 \times 1120}{2 \times 10} = 672.$$

It has, sometimes, been noticed that echoes from small reflectors are rich in upper harmonics of the incident sound. It is because the efficiency of a reflector for sounds of high pitch and consequently of short wavelength improves. The phenomenon has been studied by Rayleigh and he arrives at similar results as obtained on the scattering of light rays from small particles, for which he finds that the intensity of scattered light varies inversely as the 4th power of the wavelength.

100. Reflection of Sound from Curved Surfaces.—(*a*) **Spherical** or **Parabolic Mirrors.** The laws of reflection are obeyed in the case of spherical and parabolic surfaces as well. If a source of high-pitched sound is placed at the principal focus of a concave spherical or parabolic mirror, the rays diverging from it strike the mirror and are reflected as a parallel beam; or conversely, if a parallel beam of sound is allowed to fall on it the sound energy will converge to the focus of the mirror. By means of two concave metallic reflectors of about 10 to 12″ aperture the phenomenon of reflection can be easily demonstrated. Arrange two concave spherical reflectors on a table with their axes coincident and 15 to 20 ft. apart. Suspend a watch at the focus of one of the reflectors, say P, and a small funnel connected with a rubber tubing at the focus of the other with its mouth towards the reflector. The sound waves from the watch strike the reflector P, are rendered parallel, fall on the second reflector Q, become spherical and converge to its focus F_2. They are collected by the funnel and sent down the rubber tube to the ear applied to its other end. The ticking of the watch will be distinctly heard.

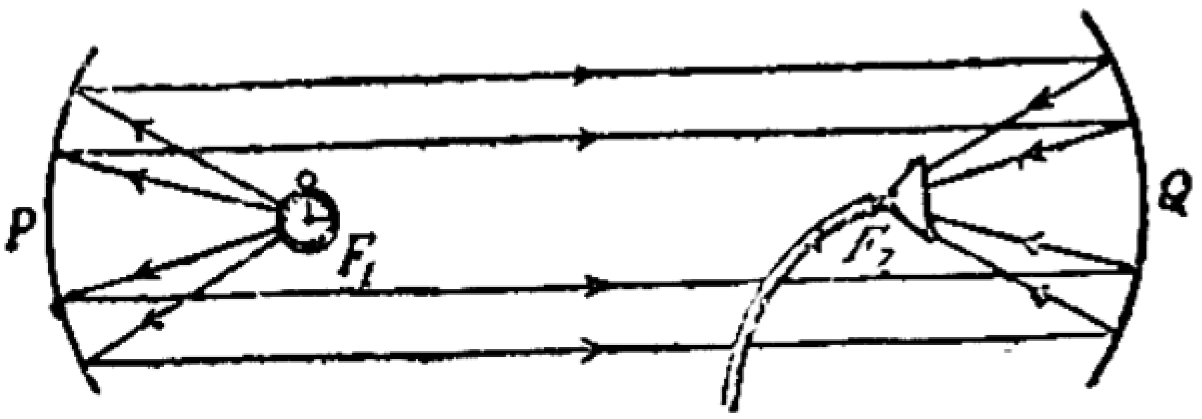

Fig. 76.

The demonstrate the focussing of the sound waves to a class we make use of a *Sensitive Flame.* It consists of a gas flame formed by a fine glass tube jet (which can be easily drawn out in a laboratory) or by a pin hole orifice, if available. Attached to the jet is a gas bag in which the pressure of the gas can be gradually increased making the flame long and narrow until it is on the point of flaring. Such a flame is sensitive to external sounds particularly to those of high pitch. If now a rattle of a bunch of keys or coins, a hiss or a sharp metallic sound is produced in its neighbourhood, the flame begins to flare and shorten. If such a flame is placed in place of the funnel in the above experiment with its orifice at the focus, we notice that it flares and dances in tune with the ticks of the watch.

(*b*) **Whispering Galleries.**—They offer a very peculiar illustration of the phenomenon of reflection of sound. The gallery of the dome of the St. Paul Cathedral in London which is running round the base in the inside of the dome in a circular form is particularly noted for it. There are similar whispering galleries in other circular buildings. It is observed that in such a gallery a whisper spoken at some point, *A* (Fig. 77) close to the wall is reflected round the wall of the gallery so as to be distinctly audible all round near the wall, although at a short distance towards the centre the sound is hardly audible. This is not a case of concentration of sound waves as in curved surfaces (for in that case whisper would be heard at a point diametrically opposite only) but of slow horizontal creeping of the sound along the curved surface of the gallery without ever getting far away from it. It is the portion of the sound energy included between the rays AP and AQ which do not make a large angle with the tangents to the wall between P and Q, that is reflected round and round in a series of equal chords touching a cylindrical surface in the interior of the dome. This cylindrical surface is such that AQ and PA produced are tangential to it. A number of intermediate rays will form short chords and will travel along similar paths but in larger circles. The walls of the gallery prevent the lateral spreading out of sound waves and consequently the intensity of the sound remains sufficient to be audible all round.

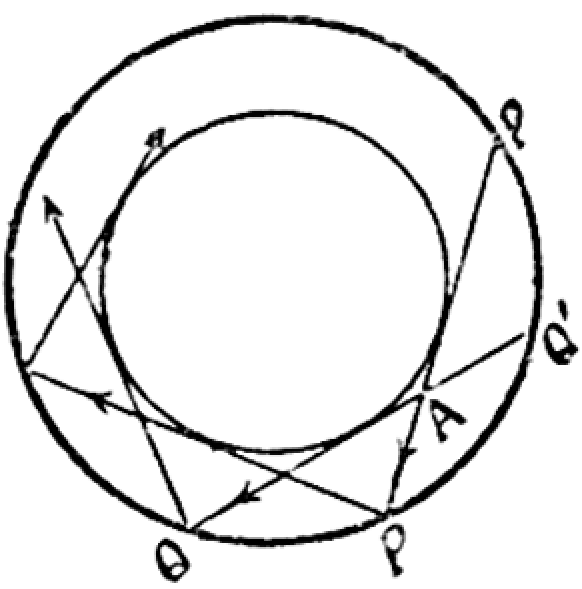

Fig. 77.

101. A few Practical Applications of Reflection.—(*a*) **Speaking Tube**. Ordinarily when we speak in open air, the intensity of the sound falls off inversely as the square of the distance so that a short distance away from the speaker the sound becomes inaudible. But when the sound is delivered at one end of a tube, the waves are prevented from spreading out by successive reflections from the sides of the tube and are confined to the air in the tube. They travel onward with very little diminution in amplitude and the sound therefore can be heard clearly at the distant end. Bends and particularly sharp turns should, as far as possible, be avoided because reflections will then arise which may break up the advancing wave, reduce the intensity and interfere with the clearness of sound.

When one end of a metallic rod or a long uniform wooden beam is struck, the sound is carried along the rod or beam just as in a speaking tube with very little loss through waves sent out sidewise, and therefore very distinctly heard at the farther end.

(*b*) **Ear Trumpet** used by deaf persons is another application of the same phenomenon. The sound waves collected by the wide end are reflected in narrower and narrower cross sections leading them to the ear. As the sound energy delivered to the wide mouth of the trumpet is subsequently thrown in a much smaller space, the amplitude of the vibrating layers of air becomes much greater so that the intensity of the sound reaching the ear is considerably increased.

(c) **The Sounding Board**, which consists of a reflecting surface, is sometimes placed over the speaker's head in a large hall. It not only prevents spreading out of the sound waves in the upward direction but also sends them by reflection towards the audience thus assisting in making the speech audible at a distance.

102. Refraction of Sound Waves.— When sound waves pass from one medium into another, there is generally a change in velocity and as a consequence they are refracted according to the same laws as in optics, namely :—

(1) The incident ray, the refracted ray and the normal to the refracting surface lie in the same plane.

(2) The sine of the angle between the incident ray and the normal bears a constant ratio to the sine of the angle between the refracted ray and the normal,

i.e.,
$$\frac{\sin i}{\sin r}=\frac{v_1}{v_2},$$

where v_1 and v_2 are wave velocities in the first and the second medium. The path of the refracted ray is obtained by Huygen's Principle*.

Let AB represent a plane wave-front incident on a second medium, where velocity is less. While the wave-front advances from B to C in the first medium, the wave from A will have gone a distance $AD=v_2t$, where v_2, is the velocity in the second medium and, t, the time taken by the wave in going from B to C. Each point of AC will become, in succession, a centre of disturbance sending out secondary wavelets in the second medium. CD will become the new wave-front being a tangent plane to all the secondary wavelets originating from AC. The direction of advance of the refracted wave front is bent

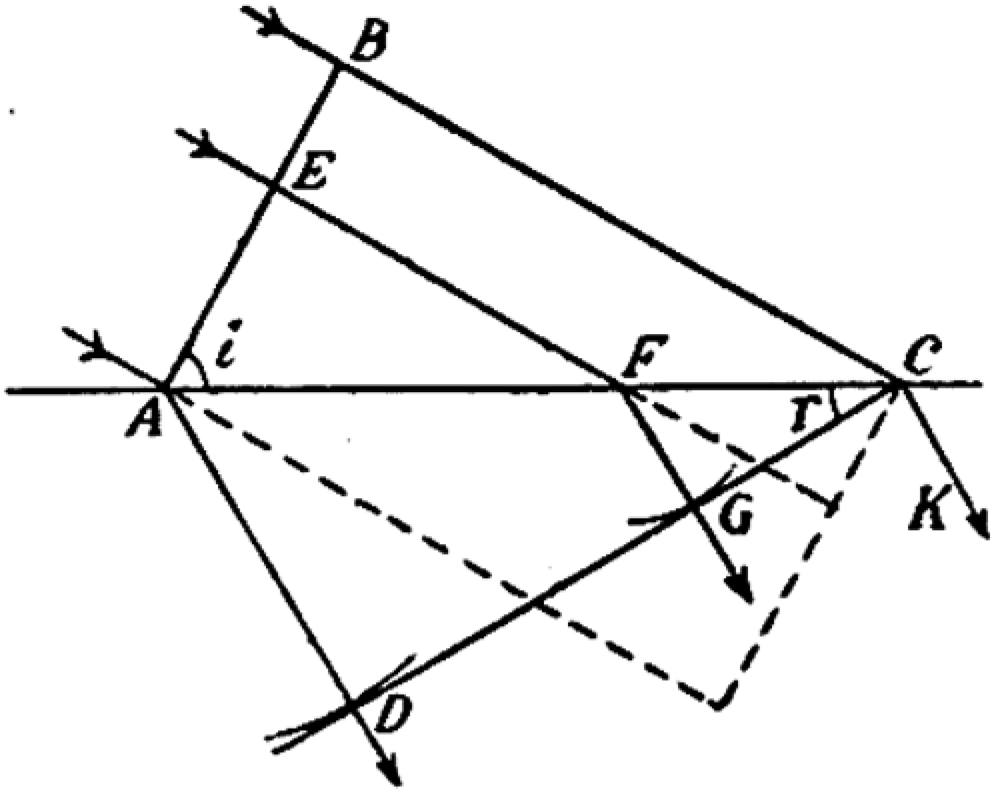

Fig. 78.

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*See foot note on page 138.

towards the normal from BC to CK. If the velocity in the second medium were greater than the first, the rays on refraction would bend away from the normal becoming more oblique to the surface of separation.

103. Demonstration of the Refraction of Sound Waves.—To demonstrate the refraction of sound waves a lens shaped vessel such as an India-rubber balloon is taken and filled with CO_2 in which sound travels more slowly than in air, so that spherical sound waves starting from a source of sound P, such as a high-pitch Galton's whistle or a watch placed on one side of the lens, will be found to converge to a point Q on the other side just as light waves are converged by a glass lens. In passing through such a lens the central part of the wave-front AB advances more slowly than at the edges, so that when it comes out, it is concave towards the ear at Q.

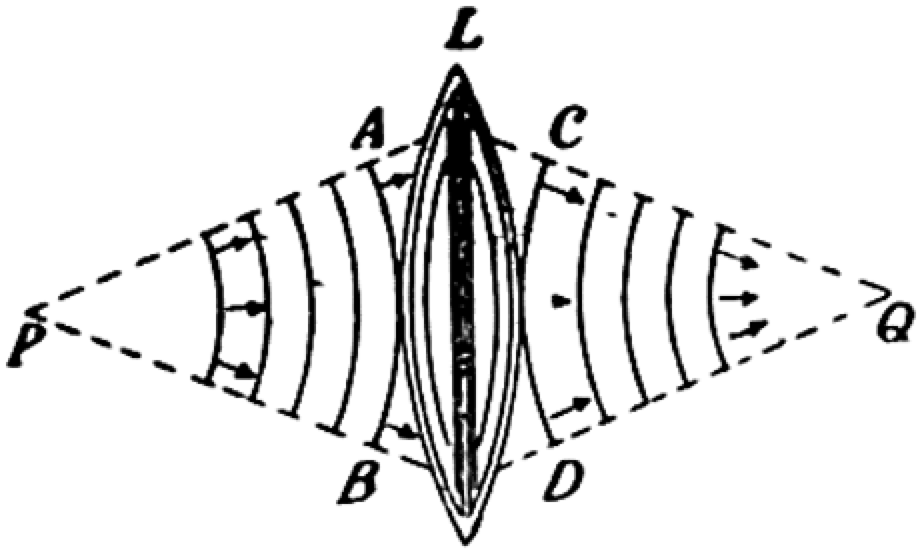


Fig. 79.

104. Total Internal reflection of Sound Waves.—When sound waves are incident on a surface separating two media A and B, such that the velocity of sound in A is less than in B, on refraction they bend away from the normal as light rays do. By the second Law of Refraction, $\frac{\sin i}{\sin r}=\frac{v_1}{v_2}$ where v_1 and v_2 are the velocities of sound in A and B respectively.

As the angle of incidence goes on increasing, the refracted ray bends away and away from the normal till for a certain angle of incidence the angle of refraction becomes 90°. This angle of incidence is called *critical angle* for medium A. If C denotes this critical angle

$$\frac{\sin C}{\sin 90}=\frac{v_1}{v_2} \text{ or } \sin C=\frac{v_1}{v_2}$$

$$\therefore C=\sin^{-1}\left(\frac{v_1}{v_2}\right)$$

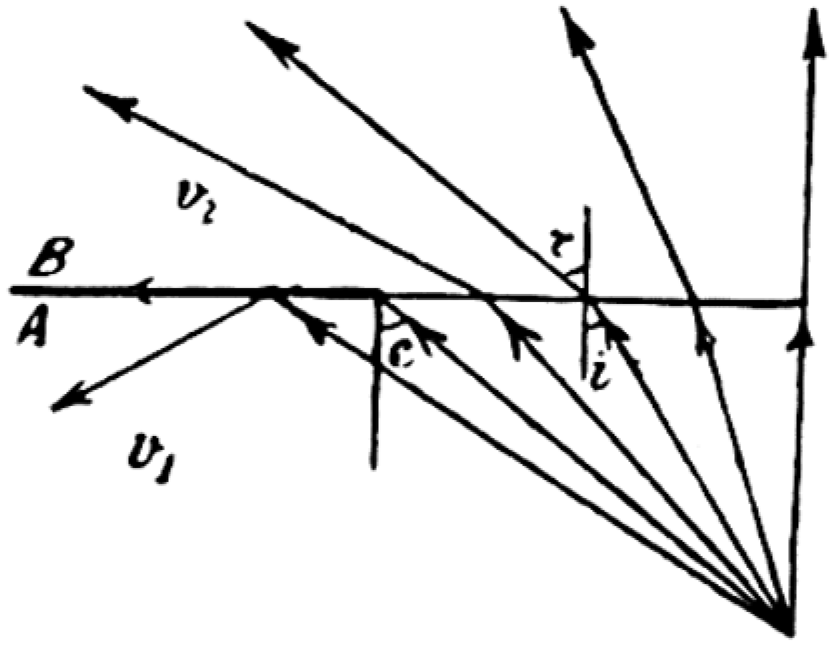


Fig. 80.

If now the angle of incidence exceeds the critical angle, the ray cannot emerge at all into the upper medium. It is then totally reflected at the refracting surface and remains in the lower medium. Suppose now the lower medium is air in which the velocity of sound is 332 metres/sec. and the upper medium is water in which velocity is 1450 metres/sec., the critical angle

$$C=\sin^{-1}\left(\frac{332}{1450}\right)=13^\circ\ 14'.$$

We find therefore, that if sound waves are incident on water surface in a narrow cone with a semi-vertical angle 13°14′ there shall be a partial refraction. If, however, the angle of incidence is greater than about 13°, they are totally reflected and sent back in the air. In the case of iron and wood the velocity is still greater and the critical angle will be much less. This explains why sound loses so little in intensity when it travels along a speaking tube or an organ pipe, for it cannot emerge unless it strikes the walls at almost normal incidence.

105. Effect of Wind on Refraction.—It is a common observation that when the wind is blowing from the sounding body to the observer, the sound is more clearly heard than if there were no wind or if it were blowing in the opposite direction. If the wind consisted of the whole mass of air all moving with the same velocity, there is no reason why the sound should be heard more clearly in one direction than in another, the velocity of sound would, of course, be changed (being equal to the sum of the wave velocity and the wind velocity if the wind is blowing in the direction of sound and equal to their difference if wind is opposing) but not its intensity. However, the velocity of wind is not the same, it is least near the ground and increases as we go higher up. Consider a plane wave front '*a*' travelling horizontally from left to right. Fig. 81 (*a*). It will successively occupy the positions shown by the dotted lines regularly spaced for equal intervals of time when there is no wind. If, now, the wind is supposed to travel in the same direction as the wave-front, the upper parts of the wave-front will have greater velocity than the lower parts. Consequently it becomes tilted forwards more and more as it moves on, and occupies the position shown by *b*, *c*, *d*, etc. But as a sound ray always travels at right angles to the wave-front, its direction will be inclined downwards and a listener at *O* will hear sound more clearly than if there were no wind.

On the other hand, if the wind is blowing against the sound, the lower parts of the wave-front travel more rapidly than its upper parts. The wave-front therefore tilts in the opposite direction, as shown in Fig. 81 (*b*). The sound ray curves upwards and travels away from the ground. This results in the thinning off of the sound intensity near the surface and a listener at *O* will therefore have much less chance of hearing.

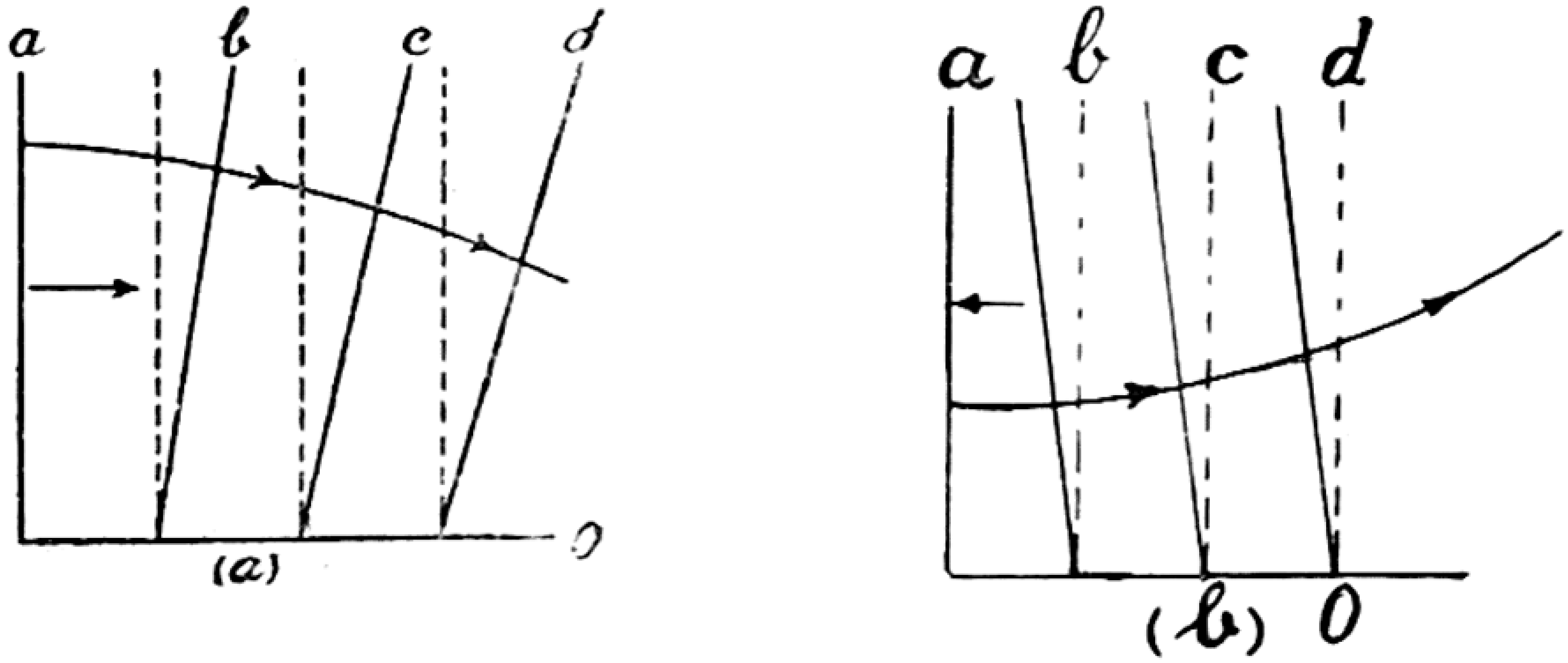


Fig. 81.

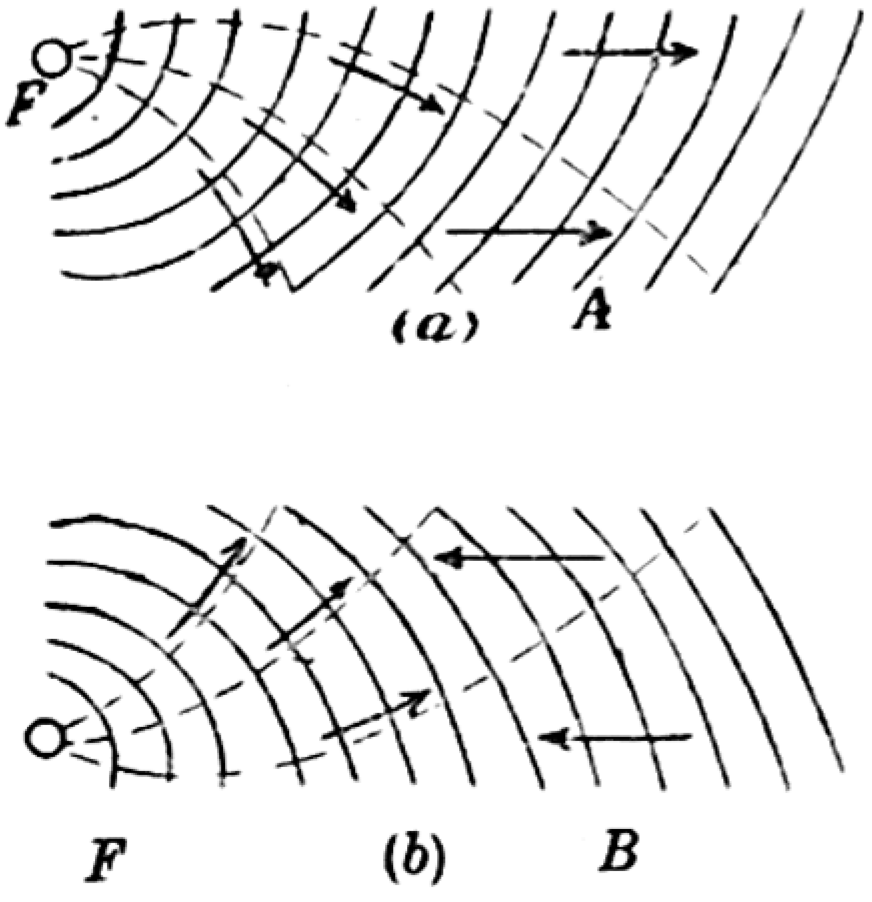


Fig. 82.

Fig. 82 illustrates the effect of wind refraction on the propagation of sound waves from a sounding body *F* blown at a height. In Fig. 82 (*a*) the wind is blowing from left to right. As explained above, *A* receives more rays than it would do in the absence of the wind, whereas, if the wind blows from right to left as in Fig. 82 (*b*) an observer *B*, similarly situated will lie in a zone of silence.

106. Effect of Temperature on Refraction.—An effect similar to that discussed above is observed due to the variations of the temperature of air at different altitudes. The velocity of sound being greater the higher the temperature, it follows that any variation of temperature from one place to another causes refraction of sound waves. During the day-time the lower layers of atmosphere are at a higher temperature than the upper layers and the velocity of the sound is, therefore, greatest near the surface of the earth and decreases as the height increases. The sound, therefore, travels in the same way as if it were travelling against a wind and the waves will be refracted upwards as in Fig.82 (*b*) and lost in the air, hence the intensity at a short distance is decreased due to this effect.

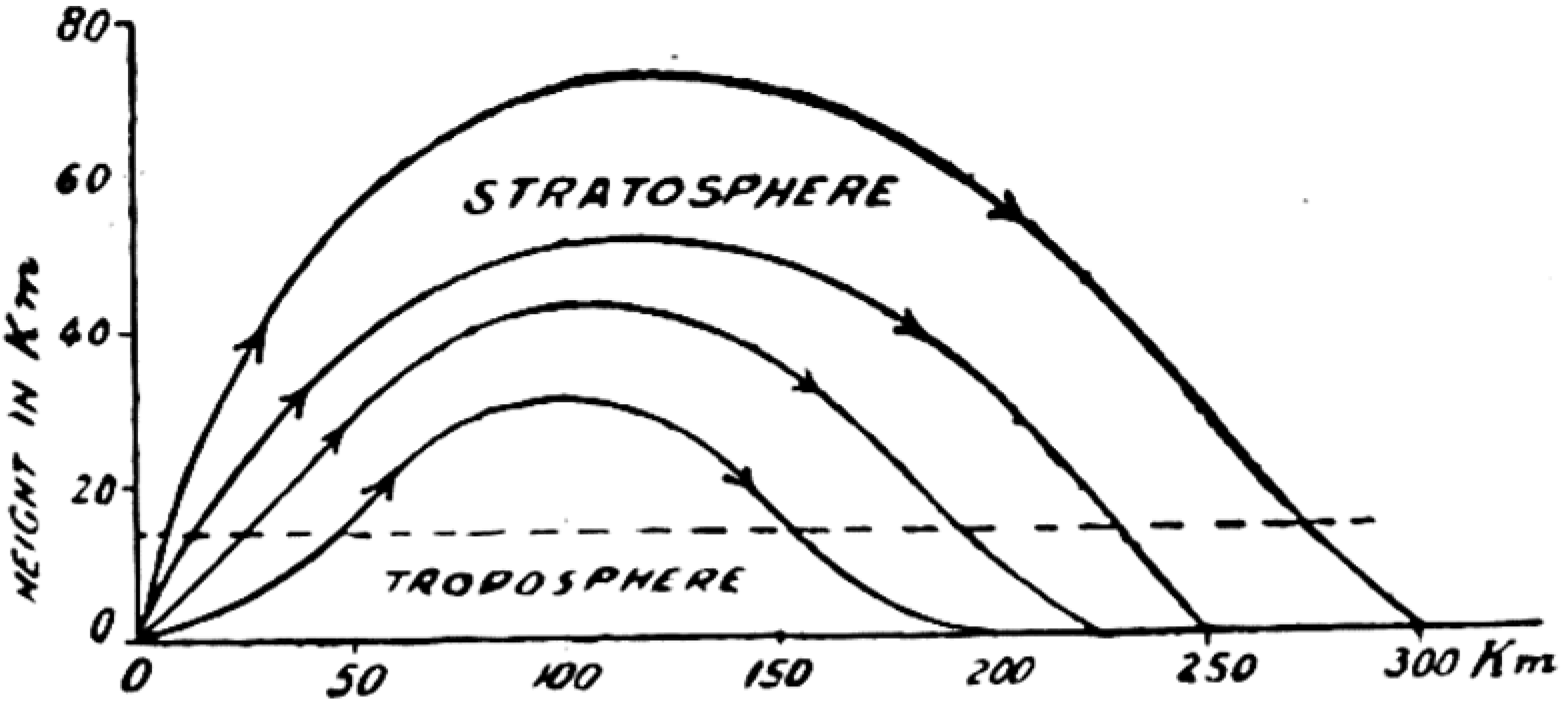
On the other hand, if the lower layers are at a lower temperature than the upper layers as frequently happens on a still evening due to the rapid cooling of the ground layers and particularly over the surface of water, the refraction of the waves is downwards as in Fig. 82 (*a*), and the sounds are found to carry to great distances along the surface. It is exactly for this reason that at dusk voices are heard with great clearness across still water. Whereas, on the contrary, on a hot summer day the sounds carry only short distances along the ground.

There is, however, another reason for the fading of the sound waves on a hot day. If the temperature gradient upwards is negative and very rapid, the air becomes unstable and convection currents are set up. This causes the air to become a heterogeneous mixture and therefore the sound waves are being continually reflected and refracted at the surfaces of separation of the various currents and quickly broken up and spread out.

Very calm air, in which the temperature is practically uniform for considerable heights, presents an almost homogeneous medium to the sound waves and therefore permits them to travel over longer distances without much loss by reflection and refraction. We have often observed that on a damp day when the air is still, the sound is carried to a longer distance than on other days as the presence of the moisture makes the medium more homogeneous.

107. Explosive Sounds and the Upper Atmosphere.—A systematic study of the propagation and the velocity of sound from explosions has been carried out during recent years and it has revealed several '**Anomalous Effects**'. It has generally been observed that sound waves of high intensity (large amplitude) travel with abnormally high velocity in the neighbourhood of the source than those of low intensity (small amplitude) while farther off the velocity assumes the normal value Besides this there is another *peculiar* phenomenon attending the explosive sounds of high intensity. It has been found that surrounding the source of sound there is a region of normal audibility which extends to about 60 miles in which the intensity of sound falls off according to the inverse square law. Surrounding this there is a zone of complete silence extending upto about 125 miles. This is followed by another zone of audibility where the sound appears with usual intensity but takes up an exceptionally long time to arrive. This is called the zone of *anomalous* audibility. Existence has been shown of a second zone of silence followed by another zone of audibility and so on. The phenomenon is analogous to 'skip distance' in the propagation of radio waves, which is considered to be due to the reflection of radio waves from ionized layers in the upper atmosphere, known as **Heaviside** and **Appleton** layers.

This anomalous behaviour can be accounted for by assuming that sound waves of high intensity break up into two parts : (1) a *Ground Wave* which travels directly over the surface of the earth and becomes sufficiently attenuated beyond a distance of about 60 miles so as to become inaudible, (2) an *Abnormal Wave* which has moved up obliquely in the upper parts of the atmosphere and is gradually bent down again to reach the surface of the earth by a curved path ; which accounts for the large interval of time between the explosion and the arrival of the sound in the second zone of audibility.



→Normal Audibility→ ←Zone of Silence←Abnormal Audibility←

Fig. 83.

Let us now see how we can explain the curved path of the waves proceeding at an inclination to the earth's surface. Research on the physics of the **Upper Atmosphere** has indicated that it is divided into two regions : (1) the **Troposphere** extending to a height of about* 12 km. above the surface of the earth in which the temperature decreases continuously at the rate of about 6° per km. of ascent until it reaches about —70° C. or —94° F. (2) the **Stratosphere** which extends beyond 12 km. In this region the temperature remains constant at first up to a height of about 20 km., where there is a *Temperature Inversion* layer and beyond that the temperature increases with height; reaches the surface temperature again at 35 km. and at 60 km. has passed 70°C. On account of the fall in temperature in the Troposphere, the velocity of a sound wave sent obliquely diminishes according to the relation $v_t=v_t\sqrt{(1+at)}$, whereby the sound is bent towards the normal as explained in the previous section and pursues a path, which is concave upwards till a height of 12 km. is reached. Then there is the region of constant temperature up to 20 km in which the inclination of the ray remains constant. Beyond that warmer regions are rapidly entered, which cause the refraction in opposite sense on account of the increased velocity. Consequently the sound waves now begin to bend towards the horizontal surface till at a certain height they become parallel to the horizontal. Further penetration in the upper regions being not possible, the waves bend downwards and ultimately reach the ground again. The paths of sound waves in the upper atmosphere are shown in Fig. 83. It is obvious. therefore, that the reversed temperature gradient above 20 km. accounts for the occurrence of the 'anomalous zone' of audibility.

Fig. 84 shows the results of a series of previously announced explosions arranged in 1923 at La Courtine in Central France. The shaded areas indicate the regions where the sound is audible. Surrounding the source there is the central zone of audibility with irregular confines which can be accounted for by the irregular conditions of wind and temperature in parts of the atmosphere. Next large areas of silence are

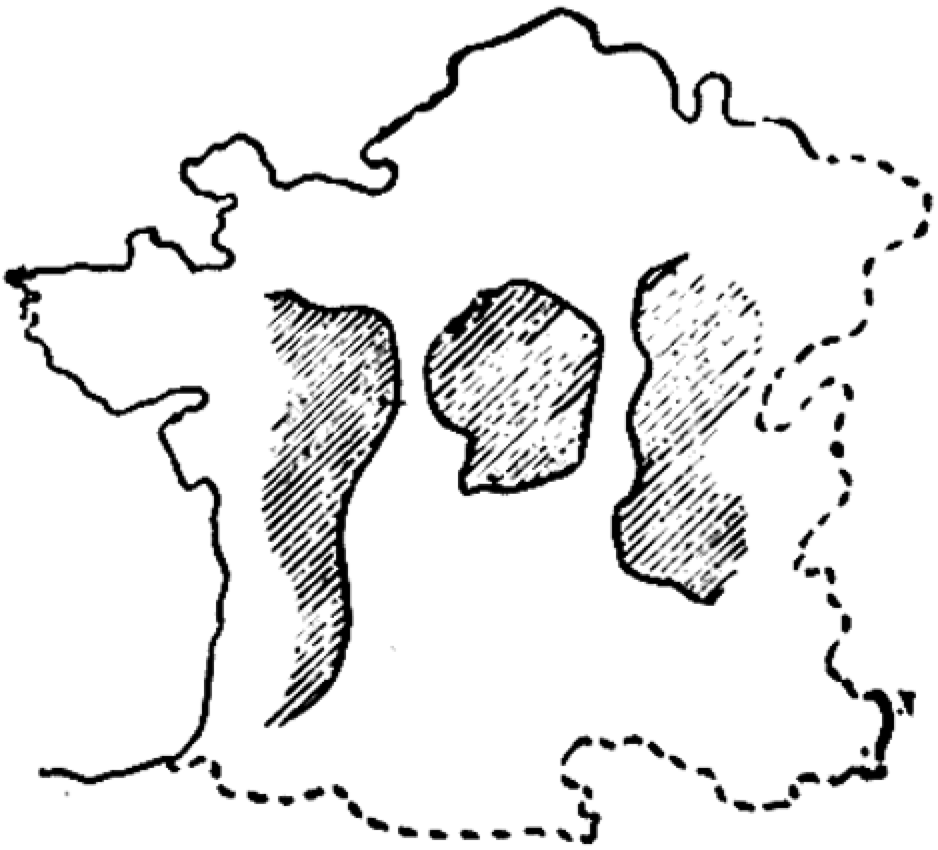


Fig. 84.

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*In our latitudes the height of the Troposphere extends to 16—17 km.

obvious. Then the second zone of audibility begins showing a fair symmetry about the source. It is into this region that sound waves have entered after refraction in the upper atmosphere.

108. Diffraction of Sound Waves (Sound Shadows).—Everyone is familiar with the fact that sound waves bend round the corners of obstacles freely and a sound is heard almost as well behind the obstacle as in front of it, whereas light waves travel almost in a straight line and if an opaque obstacle is placed in their path, no light gets to the back of the obstacle but we have a clearly defined shadow. It was this observation which led Newton to abandon the Huygen's Wave Theory of light. Later, however, carefully planned experiments with narrow slits and obstacles like fine wires, showed that light bends round corners and the shadow instead of being sharply defined is surrounded by a number of bands called **Diffraction Bands**. These experiments suggested that there was a close relation between the wavelength of the light or the sound waves and the size of the obstacle, and this explained the apparent anomaly between the behaviour of sound and light waves with regard to obstacles. It is found that a sharply defined shadow can only be cast when the dimensions of the obstacles are very large as compared with the wavelength. The frequency of the ordinary sounds varies from 200 to 500 and therefore the wavelengths will vary from 5½ ft. to 2¼ ft. To cast shadows of such sounds the dimensions of the obstacle should vary from 200 ft. to 500 ft. at least. As ordinary obstacles, like houses and walls, are of much smaller dimensions the **sound shadow** is not well-defined. To obtain sharp sound shadows with obstacles of moderate size, it is therefore necessary to use very short waves *i.e.*, sounds of high frequency, such as from a Galton's whistle.

Fresnel has shown mathematically that the extent of the bending of the waves inside the geometrical shadow increases with the increase in the wavelength. All such effects involving the bending of sound waves are described as diffraction. It is due to the mutual interference of the secondary wavelets from the same wave-front and plays a very important part in the emission, reception, reflection, refraction and scattering of sound waves from high frequency sources.

The *peculiar* effects of diffraction are also observed in the case of transmission of sound through apertures. When sound waves pass through an opening which is not more than a wave-length in diameter the waves are found to spread out in every direction from the opening. [Fig. 85 (*a*)], while if the opening is much larger, the waves on account of interference will not spread out so much but travel forwards at right angles to its own wavefront [Fig. 85 (*b*)]. In this case no sound is audible except at points from which straight lines can be drawn to the source of sound.

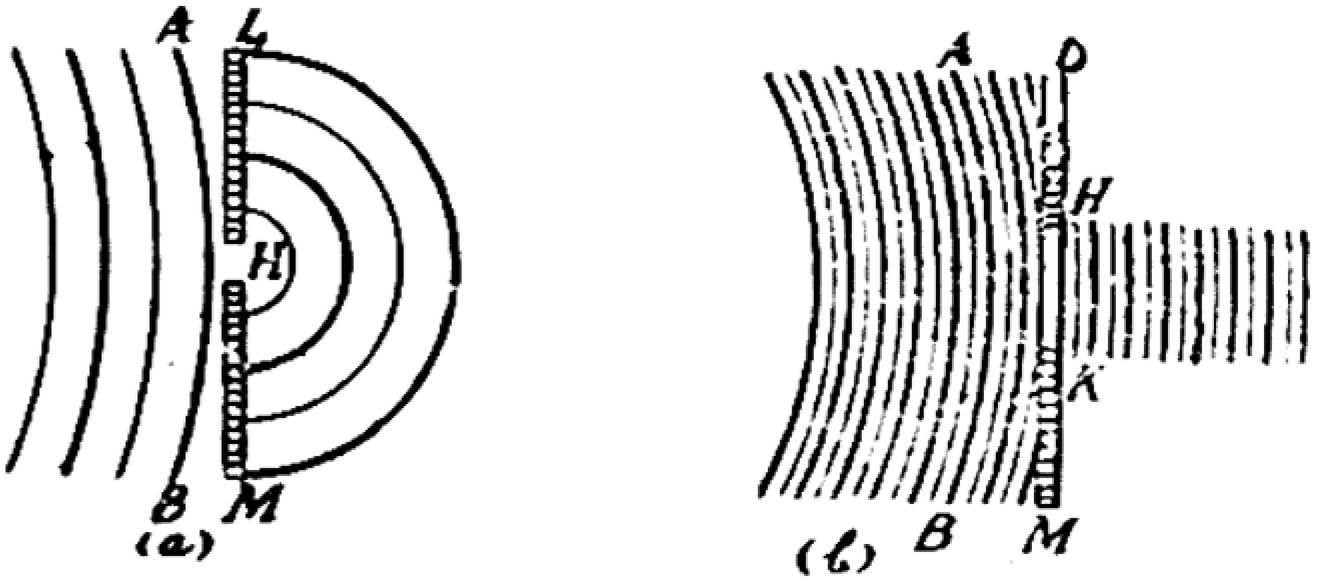

Fig. 85

The action of a **Megaphone** and **Speaking Trumpet** may now become intelligible. The apparatus is conical in shape with a narrow end where one speaks. The sound waves coming from the speaker instead of spreading out in all directions from the mouth are limited by the walls of the instrument so that the wave emerging from the wide end has the whole energy of the voice. As explained in the last paragraph, the mouth being a small opening the sound waves coming directly from it spread out in every direction while the waves emerging from the wide end of the trumpet or the megaphone, which corresponds to the larger opening, do not spread out so much and produce a more intense effect directly in front.

CHAPTER IX

FREE, FORCED AND RESONANT VIBRATIONS

109. **Free and Forced Vibrations**.—We have so far been considering the oscillatory motion of the type in which the bodies have been vibrating with their *natural* rate of vibration which depends upon their dimensions and the elastic constant of the material of which the bodies are made. A tuning-fork vibrating freely, the strings of the musical instruments and the reeds of a harmonium have all natural vibration rates. A pendulum displaced from its position of equilibrium and left to itself, vibrates under the action of gravity with a definite period depending upon its length. The period of vibration of such bodies is called *Free Period* and the vibrations are called **Free Vibrations**. Such vibrations diminish in amplitude according to certain simple laws and ultimately die away.

We next consider a very important case where a body is maintained in a state of vibration by a periodic force which may or may not have the same period as the free vibration. The body in this case, will ultimately vibrate with the same period as that of the force, its vibrations being designated **Forced Vibrations** since their frequency is determined by that of the applied impulses rather than by the characteristics of the body itself. To illustrate it, hold the shank of a tuning-fork against the top of a table. You will notice that the sound is intensified, no matter what the frequency of the fork is. We say that the table is forced to vibrate in response to the periodic force applied. Both resonant air chambers and sounding boards in stringed instruments act precisely as the table top does, and the loudness of sound is due to these forced vibrations.

When a body vibrating freely has no resistance offered to its motion, its amplitude remains constant. But in general there is always some resistance to be overcome which is called *Damping*. The effect of damping is two-fold, (*i*) a decrease in amplitude and (*ii*) a slight change in frequency. It is usually a slight increase in the time-period (*See* Art. 122). If the resistance is very large, motion becomes *aperiodic*.

110. Resonance.—When the period of the applied force coincides with the free period of the body, the force helps to increase the amplitude of the body at each step of its vibration being always in phase with it. The effects of the successive impulses or pressure changes due to the periodic force may, therefore, be added up together and almost the whole of the work done by the force will be used up in increasing the amplitude which would become considerable. Such forced vibrations greatly exceeding in extent those which would be due to pressure changes actually occurring and resulting from accumulated pressure impulses, are called sympathetic or resonant forced vibrations or simply **Resonance.**

111. Demonstration of Forced and Resonant Vibrations.—Suspend from a flexible support, say, a light wooden stick four pendulums, *B* and *C* of equal lengths and *A* and *D* of different lengths. If one of the equal pendulums, say *B*, with rather a heavy bob is set swinging. *C* will soon gain an equal amplitude but *A* and *D* will never swing more than a small arc and if you notice carefully you will find that they keep getting up to a certain amplitude and then dropping to zero and so on. As *B* swings to and fro, it sends periodic impulses along the stick which pass on to *A*, *C* and *D*. These impulses are due to the sudden motion to one side of the point of support. The application of these periodic disturbances tends to set other pendulums in vibration.

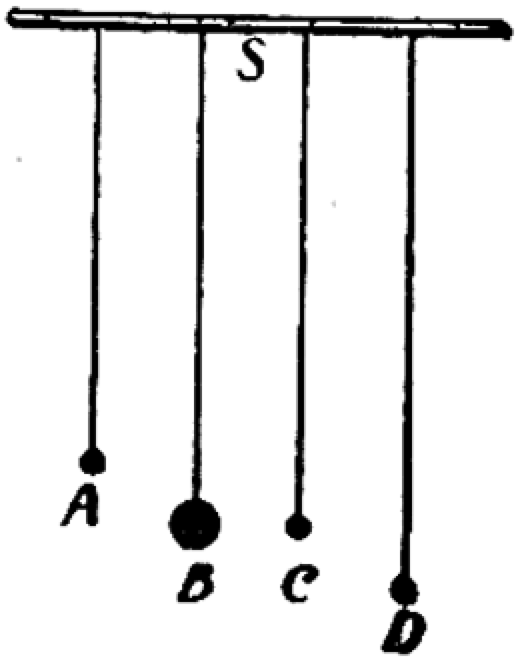

Fig. 36.

Consider first the effect on *C*. The first impulse disturbs it from rest and it swings out and back again. But when it has just gone through a complete swing, the second impulse arrives and the motion being in the same direction as that produced by the first impulse, the second impulse does work on *C* *i.e.*, it gives its energy and increases the swing. As the successive impulses always arrive in phase with its own natural period. *C* is quickly set into vibration. This is the case of *Resonance.*

Now let us consider the effect on *A* or *D*. Suppose there is not much difference between the time-periods of *B* and *A* or *D*. As in the previous case the first impulse disturbs *A* or *D* and the bob swings out and back again. But before the second impulse arrives, *A* has more than completed the swing (being shorter than *B*) and *D* has nearly completed it (being longer). So they are still approximately moving in the original direction and have work done upon them *i.e.*, receive energy. For the first few swings the impulses due to *B* will arrive to find the pendulums *A* and *D* to be approximately in phase with them and their amplitudes will increase. But as the impulses get more and more out of phase with the pendulums, a time will come when *A* or *D* will be moving in the opposite direction when they arrive and now they oppose the motion and take energy away from *A* or *D* and for several swings the amplitude gets less and less till *A* or *D* are at rest. But then the impulses act exactly as before and the swings grow again. These rises and falls of amplitude will continue for some time as though the vibrating pendulums are struggling to assert their own natural periods against the applied periodic force till ultimately, as we shall explain later, the pendulums will make a sort of compromise and settle down to a vibration that has the same period as that of *B* but the amplitude will remain small. The vibrations in *A* and *D* are called *Forced Vibrations.*

112. Some Examples of Forced and Resonant Vibrations.—(1) A remarkable instance of forced vibrations occurs when the two clocks which keep approximately the same time when placed on different stands, maintain the same time when on the same stand. The pendulum of the clock which gains normally exerts a periodic force on the second so that the two periods tend to become equal ; the second clock exerts a similar effect on the first so that eventually the two periods are equal and the clocks synchronise.

(2) The rythmic marching of troops over a suspension bridge sets it swinging if the natural period of the bridge is an integral multiple of that of the marching 'tempo'. Then the impulses always occur in the same phase as the swing of the bridge and it oscillates more and more violently till its amplitude assumes dangerous proportions. On this account the troops when marching over a bridge are ordered to break step.

(3) Suppose we have a motor, of which the speed can be gradually varied, mounted on a soft pad. Now the armature of a motor is never quite balanced and when rotating, will produce a small back and forth vibration of the motor for every revolution. Suppose a thin steel spring is clamped at one end of the

base and the motor is started with a slowly increasing speed. While it is low, the vibrational frequency of the base is far below the natural frequency of the spring which, therefore, is set into forced vibrations with a small amplitude. As the vibration frequency of the base approaches the natural frequency of the spring, its amplitude commences to increase rather sharply. When the periods exactly coincide, we have resonance and the spring may swing through an arc 15 to 20 cm. With further increase in speed the amplitude dies down again to a few mm.

(4) A child soon learns that in order to 'work up' in a swing *i.e.*, to gain a large amplitude, he must tune the force he exerts to the natural period of vibration of the swing

(5) Often when one speaks or sings near a piano, a string may be set in audible vibration. This is due to the fact that the frequency of the sound waves striking the string is equal to the natural frequency of the string. In addition to these illustrations the phenomenon of resonance plays an important part in other branches of science, namely, in Resonant Spectra and Wireless Reception.

113. Resonance Box of a Tuning-fork.—Another example of everyday use of resonance is the resonance box of a tuning-fork. As a tuning-fork is generally used as a standard of constant known frequency, it is desired that it should emit a loud sound. For that purpose the shank of a tuning-fork is mounted on a box called **Resonance Box.** It is a thin wooden box open at one end. The dimensions of the box are such that the natural time-period of the air in it agrees with that of the fork. Ordinarily, when a tuning-fork is struck, its sound can be heard only when placed close to the ear. But when it is mounted on the box, the air in the box is set into sympathetic vibration and the intensity of sound is considerably increased. Owing to the rapid loss of energy in the form of sound waves the duration of the vibration of the fork will be much less than when it is held in the hand.

Perhaps the most remarkable case of resonance is that of two tuning-forks of exactly the same pitch mounted on suitable resonance boxes (Fig. 87). The boxes are placed on a table close together with their open ends facing each other. One of them is bowed and after a few seconds it is silenced, when we will find that the other one is sounding feebly. Here, though the energy of the air waves emitted by the excited fork is small, yet their cumulative effect is to set energetic vibrations in the other. The excited fork impresses its periodic movements on its own resonance box. This sends out waves which excite sympathetic vibrations in the resonance box of the other making its top vibrate which in turn sets its own fork to vibrate.

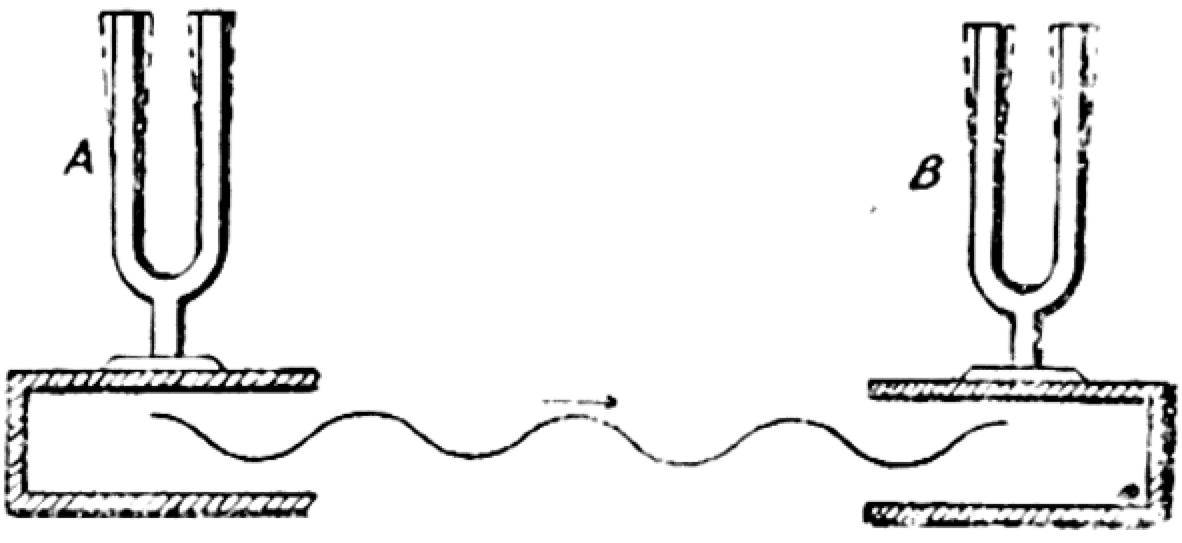

Fig. 87.

114. Sounding Board and Forced Vibrations.—A sounding board plays a very important part in musical instruments like pianoforte, harp, violin, etc. as it helps to increase the loudness of the sound emitted by the vibrating body. If we stretch a wire between two isolated rigid supports lying on a hard floor and pluck it, the sound heard is very feeble for the air merely slips round the string and no proper condensations and rarefactions are set up. As the string alone is in contact with a small quantity of the air, it sets only a small quantity in motion The intensity of the sound is further reduced on account of the interference of the compression set up on one side of the string and a simultaneous rarefaction on the other. To increase, therefore, the energy transmitted to the surrounding medium, the strings are fixed by suitable flexible supports on a sounding board to which its vibrations are communicated. The strings exert a periodic force on the supports which in turn set up forced (and not resonant) vibrations in the box. As the damping is very large the sounding board is able to take up vibrations over a very wide range of frequencies. In this manner a much larger vibrating area is brought in contact with the air and the rate of emission of energy is greatly augmented The duration of sound is, however, proportionately decreased. It is desirable in good stringed instruments that the natural tones of the sounding board should not be close to the frequencies of the strings, otherwise there will be a disproportionate reinforcement of these frequencies.

Let us now *clearly* understand the action of the sounding board *i.e.*, the manner in which a stretched string communicates its vibrations to it.

In Fig. 88, *AB* represent a vibrating string attached by means of two flexible supports to a sounding board *PQ*. The string during its up and down motions exerts a force on the supports drawing them a little closer when it is at the upper and at the lower limits of its swing. The supports, therefore, vibrate with twice the frequency of the string. The sounding board, however, follows the movements of the string and vibrates with the same frequency as the string. When the string is at the lower limit of its swing, the supports are drawn inwards and the corresponding portion of the sounding board is concave downwards as shown in 1′. When the string recovers the normal position, the board straightens but on account of inertia swings to the other side becoming concave upwards. This concavity and consequently the amplitude of its vibration is further increased by the ends of the supports drawing closer again by the action of the string which has in the mean time reached its upper limit, the board *PQ* now occupying the position as shown in 2′. We learn that the string and the sounding board though vibrating in opposite directions are always in tune with each other and these forced vibrations continue so long as the string is bowed.

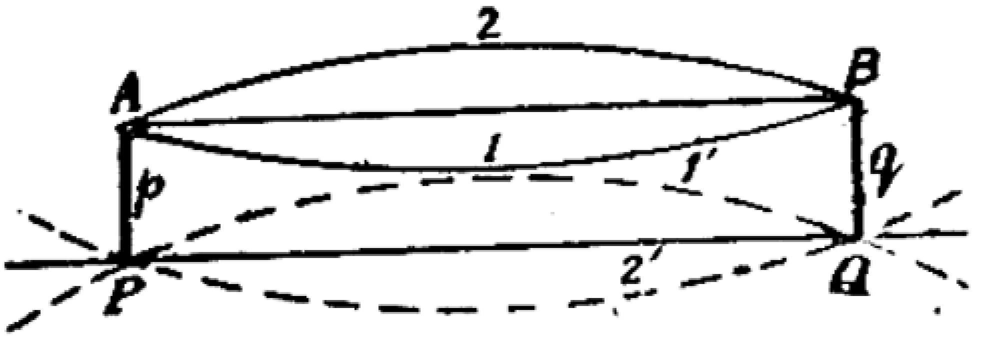

Fig. 88.

115. Further Consideration of Forced Vibrations—In order to clearly understand the phase and amplitude relations in the case of forced vibrations, let the experiment described in Art. 111 be varied by suspending a small pendulum *B* from a heavy pendulum *A* (Fig. 89). When *A* is set oscillating it will impress on *B* a harmonic force of its own

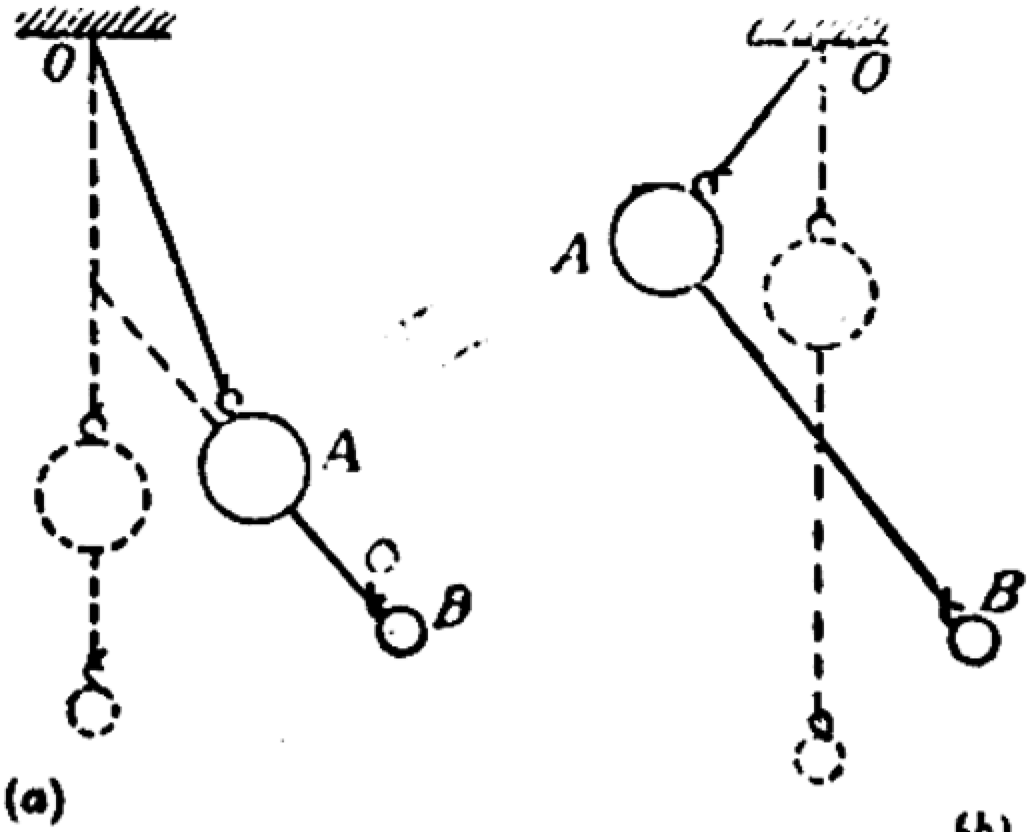

Fig. 89.

period and produce forced oscillations in it. The amplitude of *B*, however, will depend upon the relation of the natural period of *B* to that of *A*. The following three cases will now be considered :—

(*i*) The natural period of *A* is greater than the natural period of *B* *i.e.*, the frequency of the applied force is less than the frequency of the body. This is obtained by making the length of *A* greater than the length of *B* [Fig. 89 (*a*)] We find that soon *B* settles down to a regular vibration of the same period as *A* and in the same phase as *A*. Its amplitude is slightly larger than that of *A*.

(*ii*) The time-periods and hence the frequencies are exactly equal. In this case the amplitude of the forced vibration becomes very great. Theoretically speaking the amplitude should be infinite but on account of friction etc. it is never so. This is the case of *Resonance*,

(*iii*) Time-period of *A* is less than that of *B* *i.e.*, the frequency of the applied force is greater than that of the driven. The motion in this case is opposite in phase as shown in Fig. 89 (*b*).

We have seen that in all the cases, there is a certain amount of vibration set up. This, however, soon dies away leaving the forced vibration only. The amplitude of the forced vibration may be calculated in terms of the characteristics of the applied force. It is equal to

$$f/\sqrt{(n^2-p^2)^2+4k^2p^2},$$

where f is the intensity of the applied force. n the natural frequency of the body, p the frequency of the applied force, k the 'coefficient of damping.' The expression is deduced from dynamical considerations discussed in Art. 123 [see equation (22).]

Fig. 90 shows in a graphical manner the result of some simple cases for various degrees of damping. The frequency (p) of the applied force in terms of the natural frequency (n) of the body is plotted along the abscissae while the ratio of the amplitude of the forced vibration to that of the applied force along the ordinate If p is small as compared to n, *i.e.*, the length of the pendulum *A* is very large compared to *B* the resonator (pendulum *B*) follows the motion of the point of suspension (the ratio between the amplitudes $A=1$.) As p increases the amplitude of the resonator increases more and more until for $p=n$, it becomes infinitely great being ***asymptotic*** to the y-axis shown dotted. This is the important case of sympathetic resonant vibration. But as some damping due to friction etc., is always present in a vibratory system, an infinite amplitude is never attained as the theoretical curve indicates but a curve like 1 is obtained. As p is further increased, the amplitude of the forced vibration decreases again tending to zero for large frequencies of the applied force. The motion of B consists of rapid tremblings with very small amplitude. It may be noted that the two vibrations for values of p greater or less than n are in opposite phase.

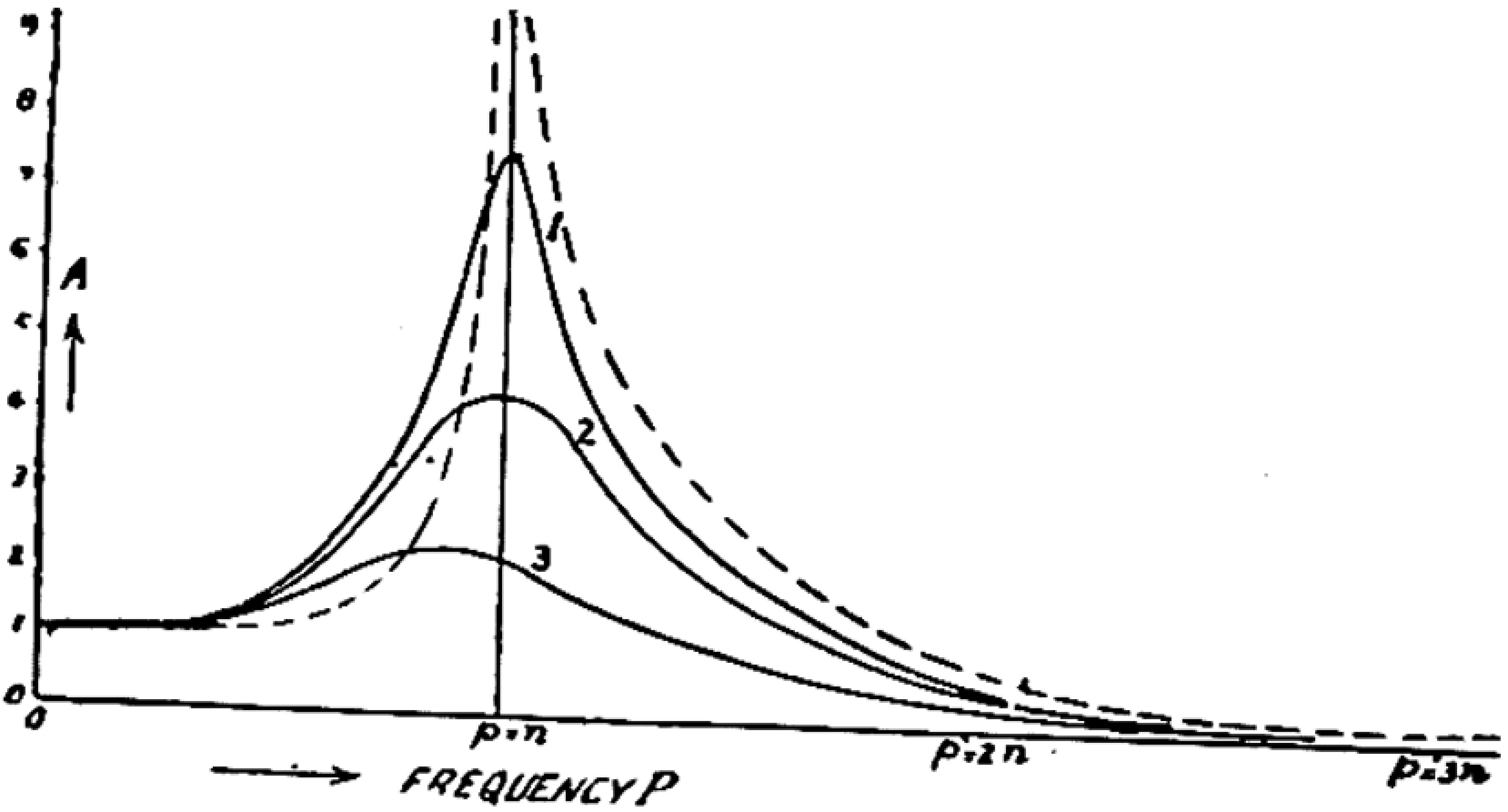

Fig. 90.

116. Phase of Forced Vibrations.—If θ be the phase difference between the applied force and the resulting displacement of the forced vibration then

$$\tan\theta = \frac{2kp}{n^2 - p^2}$$ [Art. 123 equation (19 *a*)

This relation shows that the phase between the impressed force and the resultant forced vibration depends upon the amount of damping and the relative magnitudes of p and n. If damping be negligible, the phase of vibration coincides nearly with that of the impressed force for all values of p smaller than n, since $\tan\theta$ is a small positive quantity. For all values of p greater than n, however, $\tan\theta$ is a small negative quantity, *i.e.*, the forced vibration differs in phase by π nearly, *i.e.*, half a period.

If $p=n$, $\tan\theta$ is infinite and θ is, therefore, $\pi/2$, *i.e.* the impressed force and the vibration differ in phase by quarter period whilst force and velocity will be in the same phase.

This is the case of exact *Resonance*. For appreciable damping, it is seen that the phase difference gradually increases. For

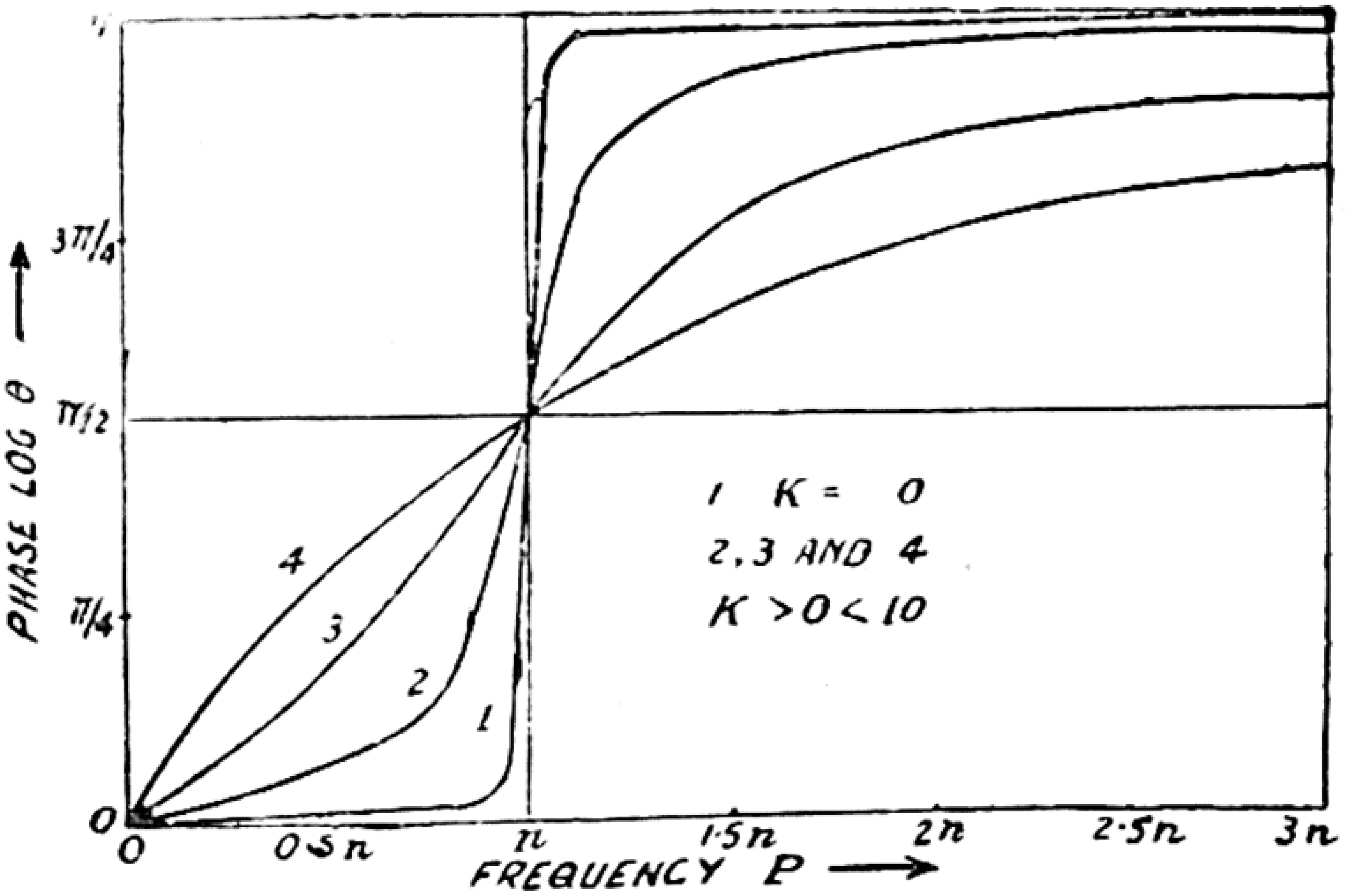

Fig. 91.

slow forced vibrations (*i.e.*, $p<n$) it is very small, increases with increasing frequency of the impressed force in a manner dependent upon the degree of damping and finally always reaches the value $\pi/2$ at resonance. The forced vibration is in its position of greatest displacement when the impressed force is just passing through its equilibrium position. Fig. 91 shows phase differences as a function of impressed frequencies for different damping factors marked 1, 2, 3, etc.

117. Effect of Mistuning on the Amplitude of Resonant Vibrations—Sharpness of Resonance.—Experiment shows that a given amount of mistuning between the impressed and the natural frequencies has not always the same harmful effect on the intensity of resonant vibrations. When the frequencies agree, it is only the presence of damping which prevents the vibrations from attaining an infinite amplitude. The damping becomes more and more powerful in its influence as resonance is approached. Further, the greater the dependence of the intensity of forced vibration on the exact equalities of the frequencies of the external force and the body, the *sharper* is the resonance said to be.

The mathematical investigation shows that sharpness of resonance varies inversely as the square of damping coefficient.

In other words the smaller the frictional damping, the more important does exact tuning become in order to obtain greater energy in the forced vibration.

For smaller values of damping coefficient, the peak value of the amplitude is greater and moreover, the amplitude falls rapidly on either side of the resonance frequency as shown in curve (1) of Fig. 90. This corresponds to *sharp* resonance, *i e.*, here the tuning must be close to get powerful resonance. It is beautifully illustrated by a tuning-fork which may be excited through the medium of surrounding air by the vibration of a second tuning fork. When the two forks are closely in tune, there will be a very marked response, otherwise there is practically no effect.

On the other hand, we find that in a sounding box or in an air column inside a pipe where the damping is considerable, the resonant vibrations can be easily excited for a wide range of frequencies on either side of the resonant frequency as shown in curves (2) and (3) of Fig. 90. This type of resonance is said to be *flat.*

The explanation of these facts is very simple. Resonance is due to the cumulative effect of successive impulses. If the damping is very large as in the case of vibrating air columns or a sounding box, the natural vibrations will decay quickly so that by the time, say, the 15th impulse is coming, the effect of the first impulse has already disappeared. This means there is practically no chance for the impulses to get appreciably out of phase and thereby diminish the amplitude. For maximum amplitude, therefore, the body and the impressed force have to be in phase for 15 periods only, that is, a slight amount of mistuning will not produce a harmful effect on the amplitude of resonant vibrations. If, on the other hand, the natural vibrations die away slowly as in a tuning-fork there is ample opportunity for the successive impulses to get out of phase soon and neutralize the effect of earlier impulses. In such a case much closer tuning would therefore be necessary for maximum intensity of resonance so that the impressed force and the natural vibration may practically always remain in phase.

118. Resonators.—A 'resonator' is a gas column having a natural vibration frequency of its own. There are various resonators. They will resound energetically to tuning-forks of appropriate pitches. Helmholtz, for his work on complex sounds and other acoustical determinations, devised special types of resonators known after him as **Helmholtz's resonators**. Two forms of Helmholtz's resonators in general use are shown in Fig. 92. They are either globular or cylindrical boxes made of brass with a wide mouth at one end to receive the exciting waves and a narrow neck at the other to conduct the vibrations to the ear. The air cavity is almost completely enclosed with the result that only a small portion of the energy is radiated into the medium. The damping is, therefore, small and the tuning sharp. For this reason it is specially suitable for detecting sounds of a definite pitch and feeble intensity. If a compound note is sounded near such a resonator and if it contains the component to which the resonator can resound, it will 'pick out' and magnify the component. Such resonators are made in sets the pitch of each being known, and they can be used to detect the components of a complex note although these components may be too feeble to be detected by the ear alone. The form (*b*) is a cylindrical resonator made in two pieces to slide one inside the other so that the volume of the air inside is varied at will and thus the same resonator can be tuned to different frequencies.

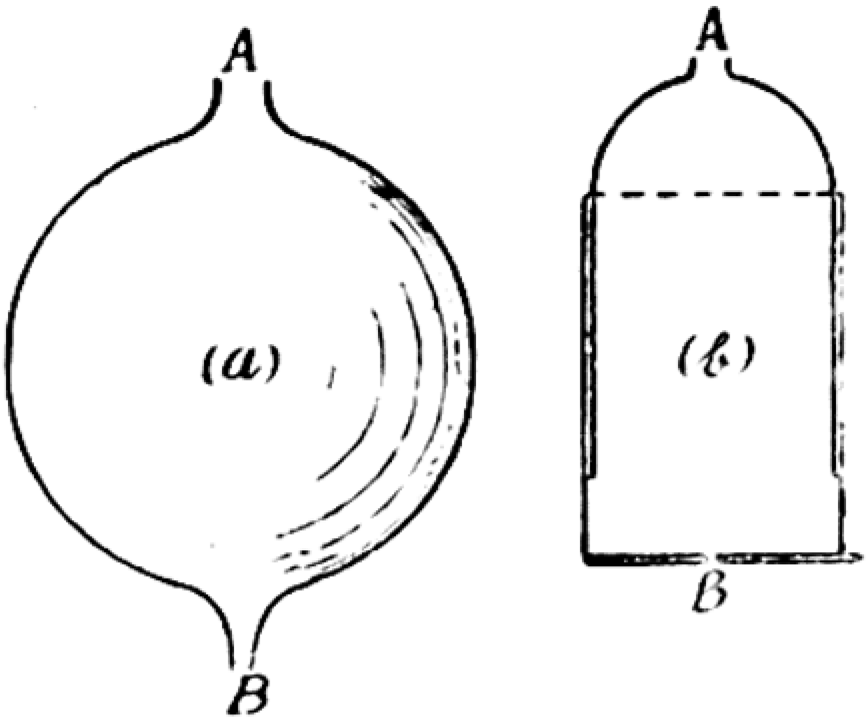

Fig. 92

***119. Theory of Resonators**.—Let us now see how oscillations are caused in resonators and what is the frequency of such oscillations. Rayleigh, who developed the theory of these resonators, regards the air in the mouth as acting like a reciprocating piston alternately compressing and rarefying the air contained in the resonator. Further, it is assumed that the dimensions of the resonator are small as compared to the wavelength of sound in the free air and therefore, at any instant, a condensation will be uniform throughout. The action of a resonator may be described as follows. Let us suppose that the stream of air rushes in by the mouth A. It increases the pressure of the air inside. The increase of pressure checks the stream of air and causes an outward rush and the momentum, thus acquired, will carry the stream past its position of equilibrium and a deficit of pressure is thereby acquired inside the resonator Consequently a stream flows in the reversed direction and thus periodic oscillations are set up (*see* Art. 4). Comparing these oscillations with a mass attached to a spring we can say that the air piston in the mouth is the mass and the air in the cavity is the spring. Let l and a denote the length and the sectional area of the mouth and ρ the normal density of the gas in the mouth. Then the mass (M) of the air piston is ρla. For adiabatic changes in the volume of the cavity, we have $pv^{\gamma}=$constant, where p and v are the pressure and volume respectively, and γ is the ratio between the two specific heats of air.

Differentiating we have,

$$\gamma pv^{\gamma-1}dv+v^{\gamma}dp=0, \text{ or } dp=-\gamma p\frac{dv}{v}.$$

For a small change in the volume the total force on the piston is $dp\times a$ and the change of volume dv per unit displacement of the piston is $a\times 1=a$. Thus the restoring force (F) per unit displacement of the piston $=dp\ a^2=-\gamma p\frac{a^2}{v}$ or $E_{\phi}\ \frac{a^2}{v}$ where E_{ϕ} $(=\gamma p)$ is the adiabatic elasticity. The differential equation of motion is

$$M\frac{d^2y}{dt^2}+Fy=0.$$

which indicates a frequency of oscillation

$$N=\frac{1}{2\pi}\sqrt{\frac{F}{M}}.$$

Putting the values of F and M we have,

$$N=\frac{1}{2\pi}\sqrt{\frac{E_{\phi}a^2}{v\rho la}}=\frac{u}{2\pi}\sqrt{\frac{a}{lv}}, \text{ where } u=\sqrt{\frac{E_{\phi}}{\rho}}$$

***120. Mathematical Treatment of the Phenomenon of Oscillatory Motion.**—So far we have treated the phenomenon of oscillatory motion only experimentally. We shall now investigate the dynamical basis for such motion. Let us recall some of the simple cases, namely—the motion of a simple pendulum, or that of a vibrating string of a violin. The system when displaced from its stationary position would return to it on account of the restoring force called into play but the momentum acquired carries it past that position to the other side till its kinetic energy is wholly converted to potential and the system comes to a momentary rest. Again the force brings it back and the to-and-fro motion is repeated which, but for resistance, may continue indefinitely. In all such cases an interchange of kinetic energy to potential energy and *vice versa*, is taking place. Assuming that neither energy is received nor dissipated into heat by internal friction etc., we have from the Law of Conservation of Energy that the sum of kinetic and potential energies will be constant.

Let us consider the oscillatory motion of such a conservative system, say, of a mass m under the influence of a restoring force which is $-\mu$ times the displacement. Then its kinetic and potential energies are given by

$$K=\tfrac{1}{2}mv^2=\tfrac{1}{2}m\left(\frac{dy}{dt}\right)^2 \quad \dots \quad \dots \quad (1)$$

and

$$P=\int_0^y \mu y\,dy=\tfrac{1}{2}\mu y^2 \quad \dots \quad \dots \quad (2)$$

Since there is no loss or supply of energy by some external agency, we have

$$(K+P)=\text{constant} \quad \dots \quad \dots \quad (3)$$

or

$$\tfrac{1}{2}m\left(\frac{dy}{dt}\right)^2+\tfrac{1}{2}\mu y^2=C \quad \dots \quad \dots \quad (4)$$

Differentiating with respect to t, we have

$$m\frac{dy}{dt}\frac{d^2y}{dt^2}+\mu y\frac{dy}{dt}=0$$

or

$$m\frac{d^2y}{dt^2}+\mu y=0$$

or

$$\frac{d^2y}{dt^2}+n^2y=0, \quad \dots \quad \dots \quad (5)$$

This is the differential equation of the motion sought. The integration of this equation will give us the displacement of the particle under the influence of the restoring force. Multiplying this by $2\frac{dy}{dt}$, we have

$$2\frac{dy}{dt}\frac{d^2y}{dt^2}+2n^2y\frac{dy}{dt}=0$$

Integrating we get $\left(\frac{dy}{dt}\right)^2+n^2y^2=C'$

To determine C', suppose $y=a$, when $\frac{dy}{dt}=0$.

then $$C'=a^2n^2.$$

$$\therefore \quad \left(\frac{dy}{dt}\right)^2=a^2n^2-n^2y^2=n^2(a^2-y^2)$$

or $$\frac{dy}{dt}=n\sqrt{a^2-y^2}$$

or $$\frac{dy}{\sqrt{a^2-y^2}}=n\,dt$$

Integrating again $\sin^{-1}\frac{y}{a}=nt+\phi$

whence $$\frac{y}{a}=\sin(nt+\phi)$$

or $$y=a\sin(nt+\phi) \qquad \dots \qquad \dots \qquad (6)$$

where a and ϕ are arbitrary constants. This motion is Simple Harmonic with amplitude a, phase ϕ and of time-period given by

$$T=\frac{2\pi}{n}=2\pi\sqrt{\frac{m}{\mu}} \qquad \dots \qquad \dots \qquad (7)$$

Example. *As an example of the application of this method consider the case af a mass M suspended by a coiled spring, the mass of the latter being too great to be negligible. Assuming the spring of mass m to be uniformly stretched at every instant of its motion we have its potential energy*

$$P=\tfrac{1}{2}\mu y^2$$

where $\mu=q\frac{s}{l}$, q being the Young's Modulus, s the cross section and l, the length of the coiled spring.

Kinetic energy of the suspended mass

$$=\tfrac{1}{2}M\left(\frac{dy}{dt}\right)^2$$

To calculate the kinetic energy of the spring take the axis of x along the spring, the origin being at the upper or the fixed end. Then $\frac{1}{2}\left(\frac{m}{l}dx\right)\left(\frac{x}{l}\cdot\frac{dy}{dt}\right)^2$ is the K.E. of an element of the spring of length dx and abscissa x while the lower end has the speed $\left(\frac{dy}{dt}\right)$.

Hence the K.E. of the whole spring and the mass at the same instant is given by

$$K=\tfrac{1}{2}M\left(\frac{dy}{dt}\right)^2+\tfrac{1}{2}\frac{m}{l}\int_0^l\left(\frac{x}{l}\cdot\frac{dy}{dt}\right)^2dx,$$

$$=\tfrac{1}{2}M\left(\frac{dy}{dt}\right)^2+\tfrac{1}{6}m\left(\frac{dy}{dt}\right)^2.$$

whence proceeding as above, we have,

$$T=2\pi\sqrt{\left(M+\frac{m}{3}\right)/\mu}.$$

***121. Damped Vibrations.**—In the above discussion it has been assumed that the vibratory system is not subject to any dissipative or frictional forces involving loss of energy and therefore it would continue to vibrate for an indefinite time with a constant amplitude. But in actual practice we find that this does not go on for ever. The prongs of a tuning-fork vibrate with steadily decreasing amplitude; the swing of a pendulum becomes smaller and smaller. The cause of this diminution lies in the presence of frictional forces both within the metal of the tuning-fork (internal friction) and in the retarding action of the medium (air or water etc.) upon the motion. The amplitude of the oscillations gradually diminishes and ultimately the body comes to rest. The frictional forces change with the velocity of the body but for small velocities the retarding force due to a fluid may be taken to be proportional to the velocity. Much experimental work has been done to make sure how the force of fluid friction depends upon v since this is a matter of very great concern in aerodynamics. The work of the Russian Mayevski with rotating projectiles has shown that the force of air resistance upon elongated missiles can be represented by the formula,

$$F = R_n v^n.,$$

where n and R have different values in different velocity ranges. At ordinary velocities $n=1$, that is, the resistance is directly proportional to the velocity.

Returning to our problem, the motion of the particle which we now attempt to consider is subject to (*i*) a restoring force proportional to the displacement and (*ii*) a resistance varying as the velocity. Such motions are called **Resisted** or **Damped Oscillations**. All vibrating bodies are subject to these forces, for otherwise the system would be conservative and there would be no emission of energy etc.

Let us suppose that the resisting force is given by $F=-r\left(\frac{dy}{dt}\right)$, where r is a constant depending on the dimensions of the body the minus sign indicating that the force opposes the direction of y. The rate at which work would be done against friction etc.,

$$=F\left(\frac{dy}{dt}\right)=-r\left(\frac{dy}{dt}\right)^2.$$

This is the rate at which the total energy of the system $(K+P)$ would be dissipated, therefore,

$$\frac{d}{dt}(K+P)=-r\left(\frac{dy}{dt}\right)^2 \quad \ldots \quad \ldots \quad (8)$$

Substituting the value of K and P from (1) and (2) we have,

$$\frac{d}{dt}\left[\tfrac{1}{2}m\left(\frac{dy}{dt}\right)^2+\tfrac{1}{2}\mu y^2\right]=-r\left(\frac{dy}{dt}\right)^2$$

or $$m\left(\frac{dy}{dt}\right)\frac{d^2y}{dt^2}+\mu y\frac{dy}{dt}=-r\left(\frac{dy}{dt}\right)^2$$

or $$m\frac{d^2y}{dt^2}+r\frac{dy}{dt}+\mu y=0$$

or $$\frac{d^2y}{dt^2}+\frac{r}{m}\frac{dy}{dt}+\frac{\mu}{m}y=0.$$

Put $$\frac{r}{m}=2k, \text{ and } \frac{\mu}{m}=n^2$$

$$\therefore \quad \frac{d^2y}{dt^2}+2k\frac{dy}{dt}+n^2y=0. \quad \ldots \quad \ldots \quad (9)$$

This is the differential equation for damped harmonic motion which may be solved in the form, $y=e.^{at}$ From this we have,

$$\frac{dy}{dt}=ae^{at}, \text{ and } \frac{d^2y}{dt^2}=a^2e^{at}$$

So that on substituting these values in equation (9) we get,

$$a^2e^{at}+2k\,ae^{at}+n^2e^{at}=0,$$

or $$a^2+2ka+n^2=0,$$

that is, $$a=-k\pm\sqrt{k^2-n^2}$$
and there are two solutions

$$y=A'e^{(-k+\sqrt{k^2-n^2})\,t} \text{ and } y=B'e^{(-k-\sqrt{k^2-n^2})t}$$

where A' and B' are any arbitrary constants.

Now when two solutions such as,

$$y-A'e^{(-k+\sqrt{k^2-n^2})t}=0, \text{ and } y-B'e^{(-k-\sqrt{k^2-n^2})t}=0,$$

have been found, it obviously follows that,

$$2y-A'e^{(-k+\sqrt{k^2-n^2})t}-B'e^{(-k-\sqrt{k^2-n^2})t}=0,$$

or writing A for $\frac{A'}{2}$ and B for $\frac{B'}{2}$, we get

$$y=Ae^{(-k+\sqrt{k^2-n^2})t}+Be^{(-k-\sqrt{k^2-n^2})t} \quad \ldots \quad (10)$$

Let us define the *limiting* conditions.

If the system starts with maximum displacement but no velocity, then

and $$\left.\begin{array}{r} y=y_0 \\ \frac{dy}{dt}=0 \end{array}\right\} \text{ when } t=0.$$

Hence $$A+B=y_0 \quad \ldots \quad \ldots \quad \ldots \quad (i)$$

and $$(-k+\sqrt{k^2-n^2})A+(-k-\sqrt{k^2-n^2})B=0 \quad \ldots \quad (ii)$$

or $$-k(A+B)+\sqrt{(k^2-n^2)}\,(A-B)=0$$

But
$$A+B=y_0$$

$$\therefore \quad -ky_0+\sqrt{k^2-n^2}\,(A-B)=0$$

or
$$A-B=\frac{y_0k}{\sqrt{k^2-n^2}} \quad \ldots \quad \ldots \quad (iii)$$

From (*i*) and (*iii*)

$$2A=y_0\left(1+\frac{k}{\sqrt{k^2-n^2}}\right)$$

and
$$2B=y_0\left(1-\frac{k}{\sqrt{k^2-n^2}}\right)$$

Substituting the values of A and B, we have

$$y=y_0\left\{\tfrac{1}{2}\left(1+\frac{k}{\sqrt{k^2-n^2}}\right)e^{(-k+\sqrt{k^2-n^2})t}+\tfrac{1}{2}\left(1-\frac{k}{\sqrt{k^2-n^2}}\right)e^{(-k-\sqrt{k^2-n^2})t}\right\} \quad (11)$$

The equation (11) indicates that the motion depends upon the relation of the magnitudes of k and n.

Case I.—When $k^2>n^2$, the roots will be real and negative.

The expression for y shows that the displacement decays asymptotically to zero, Fig. 93 (*a*). The retarding force is so great that the particle does not vibrate. Such a motion is called *aperiodic* motion and is realized in a dead-beat galvanometer or in a pendulum swinging in a very viscous fluid.

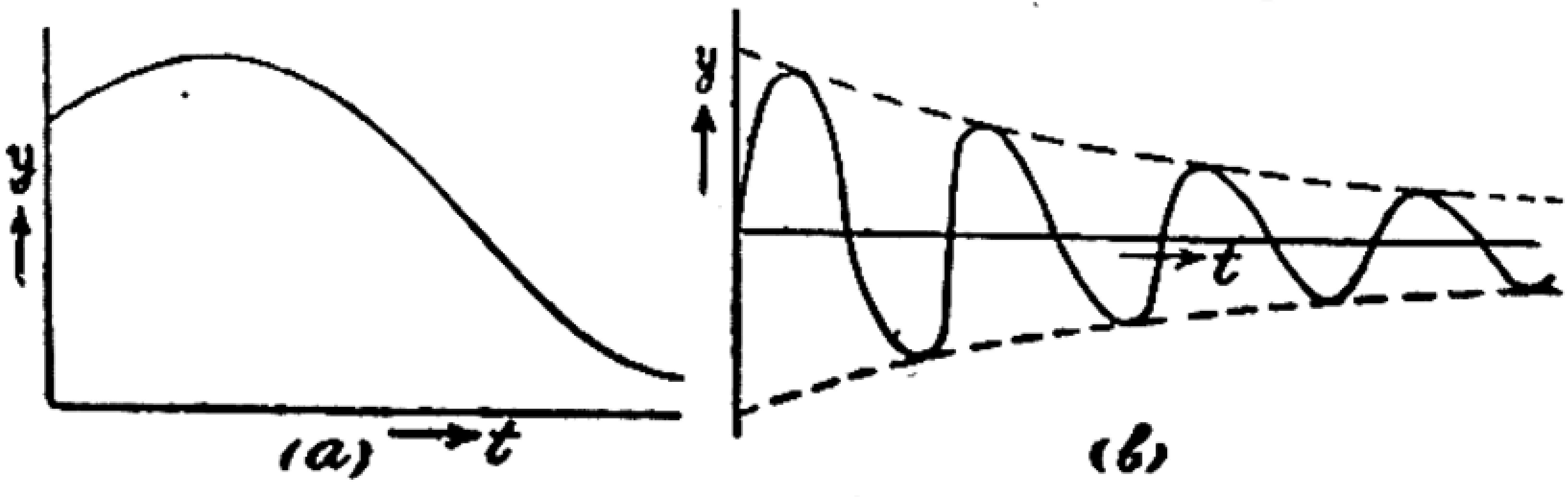

Fig. 93.

Case II.—When $k^2=n^2$, equation (11) evidently breaks down for in this case two of the coefficients become infinite. Returning to equation (10), let us see what form will this take when $\sqrt{k^2-n^2}$ has not vanished but is reduced to a small quantity h.

Then
$$y = Ae^{(-k+h)t} + Be^{(-k-h)t}$$
$$= e^{-kt}[Ae^{ht} + Be^{-ht}]$$

Writing e^{ht} and e^{-ht} in the form of series,
$$e^{ht} = 1 + ht + \frac{h^2t^2}{2\,!} + \frac{h^3t^3}{3\,!} + \ldots$$
$$e^{-ht} = 1 - ht + \frac{h^2t^2}{2\,!} - \frac{h^3t^3}{3\,!} + \ldots$$

we have,
$$y = e^{-kt}\left\{A\left(1 + ht + \frac{h^2t^2}{2\,!} + \ldots\right) + B\left(1 - ht + \frac{h^2t^2}{2\,!}\right)\right\}$$

Now h being small, the terms h^2 and higher powers may be neglected but the quantities Ah and Bh have unknown magnitudes since A and B are unknown.

Then,
$$y = e^{-kt}\{(A+B) + (A-B)ht\}$$

Denoting the two constants $(A+B)$ and $(A-B)h$, by G and H respectively, we have
$$y = e^{-kt}(G + Ht)$$

This is the transition case of '*Critical Damping*' the motion being just aperiodic or non-oscillatory.

If now
$$y = y_0 \text{ when } t = 0,$$
$$G = y_0$$

and if
$$\frac{dy}{dt} = 0 \text{ when } t = 0,$$
$$H = ky_0$$
$$\therefore\ y = y_0e^{-kt}(1 + kt) \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

The chain curve in Fig. 93 (b) represents the equation in the given case.

Case III.—A very important case arises when $k^2 < n^2$. The roots are imaginary since $k^2 - n^2$ is a negative quantity. Let it be written as $\sqrt{-1}\ \sqrt{n^2-k^2}$ or $i\beta$, where $i = \sqrt{-1}$ and $\beta = \sqrt{n^2-k^2}$ so that the quantity under the root sign is again real.

Equation (11) then becomes
$$y = y_0\left\{\tfrac{1}{2}\left(1 + \frac{k}{i\beta}\right)e^{(-k+i\beta)t} + \tfrac{1}{2}\left(1 - \frac{k}{i\beta}\right)e^{(-k-i\beta)t}\right\}$$
$$= y_0e^{-kt}\left\{\left(\tfrac{1}{2} + \frac{k}{2i\beta}\right)e^{+i\beta t} + \left(\tfrac{1}{2} - \frac{k}{2i\beta}\right)e^{-i\beta t}\right\}$$

$$=y_0e^{-kt}\left\{\frac{e^{i\beta t}+e^{-i\beta t}}{2}+\frac{k}{\beta}\,\frac{e^{i\beta t}-e^{-i\beta t}}{2i}\right\}$$

Now, the exponential form of

$$\cos\beta t \text{ is } \frac{e^{i\beta t}+e^{-i\beta t}}{2}$$

and of

$$\sin\beta t \text{ is } \frac{e^{i\beta t}-e^{-i\beta t}}{2i}$$

Putting these values in our equation we get,

$$y=y_0e^{-kt}\left\{\cos\beta t+\frac{k}{\beta}\sin\beta t\right\}$$

$$=y_0\frac{e^{-kt}}{\beta}\left\{\beta\cos\beta t+k\sin\beta t\right\}$$

Let ϕ' be an angle such that

$$k=r\cos\phi' \text{ and } \beta=r\sin\phi'$$

$$\therefore\ y=y_0\frac{re^{-kt}}{\beta}\left\{\sin\beta t\cos\phi'+\cos\beta t\sin\phi'\right\}$$

$$=y_0\frac{r^{-}e^{k_t}}{\beta}\sin(\beta t+\phi')$$

$$=a'e^{-kt}\sin(\beta t+\phi')$$

where $a'=\frac{y_0r}{\beta}$ and $\beta=\sqrt{n^2-k^2}=\sqrt{\frac{\mu}{m}-\frac{r^2}{4m^2}}$ ... (13)

This relation for y represents geometrically a Simple Harmonic curve of which the amplitude $a'e^{-kt}$ diminishes exponentially to zero with increasing time. Since the periodic term $\sin(\beta t+\phi')$ alternates between the limits ± 1, the space-time displacement y of the curve lies between the curves $y=\pm a'e^{-kt}$ [*see* Fig. 93 (b)]. The measure of the decay of the amplitude is the constant 'k' called *Damping Co-efficient*. Note that the equation differs from the equation of the undamped motion by the introduction of the factor e^{-kt}.

122. Effect of Damping.—The damping produces two effects (i) the decrease in amplitude as explained above and (ii) the increase of time-period. In the case of undamped oscillations, we have seen already that

$$T=\frac{2\pi}{n} \text{ or } T^2=\frac{4\pi^2}{n^2}$$

whereas in the case of damped oscillations

$$T'=\frac{2\pi}{\beta}=\frac{2\pi}{\sqrt{n^2-k^2}} \text{ or } T'^2=\frac{4\pi^2}{n^2-k^2} \quad \dots \quad (14)$$

which shows that time-period has become greater. In most cases occurring in practice, however, the difference between n and $\sqrt{n^2-k^2}$ is a very small quantity of the second order and may usually be neglected, *i.e.*, the effect of damping on the period is usually negligible except in a few extreme cases where the vibrations are strongly damped. Experiment has shown that in the case of a tuning-fork the damping is so small that it lowers the frequency by 1 part in 5 millions. But, on the other hand, in the case of air columns k is so large that vibrations die out after a few oscillations.

***123. Forced Vibrations, Resonance**—So far attention has been confined to the *natural* oscillations of an elastic system, *i.e.*, the oscillations that occur when it is displaced and then released. Under such circumstances every elastic body vibrates with a natural frequency that is wholly determined by the elastic constants and the vibrating mass.

We have seen in Arts. 120 and 121 that in the absence of the friction the frequency of the vibrating mass m is given by

$$n=\frac{1}{2\pi}\sqrt{\frac{\mu}{m}}$$

and in the presence of the friction it is given by

$$\beta=\frac{1}{2\pi}\sqrt{\frac{\mu}{m}-\frac{r^2}{4m^2}}\,.$$

But when the mass is subjected to an oscillatory external force such as a bridge which vibrates under the rhythm of marching soldiers, the prongs of a tuning-fork which is exposed to regular impulses of a sound wave or when one sings near a piano, the oscillations that ensue are called *forced* oscillations.

Use is very often made of an external periodic force to maintain the vibrations of a system. The impressed periodic force may or may not have the same period as the free vibration. Note that in this case our vibrating system is subjected to damping and to an external periodic force. The friction tends to retard the motion and the impressed force tends to maintain it. Evidently the result should be a settling down to a certain definite amplitude and phase which will remain constant while the impressed periodic force continues, the frequency being that of the impressed force. In the special case when the period of the free vibrations coincides with the period of the force, we have the phenomenon of **Resonance**.

Let the external periodic force be represented by $F \sin pt$ where F is the maximum value of the force of frequency $p/2\pi$. Then we may write the equation of motion as follows :—

$$m\frac{d^2y}{dt^2}+r\frac{dy}{dt}+\mu y=F\sin pt \qquad \ldots \; (15)$$

or
$$\frac{d^2y}{dt^2}+\frac{r}{m}\frac{dy}{dt}+\frac{\mu}{m}y=\frac{F}{m}\sin pt$$

Writing $\frac{r}{m}=2k$, $\frac{\mu}{m}=n^2$ and $\frac{F}{m}=f$, we get

$$\frac{d^2y}{dt^2}+2k\frac{dy}{dt}+n^2y=f\sin pt \qquad \ldots \qquad \ldots \; (16)$$

The vibration maintained by a periodic force has the same period as that of the force *viz.*, $2\pi/p$ and the amplitude of the maintained vibration almost remains constant. Therefore we write for the maintained vibration

$$y=A\sin(pt-\theta) \qquad \ldots \qquad \ldots \; (17)$$

where A represents the amplitude of the maintained vibration and θ the lag in phase of the same *i.e.*, θ represents the phase difference between the force and the resulting displacement of the system.

Substituting the value of y and its differential coefficients in (16) and writing $\sin pt=\sin(pt-\theta+\theta)$, we obtain

$$-Ap^2\sin(pt-\theta)+2kp\,A\cos(pt-\theta)+n^2A\sin(pt-\theta)$$
$$=f\sin(pt-\theta+\theta)$$

or
$$A(n^2-p^2)\sin(pt-\theta)+2kp\,A\cos(pt-\theta)$$
$$=f\cos\theta\sin(pt-\theta)+f\sin\theta\cos(pt-\theta) \qquad \ldots \; (18)$$

Now equation (18) holds good for all values of t, we may equate the coefficients of sine and cosine of $(pt-\theta)$.

Hence, we get

$$A(n^2-p^2)=f\cos\theta, \text{ and } 2kp\,A=f\sin\theta \qquad (19)$$

$$\text{thus} \quad A=\frac{f}{\sqrt{(n^2-p^2)^2+4k^2p^2}}, \text{ and } \tan\theta=\frac{2kp}{n^2-p^2} \qquad (19a)$$

$$\therefore\ y=\frac{f}{\sqrt{(n^2-p^2)^2+4k^2p^2}}\sin(pt-\theta) \qquad \ldots\ (20)$$

Equation (20), however, does not represent the complete solution of equation (16). We may add to this another value of y as found in the preceding article, which makes the left side of (16) equal to zero without impairing its validity as a solution. Hence, we have the complete solution,

$$y=\frac{f}{\sqrt{(n^2-p^2)^2+4k^2p^2}}\,\theta\sin(pt-\theta)+a'e^{-kt}\sin(\beta t+\phi') \qquad (21)$$

where $\tan\theta=\dfrac{2kp}{n^2-p^2}$ and $\beta=\sqrt{n^2-k^2}$, a' and ϕ' are arbitrary constants depending on the initial conditions.

Note that the second term is only important in the beginning and it opposes the forced oscillations but after some time the natural or free oscillations die away and only the forced oscillations remain. So that the first part is the only important part of the solution.

This shows that after some time when the free vibrations have died out, the system begins to oscillate with the period of the exciting force. The amplitude A of the forced vibration has been obtained by squaring and adding the two components of equation (19)

$$\text{whence} \qquad A^2\{(n^2-p^2)^2+4k^2p^2\}=f^2$$

$$\text{or} \qquad A=\frac{f}{\sqrt{(n^2-p^2)^2+4k^2p^2}} \qquad \ldots \qquad \ldots\ (22)$$

We notice from the expression given above that the amplitude with which the body is made to oscillate depends upon the relation of n and p. The more they differ from each other, the greater n^2-p^2 will be and the smaller the amplitude. It is maximum when $n=p$ *i.e.*, when the free period of the body is the same as that of the impressed force. This is the case of ***Resonance***. It is, moreover, clear that if there is no damping, *i.e.*, k is equal to zero, then for resonant vibration, *i.e.*, $p=n$, the amplitude will be infinite. Actually this never happens for the damping co-efficient k is never zero, (*see* Fig. 90).

The meaning of the phase angle θ is best demonstrated by means of the right triangle of Fig. 93 (c).

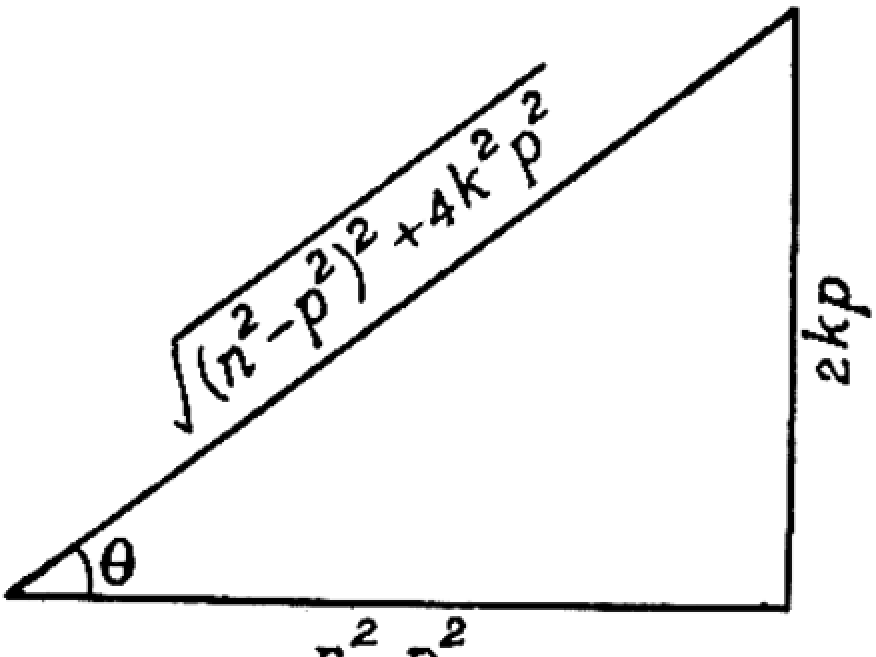

Fig. 93. (c)

From it we see that

$$\sin\theta=\frac{2kp}{D},$$

$$\cos\theta=\frac{n^2-p^2}{D}$$

and $$\tan\theta=\frac{2kp}{n^2-p^2} \qquad \ldots(23)$$

provided

$$D=\sqrt{(n^2-p^2)^2+4k^2p^2} \qquad \ldots(24)$$

The equation (20) may be written as

$$y=\frac{f}{D}\sin(pt-\theta) \qquad \ldots(25)$$

The phase angle θ depends upon the values of the damping coefficient k and the relative magnitudes of p and n, that is, on the entire physical condition of the system. But for any impressed frequency it is zero when damping is negligible. For all values of p smaller than n, $\tan\theta$ is a small +ve quantity and for all values of p greater than n, however, $\tan\theta$ is a small negative quantity *i.e.*, the forced vibration differs in phase by π nearly.

For resonance $p=n$, so $\tan\theta$ is infinite and θ is, therefore $\frac{\pi}{2}$, *i.e.*, the impressed force and the vibration differ in phase by quarter period.

***123a. Impedance and Resonance** :—In the problem of forced oscillations the velocity of vibration is usually of greater interest than the displacement. (In alternating current work, it is the current that corresponds to velocity). It is obtained at once by differentiating equation (25),

$$v=\frac{fp}{D}(\cos pt-\theta) \qquad \ldots \qquad \ldots (26)$$

The quantity $\frac{D}{p}$ is called the *Mechanical Impedance* of the system and is usually denoted by Z. From equation (24)

$$Z=\frac{D}{p}=\sqrt{\left(\frac{n^2}{p}-p\right)^2+4k^2} \quad \ldots \quad \ldots \quad (27)$$

Substituting Z for $\frac{D}{p}$ the equation (26) reduces to

$$v=\frac{f}{Z}\cos(pt-\theta) \quad \ldots \quad \ldots \quad (28)$$

So that if $p=n$, $\tan\theta$ is infinite and θ is therefore $\frac{\pi}{2}$,

whence $$v=\frac{f}{Z}\cos\left(pt\pm\frac{\pi}{2}\right)$$

$$=\pm\frac{f}{Z}\sin pt \quad \ldots \quad \ldots \quad \ldots \quad (29)$$

that the force and the velocity will be in the same phase.

The maximum value of v during any period of motion is $\frac{f}{Z}$ and is called the *velocity amplitude.* Let us see how it varies with the impressed frequency. To do so we shall investigate the variation of Z with p.

(1) Assume that $k=0$; that is there is no damping In that case $Z=\infty$ for $p=0$ and for $p=\infty$. But it has zero value when $\frac{n^2}{p}=p$ *i.e.*, the frequency of the impressed force is equal to the natural frequency of the system. The velocity then becomes infinite and what is called the "**Resonance Catastrophe**" takes place. It is indeed a dangerous condition and the designers of the moving machinery like aeroplane etc. have to be very careful never to have driving frequencies which are close to the natural frequencies of the structural parts. The phenomenon of resonance in general is the excitation of large amplitudes of motion or velocity by means of small periodic forces applied at the proper rate. We have heard that a suspension bridge may collapse due to the large swing produced by the regular steps of the soldiers marching in formation when their tramp happened to keep time with the natural period of the

bridge unless their step is broken. The appearance of rattles in a motor car at definite driving speeds may have been experienced by several readers.

(2) Assume that k is finite, that is damping is present. In this case also resonance occurs but resonance catastrophe is reduced to limited proportions, v though maximum, has a finite value. The value of p corresponding to this maximum is the one that minimizes Z and is still given by $p=n$. as the differentiation of Z and equating it to zero will show.

CHAPTER X

CHARACTERISTICS OF A MUSICAL SOUND: DOPPLER'S PRINCIPLE

124. Noises and Musical Sounds.—We have seen that sound is due to some sort of mechanical disturbance propagating through a material medium. This mechanical disturbance may be either of a periodic or of a non-periodic character. If it is periodic, regular and long continued, it produces a pleasing effect; but, if it is non-periodic, irregular and of very short duration, it produces a displeasing effect. The sounds produced by the former type of disturbance are called **musical sounds** and can be obtained by blowing into an organ pipe, by plucking the strings of a *sitar* or by playing upon a harmonium. Those produced by the latter type are called **noises**. They are abrupt sounds of a complex character with an irregular period and amplitude originating from a source in non-periodic motion. The report of a gun, the rustling sound of dry leaves, the sound of a hammer striking an anvil, the sound produced by the clapping of two wooden blocks together, etc. are all examples of *noises*. The displacement curves (*a*) and (*b*) of Fig. 94 exhibit graphically the physical difference between a noise and a musical sound. The curve (*a*) represents the wave disturbance such as would be associated with a particular kind of noise. It has no definite period or amplitude and is very irregular and complex in character. The curve (*b*) evidently represents the displacement curve for musical sound waves. It is periodic in character and possesses regularity of shape, amplitude and frequency *i.e.*, it shows no sudden variations in pitch and amplitude for an appreciable interval of time. To sum up, the essential characteristics of a musical sound are its, *periodicity*, *regularity* and *continuity*, and it may be defined as one which is produced by a *series of similar impulses following each other regularly at equal intervals of time and in quick succesion with no sudden variations in the amplitude.* But noises, on the other hand, lack these characteristics. Below we shall deal with the musical sounds only.

Muscial sounds may differ from each other in three respects: (*i*) Intensity or Loudness, (*ii*) Pitch, and (*iii*) Quality or Timbre.

125. Intensity or Loudness.—This is the characteristic of all sounds whether musical or non-musical. It relates to the magnitude of the sensation produced on the organ of hearing. It may be noted that the intensity of sound waves is a purely physical quantity independent of the ear and is measured by the amount of the wave energy crossing per unit time through a unit area taken perpendicularly to the direction of propagation. Mathematically we have seen (Art.44) that the energy of a wave and hence the intensity at a point is proportional to the square of the amplitude of vibration of the point. But the distinction between this physical quantity called *Intensity* and the meaning to be understood by the term, *loudness*, must be clearly noted. The loudness of a sound is not the intensity of waves but something which depends upon the intensity of waves producing it, and increases and decreases with the intensity of sound not in a simple way but it varies directly as its logarithm, *i.e.*, $L \propto \log I$ (Weber and Fechner's Law*) where L represents the sensation of loudness and I, the intensity of sound. To sum up the intensity of sound is a purely physical quantity independent of the ear and has objective existence. Loudness, on the other hand, is the degree of sensation and is not wholly physical, but partly subjective and does depend upon the ear and the listener.

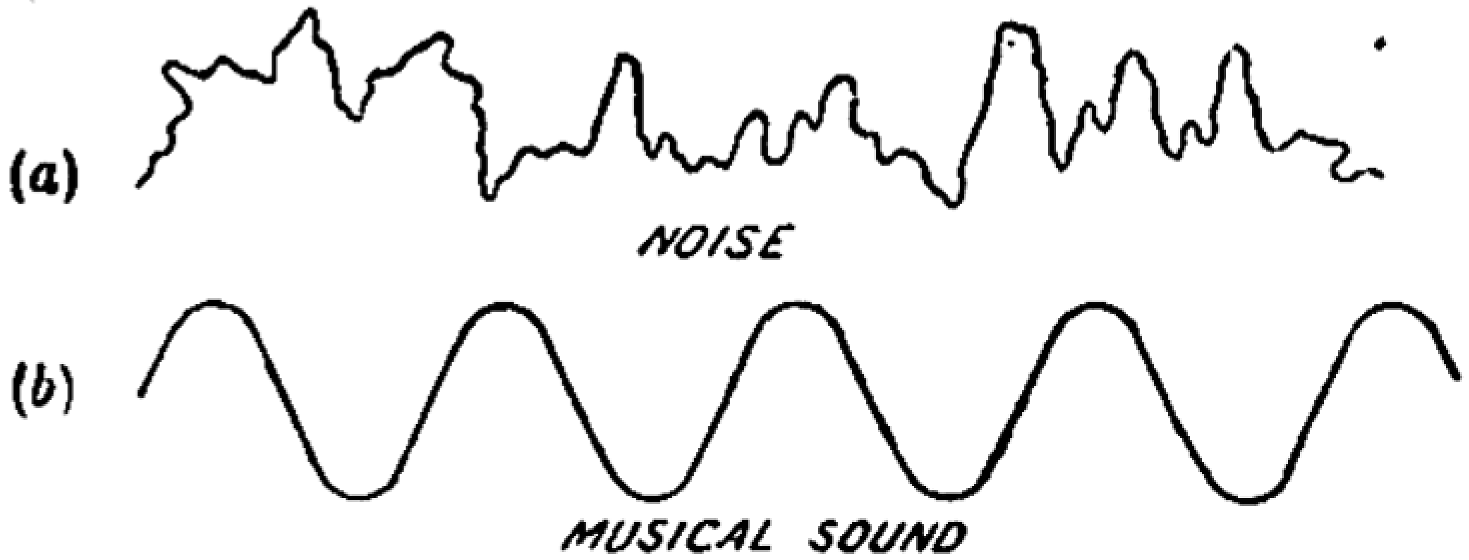

Fig. 94.

The following are the various factors upon which the 'Intensity' or 'Loudness' of a sound depends :—

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* It states that the magnitude of any sensation is proportional to the logarithm of the stimulus.

(*i*) **Amplitude of vibration of the source.**—Pluck the string of a monochord, and note its amplitude as well as the loudness of sound. Again pluck it with a greater force, note that now it will vibrate with a larger amplitude and at the same time will produce a louder sound. When the amplitude decreases the loudness decreases as well. In Fig. 95 (*a*) *AB* denotes the amplitude of the upper wave and the curves (*a*) and (*b*) differ in no other respect but in amplitude. We saw in Art. 44, that the energy and hence the Intensity of a sound wave varies directly as the square of the amplitude and consequently it will be much greater for the sound depicted by the curve (*a*).

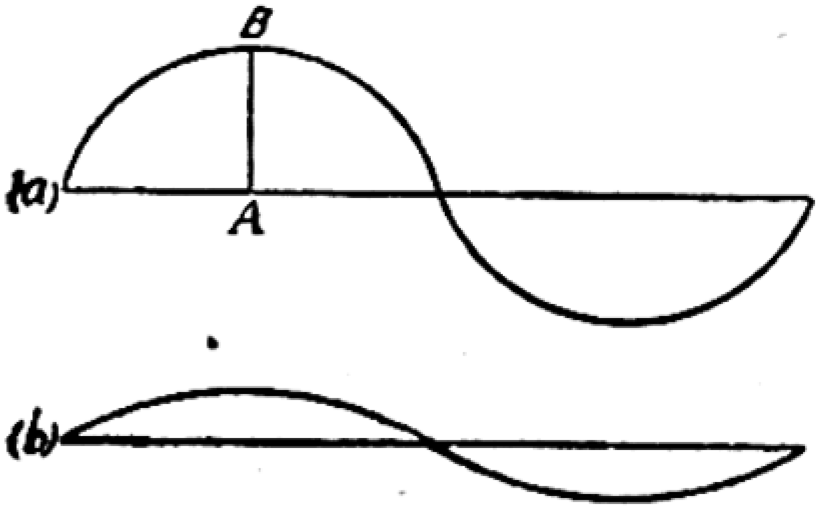

Fig. 95.

(*ii*) **Size of the sonorous body.**—A large vibrating area sends out a great amount of energy, and so the larger is the size of the sonorous body, the louder is the sound produced. This is why a larger tuning-fork produces a louder sound than a small one. For the same reason, a small tuning-fork which produces a feeble sound ordinarily produces a loud note when its stem is pressed against the top of a table. Church bells having a large vibrating area than an ordinary gong, can be heard from a great distance.

(*iii*) **Distance from the source.**—Imagine two spherical surfaces *A* and *B* with the source as centre and r_1 and r_2 as their radii respectively. Then the amount of energy flowing across unit area of the surface of the sphere *A* is given by $I_1 = \frac{E}{4\pi r_1^2}$, where *E* is the total energy emanating per second from the source. Similarly, for the sphere *B*

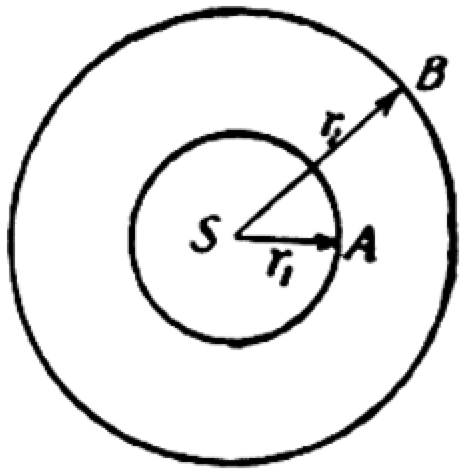

Fig. 96.

$$I_2 = \frac{E}{4\pi r_2^2}$$

$$\therefore \frac{I_1}{I_2} = \frac{E/4\pi x_1^2}{E/4\pi x_2^2} = \frac{r_2^2}{r_1^2}$$

i.e., intensity at a point is inversely proportional to the square of its distance from the source.

(*iv*) **Density of the medium.**—As the kinetic energy of the medium increases with the mass per unit volume of the medium it naturally follows that loudness will depend on the density of the medium ; the greater the density, the more intense the sound (Art 45).

(*v*) **Presence of other bodies.**—A resonant body when placed near a source intensifies sound. Even large reflectors act as sound amplifiers.

(*vi*) **Motion of the medium.**—Sound is carried to a greater distance in the direction of the motion of the air and as shown in Art. 45 the amount of energy crossing any area/sec. will correspondingly increase. In other words the intensity of sound is greater with the motion of the medium than against it.

126. How is loudness measured ? The Decibel.—We mentioned in the previous section that L $\propto$ log $_{10}I$. This result has been experimentally tested. So we may put

$$L=K\log_{10}I$$

where K is a constant of proportionality.
Now if L_1 and L_2 be the loudnesses of two sounds and I_1 and I_2 their respective intensities, we have,

$$L_1=K\log_{10}I_1$$

and

$$L_2=K\log_{10}I_2$$

Subtracting

$$L_1-L_2=K\log_{10}I_1-K\log_{10}I_2$$

$$=K\log_{10}\frac{I_1}{I_2}.$$

This gives us the method of measuring the difference in loudness technically known as *Intensity level*, between any two sounds.

If $K=1$, L is measured in '*bels*', a unit named after Alexander Graham Bell, the inventor of the telephone.
If $K=10$, L is measured in '*decibels*', one-tenth of a '*bel*'.

If

$$L_1-L_2=1 \text{ decibel}$$

$$\mathrm{I}=10\log_{10}\frac{I_1}{I_2} \text{ or } \log_{10}\frac{I_1}{I_2}=\frac{1}{10}$$

$$\therefore \quad \frac{I_1}{I_2}=1{\cdot}259.$$

i.e., two sounds are said to differ in loudness by one decibel when their intensities are in the ratio of 1·259. To build a scale of loudness, we have to fix its zero. The loudness corresponding to the 'threshold of hearing' is the zero of this scale. The following table gives the value of some sounds measured in decibels.

Threshold of hearing	0
Whisper	10–20
Average conversation	60—65
Motor trucks and heavy street traffic	70—80
Roaring lion at 20 ft.	90
Thunder	100–110
Painful sounds	130 or above.

It will be clear from above that the loudness of a sound does not increase in proportion to its intensity. For example when $I_1=100$ and $I_2=1$,

$$L_1-L_2=10\log_{10}\frac{100}{1}=10\log_{10}10^2=20.$$

and when $I_2=1000$ and $I_1=1$,

$$L_1-L_2=10\log_{10}\frac{1000}{1}=30.$$

In other words, we learn that when two sounds differ by 20 'decibels' the louder of them is 100 times more intense and when they differ by 30 'decibels', the louder one is 1000 times more intense. It is obvious, therefore, that when I is increased to ten times the former value, the difference in loudness is not 20×10 decibels, but is only 30 decibels as shown above. For Sensitivity of the Ear and the Limits of its Audibility see Art. 146.

127. Pitch—It is that characteristic of a musical sound by which a shrill sound can be distinguished from a grave one, even though the two sounds may be of the same intensity. It depends upon frequency; the greater the frequency the higher the pitch, and the lower the frequency the lower the pitch. It is often used as synonymous with frequency but it is in fact a sensation depending upon frequency. Pitch is entirely independent of both loudness and quality.

To show that pitch depends upon frequency let us consider a Savart's wheel. It consists of a toothed wheel mounted on an axle rotated by a handle. The rotation of the axle is communicated to two counters (not shown in Fig. 97. One registers every rotation and the other every hundredth of it. Turn the wheel, whilst a card is held against the teeth. Firstly separate taps are heard, but as the speed of the wheel is increased, these blend together and a single note of low pitch is heard. As the speed is further increased the note becomes more and more shrill and the pitch will be found to rise.

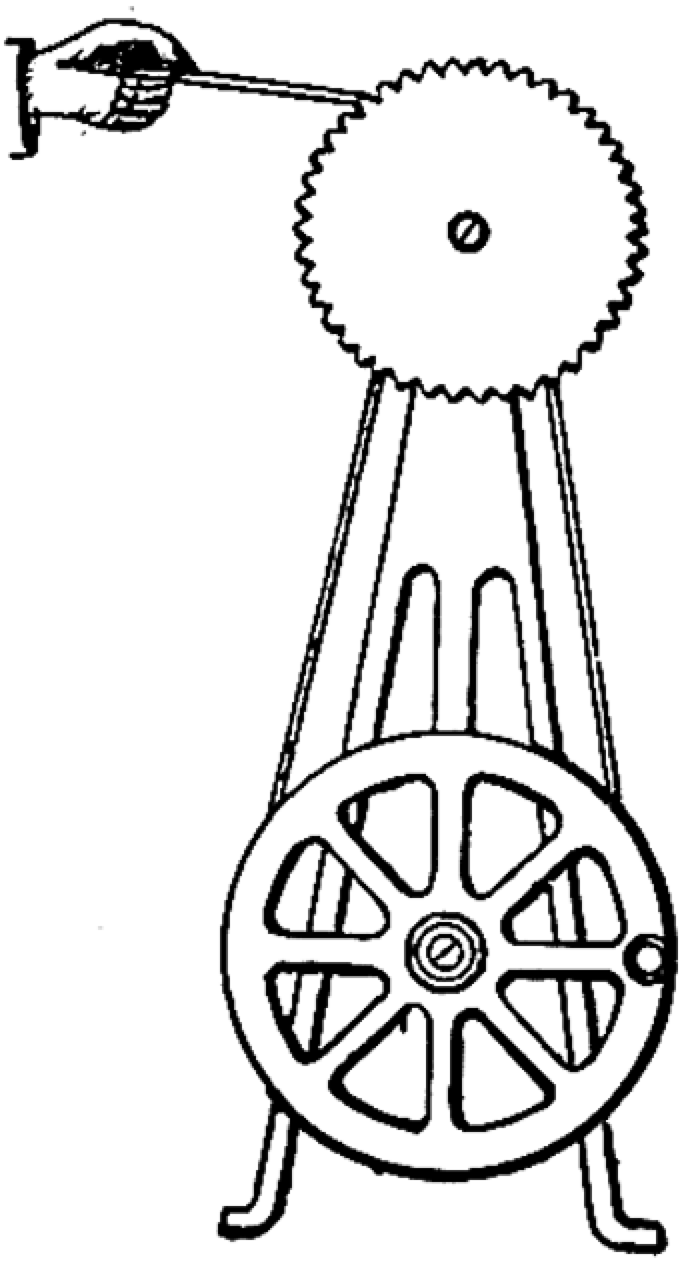
Fig. 97.

We can also show with the help of a Disc Siren (Fig. 98) that pitch of a note rises as the frequency of vibration is increased. Disc Siren consists of a circular disc pierced with several rings of small and equidistant holes. It is mounted on an axle of a motor which can be rotated very rapidly. When a narrow stream of air is forced through a glass jet directed against one of these holes, a puff of air passes through the disc. As the disc rotates every hole comes successively in front of the jet and a series of puffs are emitted in quick succession. For a certain speed of the disc these puffs blend into a musical note of a definite pitch. As the speed of the disc is gradually increased the pitch rises.

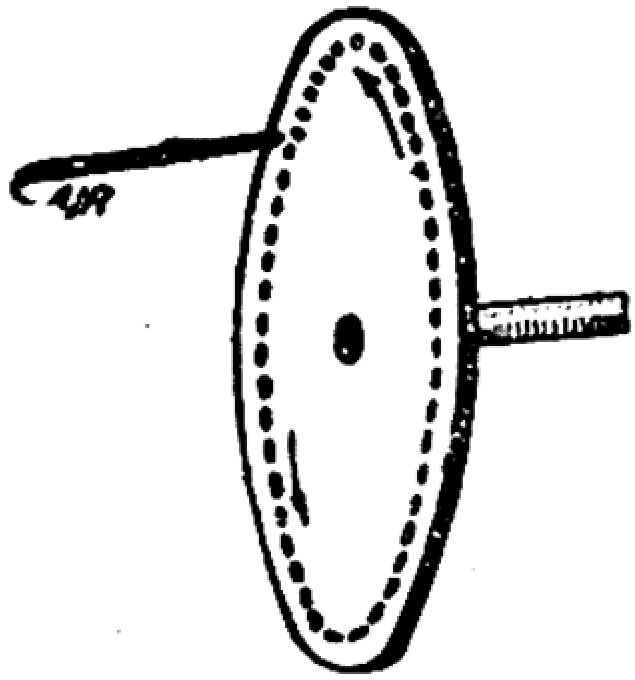

Fig. 98.

It has already been shown that the number of waves that reach the ear per second is equal to the number of vibrations per second of the sounding body. But as the waves of all lengths travel with the same velocity in the same medium, a sound of higher pitch should have a smaller wavelength as compared to that of a sound of lower pitch.

Fig. 99 represents two waves. They are of the same amplitude but of different wavelengths. The upper curve refers to a note shriller than the lower one.

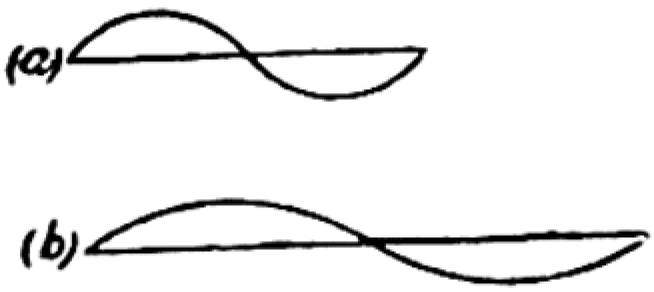

Fig. 99.

128. Doppler's Principle.—When a sonorous body is producing a note of frequency *n*, a listener at a distance receives *n* waves per second. But if there is relative motion between them, the pitch of the note is apparently altered. This may often be observed at a railway station. If a locomotive travelling at high speed with its whistle blowing, approaches an observer on the platform, the pitch of the whistle appears to rise and then to drop suddenly at the moment when the locomotive passes him. This effect can also be noticed in case of a vibrating tuning-fork mounted on a resonance box and swinging rapidly in a circle. A person who places himself in the plane in which the fork swings, will find that the pitch of the note rises when the fork is moving towards him and falls when it is moving away from him. In general, whenever the distance between the listener and the source is altered, the pitch of the note is apparently changed. A rise in pitch takes place when the distance between them decreases and *vice versa*. It was C. Doppler who in 1842 pointed out this apparent change in pitch and the principle is known after him as **Doppler's Principle.**

Let us now find the relationship between the actual pitch, the apparent pitch, the velocity of the source or that of the listener or both. We shall discuss these cases in the following sections.

129. Source in Motion and Listener at rest.—Let *S* and *L* be the positions of the Source and the listner respectively to begin with. Suppose the frequency of the sound emitted by the source is *n*, its velocity towards the listener is *a*, and the velocity of propagation of the waves is denoted by *V*.

If the source were stationary, the *n* waves emitted by it will occupy a length *V* in one second. But as it is moving towards the right (approaching towards listener) a distance *a* in one second, the *n* waves produced by it in each second are crowded in the space length $(V-a)$ [Fig 100 (*a*)] and consequently the wavelength of the sound heard by the listener will

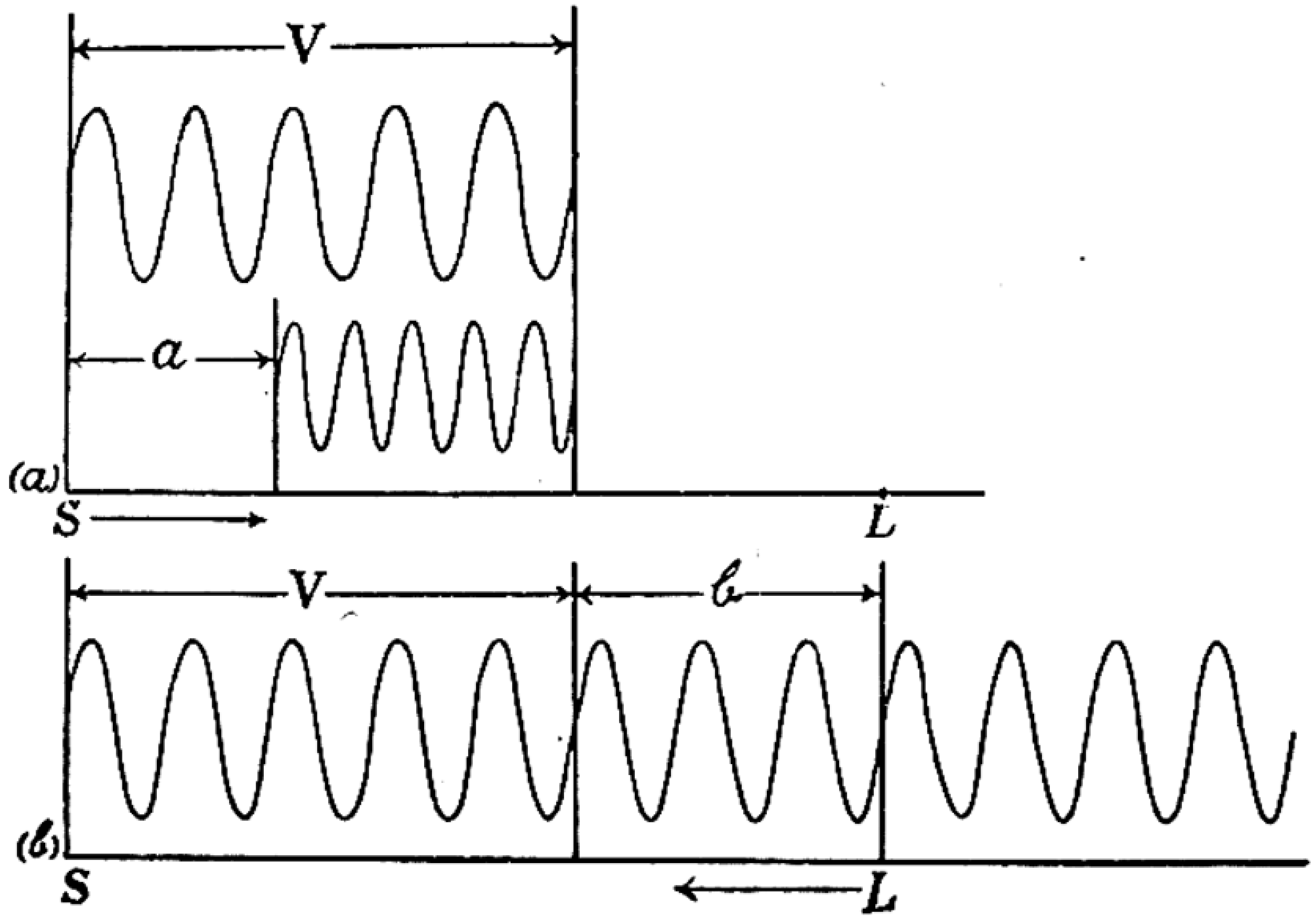

Fig 100.

be reduced from $\lambda=\frac{V}{n}$ to $\lambda'=\frac{V-a}{n}$. The pitch n' of the sound heard is then

$$n'=\frac{V}{\lambda'}$$

whence substituting the value of λ', we get

$$n'=\frac{V}{V-a}n \qquad \dots \qquad \dots \qquad (1)$$

If the source is receding, the sign of a should be reversed with the result that

$$n'=\frac{V}{V+a}n \qquad \dots \qquad \dots \qquad (2)$$

129a. Listener in Motion and Source at Rest.—When the listener is moving with a velocity b towards the stationary source [Fig. 100 (*b*)], there is no crowding of the waves but the ear receives each second not only n waves in the interval V but also an additional number $\frac{b}{\lambda}$ in the distance b which it has moved during one second.

Therefore the increase in the frequency of the note heard $=\frac{b}{\lambda}=\frac{b}{V}n$ since $\lambda=\frac{V}{n}$ and the pitch n' of the sound heard is

$$\begin{aligned} n' &= n+\frac{b}{V}n \\ &= \left(1+\frac{b}{V}\right)n=\frac{V+b}{V}n \qquad \dots \qquad (3) \end{aligned}$$

If the listener is moving away from the stationary source, the sign of b should be reversed so that

$$n'=\frac{V-b}{V}n \qquad \dots \qquad \dots \qquad \dots \qquad (4)$$

129b. Source and Listener both in Motion.—By combining the two results obtained above, we can get an expression for the pitch of the note heard when both source and listener are moving. Let the velocities a and b of the source and the listener respectively be reckoned positive when the motion is from left to right.

Then if the listener is at rest, we have as before

$$n'=\frac{V}{V-a}n \qquad \text{[by equation (1)}$$

If the listener is also moving, we have

$$n''=\frac{V-b}{V}n'$$

by the application of relation (4).

Substituting in the second of these equations the value of n' given by the first, we obtain

$$n''=\frac{V-b}{V-a}n \qquad \dots \qquad \dots \qquad \dots \qquad (5)$$

If both the source and the listener are moving to the right at the same rate, *i.e.*, when $a=b$, it is clear from equation (5), that there is no change in pitch, because they always remain the same distance apart. That is, why a person sitting in a train, finds the pitch of the whistle of the engine of the train to be the same whether the train is moving or not.

130. Effect of Wind on the Pitch of Sound.—In the formulae (1) to (5) air is taken as still. If it is moving in the direction of sound waves with velocity w, then putting $V+w$ for V in (5) we have the apparent pitch of the note

$$=\frac{V+w-b}{V+w-a}\,n \qquad \ldots \qquad \ldots \qquad (6)$$

and it is also clear from this that if $a=b$, wind does not affect the pitch. But if wind velocity changes to w', then the apparent pitch of the note is

$$\frac{V+w'-b}{V+w'-a}\,n \qquad \ldots \qquad \ldots \qquad (7)$$

which is not equal to that in equation (6), unless $a=b$.

Example 1. *Two aeroplanes are approaching each other and their velocities are 100 and 150 miles per hour. The frequency of a note emitted by the first as heard by the passengers in the other is 1000. Calculate the true frequency of the note as heard by its own passengers. Take the velocity of sound as 750 miles per hour.* (*P. U. 1939*)

The apparent pitch $n'=\frac{V-b}{V-a}n$ [Equation (5)]

Here $n'=1000$, $V=750$ miles/hr., $b=-150$ miles/hr. and $a=100$ miles/hr.

Substituting these values, we have

$$1000=\frac{750-(-150)}{750-100}.\ n=\frac{750+150}{750-100}.\ n=\frac{900}{650}.\ n$$

$$\therefore\ n=\frac{1000\times 650}{900}=722{\cdot}2 \text{ per. sec.}$$

Example 2. *Two engines pass each other in opposite directions, one of them blowing its whistle, the frequency of the note emitted being 540. Calculate the frequencies heard on the other engine before and after they have passed each other. Velocity of either of the engines in 30 miles per hour. Velocity of sound =1144 ft./sec.* (*P. U. 1940*)

(*a*) *Before the engines pass each other :*

Using equation (5) $n'=\frac{V-b}{V-a}\,n$;

Here $V=1144$ ft./sec.; $=b-44$ ft./sec. and $a=44$ ft./sec.

$$\therefore\ n'=\frac{1144-(-44)}{1144-44}\cdot 540=\frac{1144+44}{1144-44}\cdot 540=\frac{1188}{1100}\cdot 540$$

$$=583\ 2/\text{sec.}$$

(*b*) *After the engines pass each other* :

Here $b=44$ ft./sec.; $a=-44$ ft./sec.

$$\therefore\ n'=\frac{1144-44}{1144+44}540=\frac{1144-44}{1144+44}540=-\frac{1100}{1188}540=500/\text{sec.}$$

131. Difference in apparent pitches when the source or the listener approaches with the same speed.—It is clear from equation (1) and (3) on pages 187-88 that the frequencies of the notes heard for the two cases when (*a*) the source approaches the stationary listener and (*b*) the listener approaches the stationary source are respectively given by

$$n'=\frac{V}{V-a}n \qquad \ldots \qquad \ldots \qquad (1)'$$

$$n''=\frac{V+b}{V}n \qquad \ldots \qquad - \qquad (3)'$$

In both cases the apparent frequency of the note is greater than n but when the velocity of approach of the source is numerically equal to that of the listener *viz.*, $a=b$ we have

$$n'=\frac{V}{V-b}n \qquad \ldots \qquad \ldots \qquad (1)''$$

and

$$n''=\frac{V+b}{V}n \qquad \ldots \qquad \ldots \qquad (3)''$$

we find that n' is not equal to n'', because by subtracting (3)″ from (1)″ we obtain

$$n'-n''=\left(\frac{V}{V-b}\right)n-\left(\frac{V+b}{V}\right)n$$

$$=n\left[\frac{V}{V-b}-\frac{V+b}{V}\right]$$

$$=n\left[\frac{b^2}{V(V-b)}\right]$$

$=a$ positive quantity

that is $n'>n''$.

In other words when the listener is at rest and the source is approaching the listener with a certain velocity, the frequency of the note heard is greater than when the source is at rest and the listener is approaching it with the same velocity as that of the source in the first case. Thus it is not a matter of indifference whether it is the source or the listener that moves.

132. Graphical Representation of Wave-fronts in Doppler's Effect.—Let S_1, S_2, S_3 etc be the positions of the source at times 0, t, $2t$, etc. (Fig. 101) and let V the velocity of sound, be greater than a, the velocity of the source. With radii $3tV$, $2tV$, tV draw circles $c_1d_1R_1$, $c_2d_2R_2$, $c_3d_3R_3$ round S_1, S_2 and S_3 respectively. Then for a disturbance caused at $t=0$, when the source is at S_1, $c_1d_1R_1$, represents the wave surface at a time $3t$. Similarly, $c_2d_2R_2$, $c_3d_3R_3$ represent the wave surfaces at time $3t$ due to the disturbances when the source is at S_2 and S_3 respectively. Thus there is a crowding of waves on the right side and spreading out on the left side. Hence for a listener A standing on the right, there is a rise in pitch when the source approaches it, while for a listener B, there is a corresponding fall. Figure for the case when $a > V$ is given in the next section.

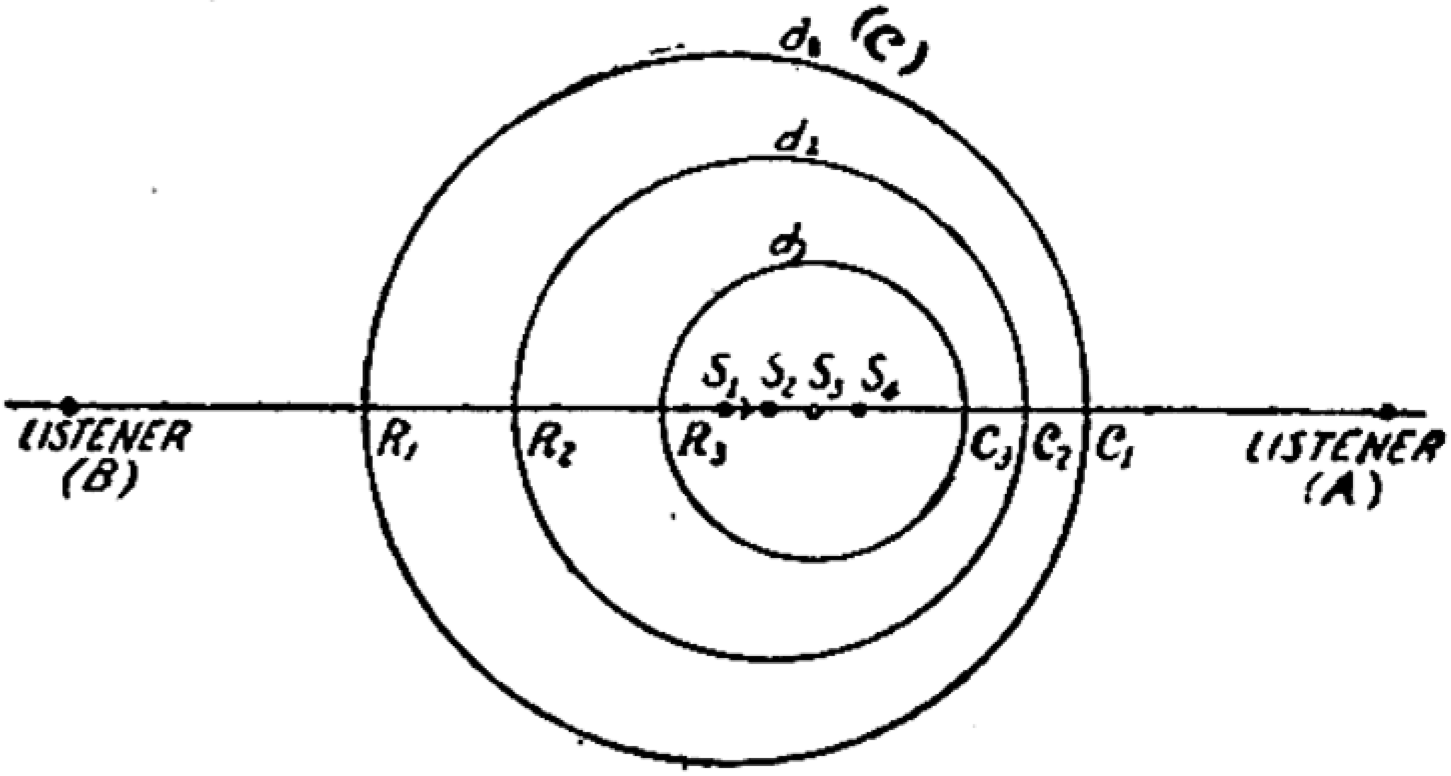

Fig. 101.

133. Flight of a Bullet.—So long as the speed of the sounding body is less than that of the sound the number of waves per second emanating from it and reaching an observer will be governed by the Doppler's Principle. In this case spheres of compression expand with the velocity of sound, while the source moves slower than the rate at which these spheres are growing. This would mean that the wave surface corresponding to the disturbance that starts at any given time lies *entirely outside* all the wave-surfaces corresponding to disturbances which start at all subsequent times (Fig. 101). On the other hand, when the speed of the sounding body is greater than that of sound just as in the case of a bullet, at every point in it's path of flight the bullet gives rise to spheres of compression which expand with the velocity of sound, while it moves faster than the rate at which these spheres are growing, *i.e.*, the whole or part of the wave-surface corresponding to subsequent disturbances lies *outside*. This will result in a conical wave front which lines along the tangent planes of these circles at any instant with the bullet at the apex (Fig. 102). Graphically we can obtain this result as shown below. As before with centres S_1, S_2, S_3 draw circles of radii, $4tV$, $3tV$, $2tV$ etc, At any given instant a tangent can be drawn to the wave surfaces. The wave surfaces will strengthen each other along these tangents but will interfere at other points, thus resulting in two plane waves inclined at a constant angle 2θ, where

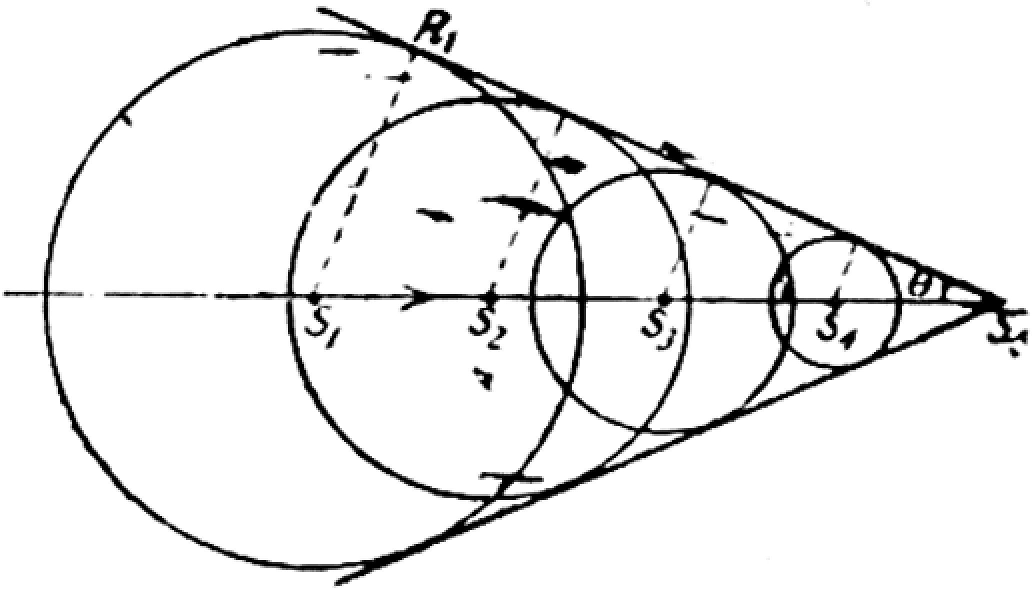

Fig. 102.

$$\sin\theta = \frac{S_1R_1}{S_1S_5} = \frac{4tV}{4ta} = \frac{V}{a}.$$

When a is large, θ will be small. Knowing angle θ and V, the velocity of sound, a the velocity of the source can be determined. If a and θ are given, V can be calculated. Fig. 103 gives the form of waves produced in air by a rifle bullet. These photographs were taken by Prof. C. V. Boys. A similar series of waves is generated on the surface of water when a steamship makes its way through a calm sea.

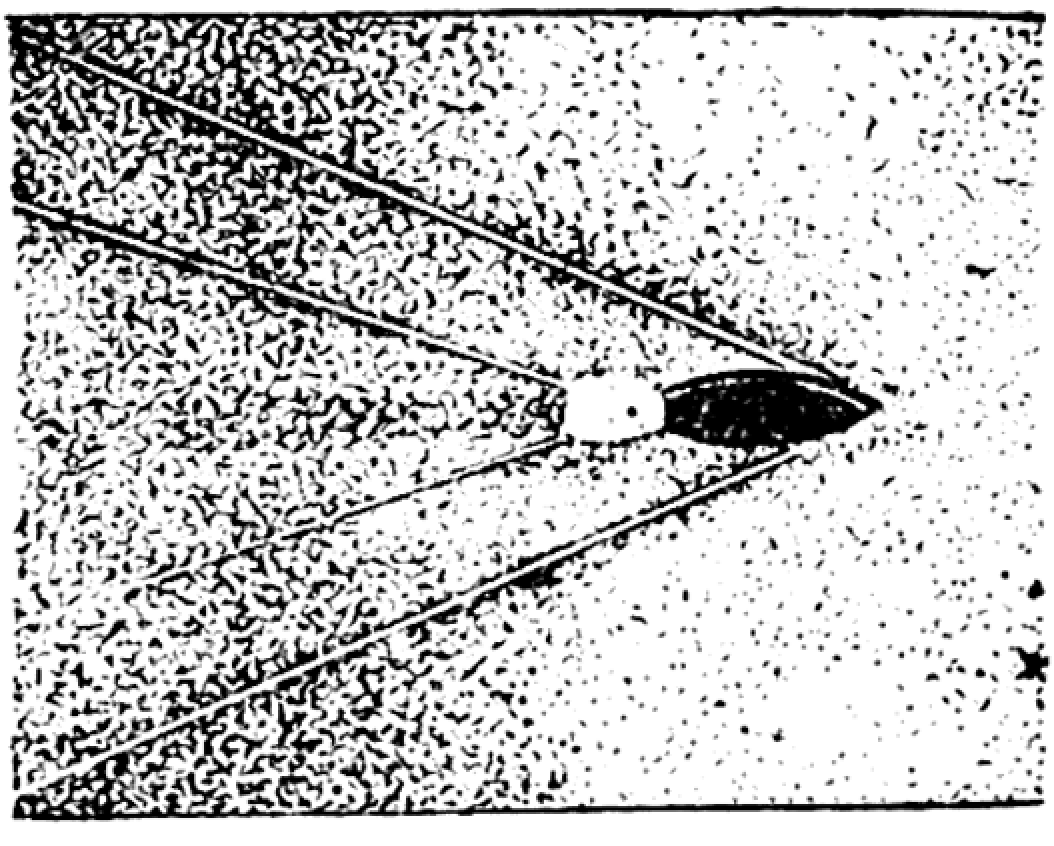

Fig. 103. Flight of a bullet.

134. Doppler's Principle applied to Light.—The spectroscopic examination of light from a star shows that its spectrum is marked with well-defined lines. As a consequence of Doppler effect we find that in the spectra of some stars photographed along side that of an arc or spark giving some of the same lines appearing in the stellar spectrum, the corresponding stellar lines are shifted towards the violet end indicating a diminution in wave length while in those of others the shift is towards the red end showing an increase in wavelength. The obvious inference is that in the first case the star is approaching the earth and in the second case it is receding from it. This apparent change in wavelength has an important application in Astrophysics for the amount of this displacement expressed as a fraction of the wavelength of the spectral line concerned affords us a measure of the velocity of approach or recession of a star. It should be noted that in light there is no physical difference between the two cases, source in motion and the observer in motion, because this distinction is based on the assumption that there is a medium which we may consider at rest such as air for sound and relative to which we express these motions. Now for light the Relativity theory has discarded such a fixed medium (ether) having a definite location in space.

If λ' be the altered wavelength, λ the actual wavelength, a the velocity of approach or recession of the star and V the velocity of light, then,

$$\lambda' = \frac{V-a}{V}\lambda \text{ for approach.}$$

$$= \frac{V+a}{V}\lambda \text{ for recession}$$

$$\therefore \quad \frac{\lambda'}{\lambda} = \frac{V \mp a}{V} \quad \text{or} \quad \frac{\lambda' - \lambda}{\lambda} = \mp \frac{a}{V}$$

i.e.,
$$\frac{d\lambda}{\lambda} = \mp \frac{a}{V}, \text{ whence } a \text{ can be calculated.}$$

Assuming the observer to be in motion we shall come to the same conclusion neglecting terms of the second and higher orders.

135. Quality.—The quality of a sound is that characteristic which enables us to distinguish two notes of the same pitch and loudness played on two different instruments. The study of vibration curves of the various musical instruments has shown that the notes emitted by them are seldom pure. A tuning-fork approximately does so, but the great majority of musical notes are complex and can be analysed into a number of constituents. These constituents besides the fundamental tone of frequency n are comprised of tones of frequencies $2n$, $3n$, $4n$, etc. Such additional tones are called *Harmonics* or *Overtones.*

Each musical instrument produces different overtones and it is the presence of these overtones which is the characteristic of each instrument. The quality of a note is due to this complex structure *i. e.* due to the presence or absence of a certain number of overtones, their relative strengths and pitches. It is the memory of this *tonal* quality which enables us to recognize an untold number of different sounds. Among these we may mention the voices of friends or acquaintances, the various sounds employed in speech, the familiar musical instruments and the cries of animals.

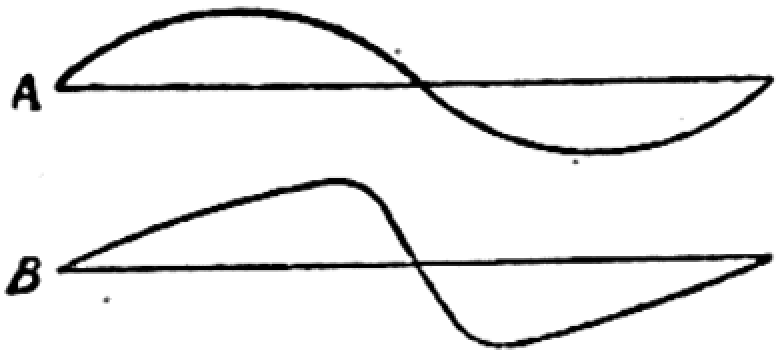

Fig. 104.

Scientifically speaking the quality of a note is judged from the shape or type of the vibration curve of the body which produces the note. Fig. 104 represents two waves A and B of the same amplitude and wavelength, but of different shapes. Hence they represent notes of the same pitch and loudness but different in quality. If the vibration curve is periodic there is a most remarkable theorem known as **Fourier's Theorem** that tells us how the curve might have been built up. It states that a *Periodic Curve* of wavelength λ may be regarded as made up of simple harmonic curves* of definite amplitudes, phases and periods, the wavelengths of the components being in the ratio $\lambda : \frac{\lambda}{2} : \frac{\lambda}{3} : \frac{\lambda}{4}$ etc. *i. e.* ; their frequencies are in the ratio $n : 2n : 3n : 4n$: etc. These simple harmonic curves represent *simple tones.* Thus the quality of a musical sound depends on the aggregate effect of the simple tones which go to form it.

---

*A simple harmonic vibration is the only kind of vibration that gives to the ear the sensation of a pure tone (Ohm's Law). A complex note si usually termed as a *note* and each of its constituents is called a *tone*, which is further incapable of analysis. A tuning-fork when struck gently gives a pure tone *viz.*, a note without overtones.

136. Displacement Curves of Complex Vibrations.—Question arises as to whether the relative phases of the component simple tones have any effect on the quality of a musical sound. To answer this let us consider the addition of two simple harmonic curves (*a*) and (*b*) producing pure tones of frequencies *n* and 3*n* under two different conditions (*i*) when they are in the *same* phase and (*ii*) when the higher component is *advanced* by $\lambda/2$.

Obviously, the two resultant displacement curves (*c*) and (*c*′) differ in shape, and it seems as if they shall differ in quality. Helmholtz concluded, on experimental evidence given later, that the relative phases of the component tones did not affect the quality of the resultant musical note. Chapin and Firestone, however, showed that some effect could be detected when sound is loud. But according to Helmholtz we can say that the quality is determined only by the *number*, the *distribution*, and the *relative intensities* of the component simple tones, and it is independent of the relative phases of the components. This is why it is a common practice to represent the components of a musical sound as forming a 'spectrum' in which only the frequencies and amplitudes of the components are shown while no attention is paid to their relative phases.

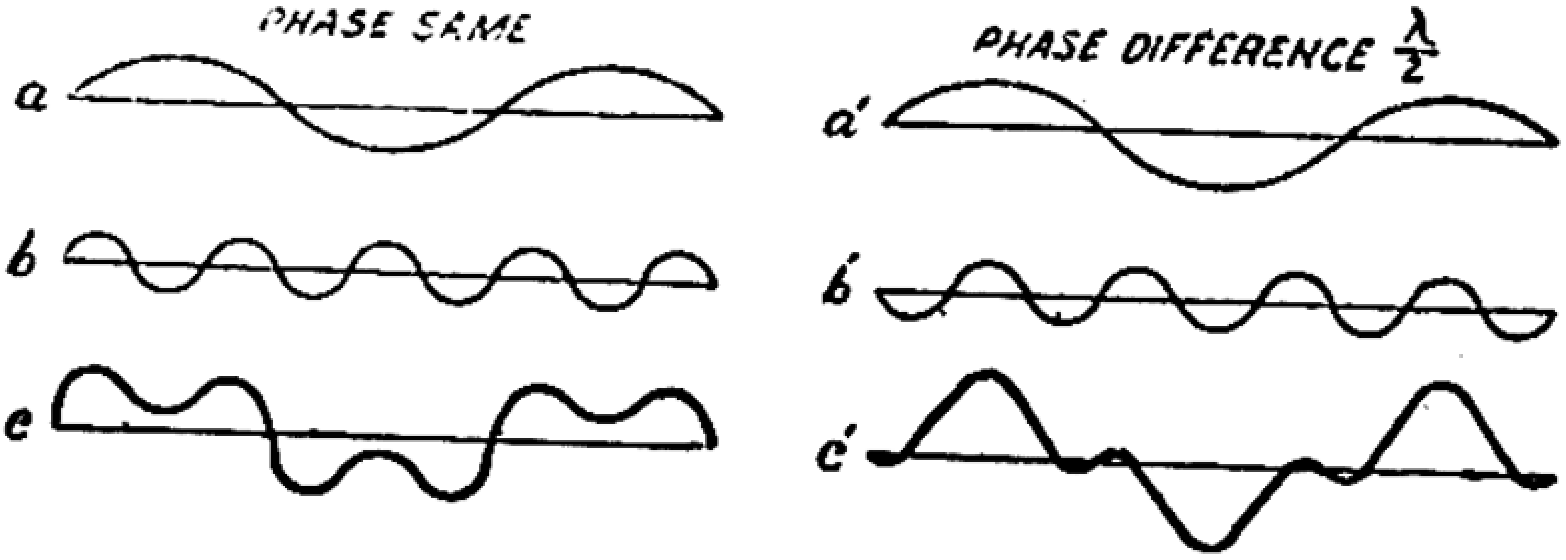

Fig. 105.

(*i*) Two curves (*a*) and (*b*) in the same phase on combining give (*c*)

(*ii*) The curve (*b*) is advanced by $\lambda/2$ giving the curve (*b*′). This combined with (*a*) or (*a*′) gives (*c*′).

137. Fourier's Theorem and its Applications.—The Theorem stated in Art. 135 was first published in 1882 by J. B. Fourier, a French Mathematician (1768—1830) in his *Analytical Theory of Heat*. The theorem is general and applicable to all types of complex periodic curves enabling us to split it up into component sine and cosine vibrations. The resulting displacement of any particle in the medium transmitting the complex periodic wave is thus given by an equation of the form,

$$y=A_1 \sin wt+A_2 \sin 2wt+A_3 \sin 3wt+\cdots$$
$$\cdots+B_0+B_1 \cos wt+B_2 \cos 2wt+B_3 \cos 3wt+\cdots$$

in which the periods and wavelengths of the components are in the ratio $1 : \frac{1}{2} : \frac{1}{3} \ldots$

Ordinarily if we are presented with a square wave of the type shown by thick lined curve in Fig. 105 (*a*) it seems incredible that such a vibration can be built up out of or analysed into a series of smooth sine curves. Yet such is the case and the mathematical analysis shows that the displacement of such a wave will be represented by

$$y=A \sin wt + \frac{A}{3} \sin 3wt + \frac{A}{5} \sin 5wt + \ldots$$

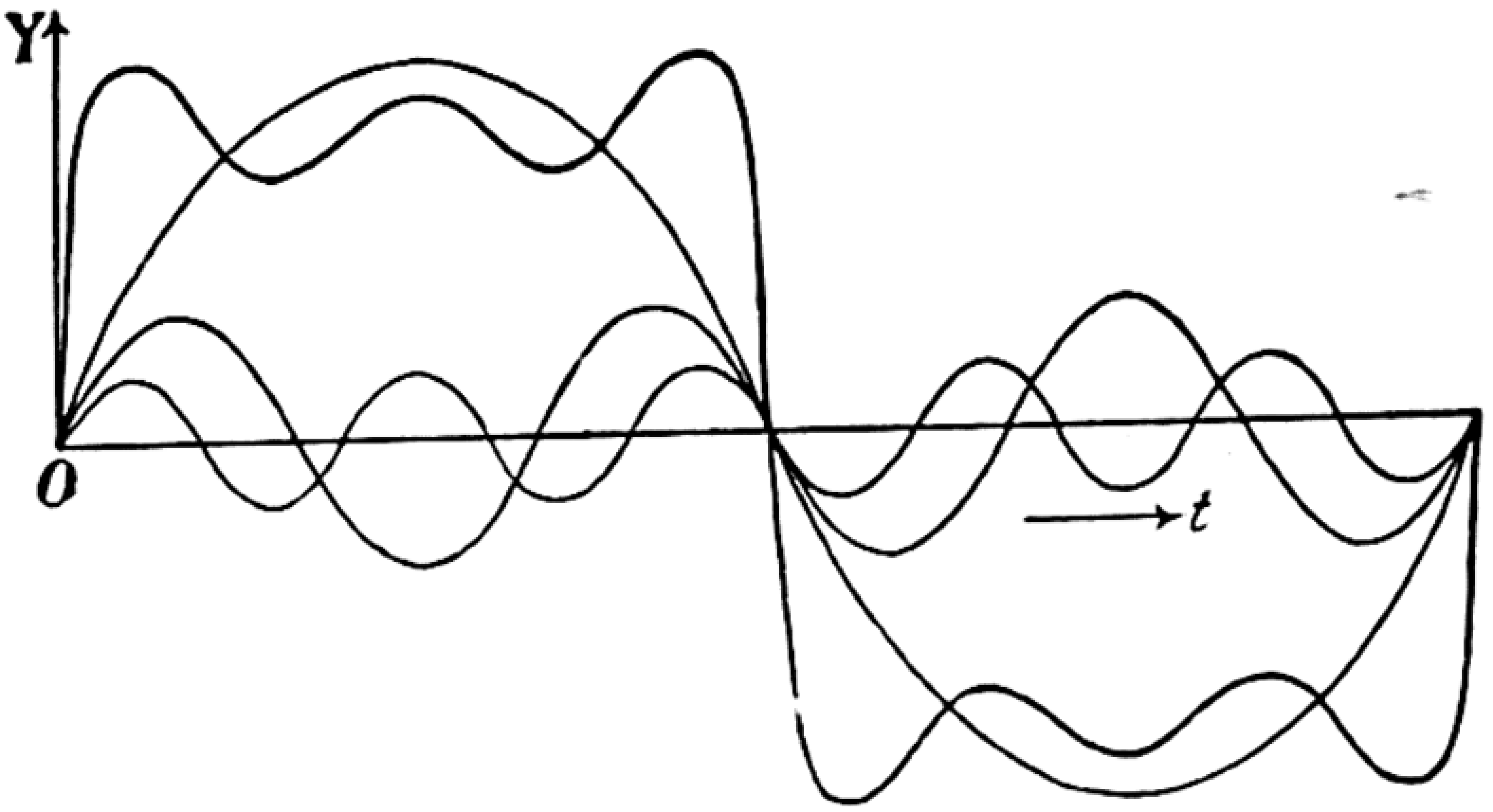

Fig. 105 (*a*)

Thus we learn that given the graph corresponding to any periodic vibration, however complicated, the constituent simple harmonic vibrations may be found together with the amplitude and the relative phase of each.

If the periodic vibration is a fairly simple one, this mathematical analysis into the proper sine and/or cosine components is not too difficult.

To study a complex sound wave we often make use of a Cathode Ray Oscilloscope.* The sound is delivered in front of a microphone and converted into an electric potential difference which is applied to the pair of plates in the Oscilloscope to give the electron beam its vertical deflections.

To obtain a stationary wave form on the screen of the Oscilloscope the "sweep" frequency must be adjusted so as to synchronise with the frequency of the complex wave or some submultiple of it.

The theorem is subject to two limitations regarding the form of the complex vibration :—

(1) The displacement should be continuous and single-valued, *i. e.*, it should not overhang anywhere as at *A* in Fig. 106.

(2) It must not have any infinite value as at *B*.

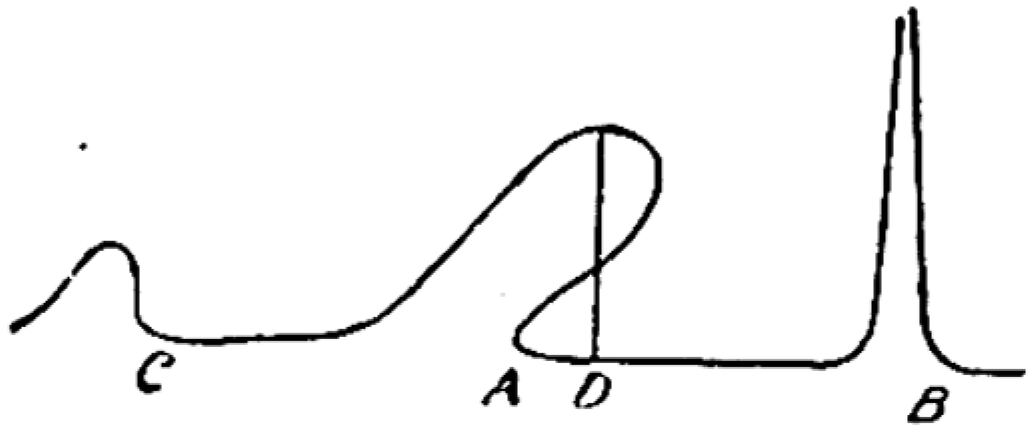

Fig. 106.

Of course while dealing with sound waves, these conditions are clearly fulfilled. For example, in the first case in actual practice an overhanging curve would mean that at some point as *D*, there are two ordinates *i.e.* a particle of air would have two different displacements at the same moment. This is an impossible condition to be realised in an elastic medium. As regards the second limitation, we cannot think of a particle of a medium thrown into an infinite displacement.

The terms required to build up any periodic curve may be limited, or may be infinite in number. They may or may not be present in the serial order. For example, when a complex vibration is due to two tones say of frequencies 300 and 301, then Fourier's analysis points out to these vibrations only, and we say that the 300th and 301st terms of the Harmonic series are present and the other terms are all absent. The frequency of the compounded vibration is unity. Each of the constituents will have completed an exact number of vibrations in 1 sec. In other words the fundamental in this case has frequency 1, but amplitude zero.

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*To understand the working of an Oscilloscope the student is advised to read some advanced book on Electronics.

In case of two harmonic constituents whose frequencies are incommensurable such as 200 and 200 $\sqrt{3}$, this theorem does not apply, because the compound vibration is not periodic and the theorem applies to periodic curves only.

If a complex curve to be analysed has slight curvature everywhere, the lower members among its constituents have greater amplitudes. For points on a curve having marked curvature, the higher terms become more important, while a curve with sharp corners possesses an infinite number of terms of the series.

Fourier's theorem also deals with the synthesis of a complex periodic vibration from simple harmonic constituents. It has a wide field of application, not only in the study of sound, but in the various other branches of Physics and has been successfully employed in the analysis of the periodic phenomena occurring in Nature.

138. Electirically-driven Tuning-fork.—For producing an almost pure tone an electrically driven tuning-fork is generally employed. Fig. 107 shows the arrangement for such a fork. The fork is clamped to a stand, and its one prong is provided with a short metallic strip *W* having a platinum stud at its end. *S* is an adjustable screw ending also in a platinum contact piece *P*, so that when it is screwed forward to touch the strip *W*, the electric battery *E* sends the current through the electromagnet *M* which pulls the prongs of the fork together slightly, thus breaking the contact at *P*. The current stops flowing, the magnetism in *M* weakens, the stiffness of the prongs swings them apart, and *W* touches *P* again, and the prongs of the fork are pulled together. This process is repeatd continually, the fork being thus maintained in a state of vibration.

During the inward motion of the prongs, the force has the same direction as the displacement and the electric battery therefore does work on the fork, while during the outward motion the displacement is in opposite direction to the force, so the fork does work on the battery. If the make and break were quite sudden

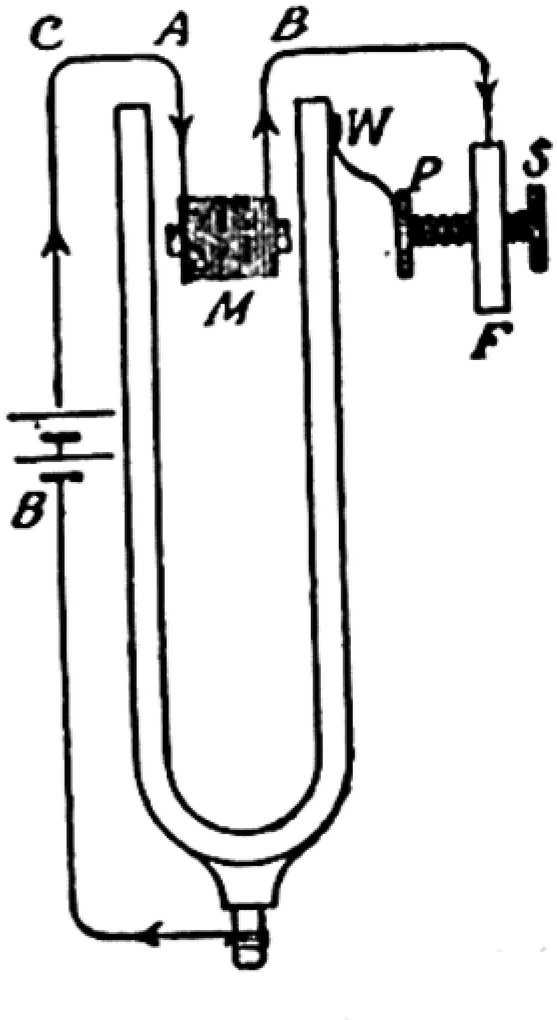

Fig. 107.

and always occurred at the same point in the swing of the prongs, the fork would gain the same amount of energy from the battery as it gives back to it and the vibrations would not be maintained, for no balance would be left to make up for the radiation of energy as sound and heat. But due to (*i*) the irregular contact and (*ii*) the self-induction of the E. M. coil, the make and break are delayed a little. These two causes introduce a lag between the establishment of contact and the phase of the maximum current and also prolong the current at the break. The result is that the work done by the battery on the fork is greater than the work done on it by the fork and so, on the whole, more energy is communicated to the fork than is given back by it, and thus vibrations are maintained. When the current is small, the period will be exactly the period of the fork.

The interruption may be accomplished by a platinum wire *P*, (Fig. 108) just dipping into a cup of mercury when the fork is not vibrating. *A* and *C* are the poles of the electromagnet *M*. When the wire is just touching the surface, current flows along *AMCPD* as shown in the figure and the electromagnet is excited. The prongs are pulled apart, causing the electrical circuit to break at *P*. This weakens the magnetism in *M* and so the prongs go back causing the current to flow again. Thus the process is repeated. Note that when the circuit breaks, the spark produced makes a good deal of noise. Hence such an interrupter fork cannot be used in the synthesis of complex vibrations. As seen above the circuit is interrupted once during each vibration of the fork, and this periodic current can be used to drive another fork at a distance from the interrupter fork so that the noise produced by the interrupter fork is out of hearing. The fork will be made to execute forced vibrations with the period of the force due to the intermittent current. If the interrupter fork and the maintained fork placed at a distance have the same pitch, resonant vibrations will be produced. In this case there

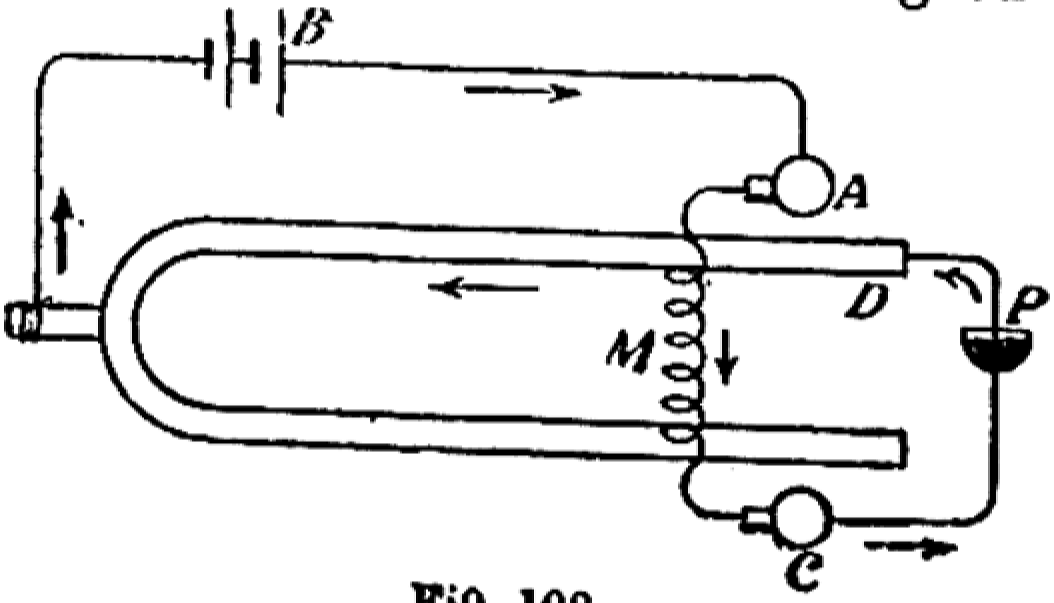

Fig. 108.

will be one impulse for each vibration. If the maintained fork has twice the frequency of the interrupter fork, there will be one impulse for each two vibrations, and large resonant vibrations will still be produced. Such electrically maintained tuning forks worked by an interrupter were used by Helmholtz for the synthesis of complex vibrations and one such is shown in Fig. 109. When held in front of a resonator it produces a tone which is almost pure.

139. Helmholtz's Theory of Quality and his Analysis and Synthesis of Complex Vibrations.—As pointed out before quality depends upon waveform but the view expressed by Helmholtz that differences of form caused by the phase differences of the constituents did not give differences of quality, was proved experimentally by him. Notes of various musical instruments were first analysed with the help of his **resonators*** described in Art. 118, to find the number, order and the intensity of the various harmonic constituents. Then tuning-forks of different frequencies representing these constituents when sounded together with proper intensities reproduced the note of original quality. The general outline of the procedure adopted by him to imitate the quality of the note of a particular instrument was as follows :

The apparatus consisted of 10 tuning-forks. which were tuned so as to give a fundamental note of 256 vibrations per second and its first nine harmonics. They were all worked electrically so as to give rise to vibrations which do not die out but are maintained continuously for any length of time. Behind each fork was a resonator tuned to unison with it (Fig. 109) and which could be placed at different distances so as to intensify the sound by olny a desired amount. Each of the resonators had a disc by which its mouth could be wholly or partially closed as

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*The air inside a resonator has a natural period of vibration, and can resound to a particular frequency only. The complex note to be analysed is played near *A*, Fig. 92. The end *B* is held in the ear or placed in front of a manometric flame or capsule devised by Koenig, in front of which is placed a revolving mirror. If this note contains a particular frequency to which the resonator responds, the flame will dentate as evidenced by the jagged edges of the images of the flame in the revolving mirror ; thus the constituent or harmonic present is picked out and sound of large intensity is produced. Thus various resonators of different frequencies are used to pick out different components or constituents and thus the *note* is analysed.

desired. The function of the resonator was to make the sound emitted by the fork loud. When the mouth of the resonator is closed the reinforcement of the sound does not take place and consequently it becomes so feeble as compared to the sound when the resonator is in action that it may practically be neglected. The electromagnets of these forks were all kept in series with the electric circuit of the interrupter fork so that the intermittent current produced by it is sent through them. Thus all the forks were kept in vibration simultaneously.

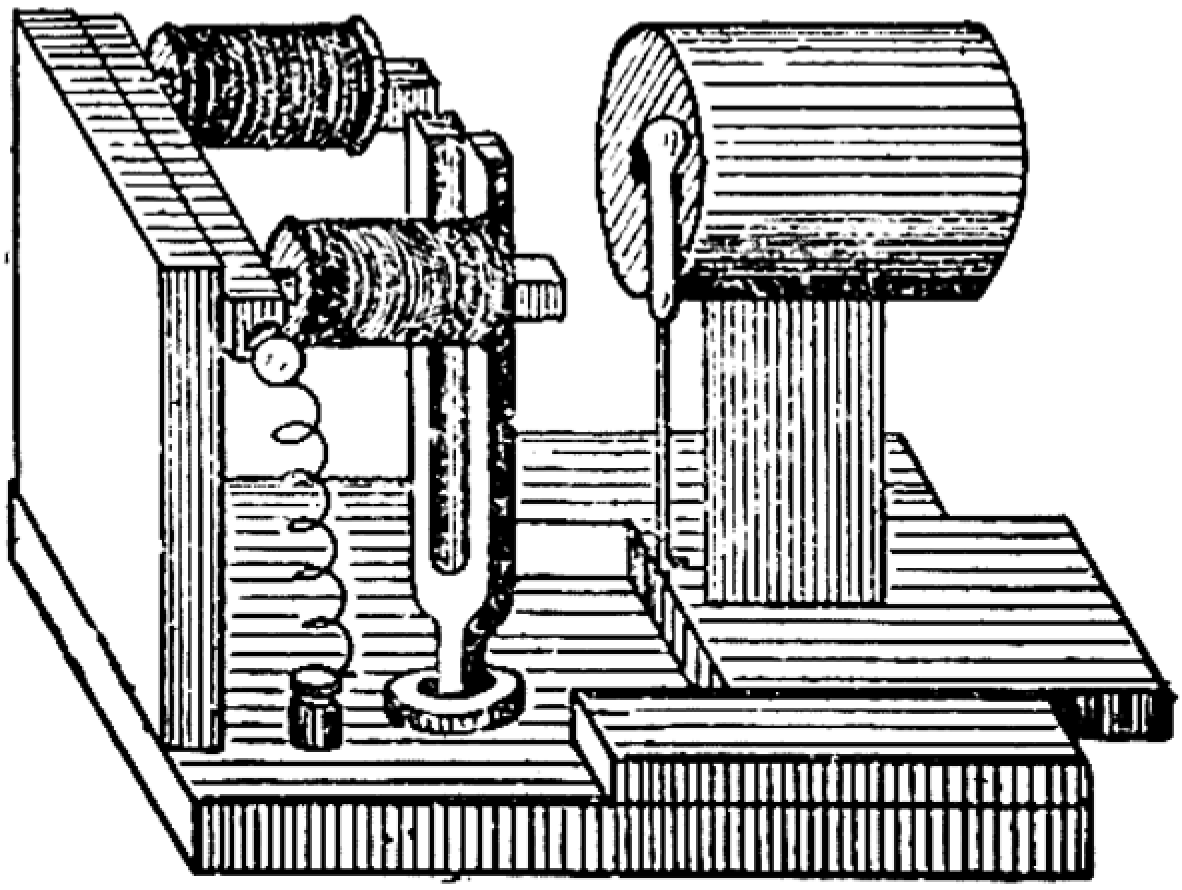


Fig. 109.

Helmholtz, with his **resonators** found on analysis that a particular organ pipe consisted of three members 1, 3, 5, of harmonic series, the other being quite inaudible. Then he used his forks as mentioned above to build up the note. The discs of all resonators except 1, 3, 5, were closed These were adjusted at proper distances from tuning-forks to obtain the exact intensity of tones as noticed in the constituents. Afterwards when these forks were worked together, it was found that the quality of the note of the organ pipe was almost fully reproduced. Thus the notes of other musical instruments were also imitated.

To introduce phase differences in the constituents Helmholtz proceeded as follows :

The mouth of a resonator was partly closed by the disc which was thus put out of tune with its fork. The intensity of sound diminished and phase difference was introduced. To restore the sound to original intensity, it was moved nearer the fork. Thus the harmonic orresponding to this fork had the original pitch and intensity but differed in phase. Thus by altering one or more constituents in phase in this way, the quality of a complex note now produced was not changed, proving thereby that the phase of the harmonics had no effect on the quality *i.e* , it depended only upon the number, distribution and strength of harmonics and not upon their relative phases. Since Helmholtz's time much has been written by the physicists and all of them do not accept this conclusion. We may say that Helmholtz's theory is right as a first approximation, and a change of relative phases of harmonics of a note should have some effect on its quality.

140. Speech-Consonants and Vowels.—Separate sounds, which make up our speech and our songs, are divided into consonant and vowels. Consonants are more or less methods of beginning and ending vowel sounds. They are only passing sounds or mere noises and are not continuous. Vowels, on the other hand, are musical sounds and can be maintained indefinitely. Each vowel has its own characteristics and maintains it. To judge its quality it is not necessary to hear its beginning or its end. For example, during a song in order to sing a word '*bat*' on a particular note, the voice begins with consonant *b* lasting for a very short duration, then at once passes on to the vowel, *a*, which can be maintained for any length of time and then ends suddenly in *t*.

140a. The Human Voice Organ.—It may be regarded as a double-reed pipe instrument (Fig 110). During speech or song the lungs force the breath upwards through the *trachea* and the *larynx* (Voice Box) into the cavities of the mouth and nose. In the larynx is a pair of stretched membranous reeds called Vocal Chords. These chords are triangular shaped masses of flesh which project from the sides of the larynx and cover nearly half of it. When not in use, their free edges are widely separated, hence no interference with the breathing. By means of muscles these chords are tightly stretched and their edges are brought together.

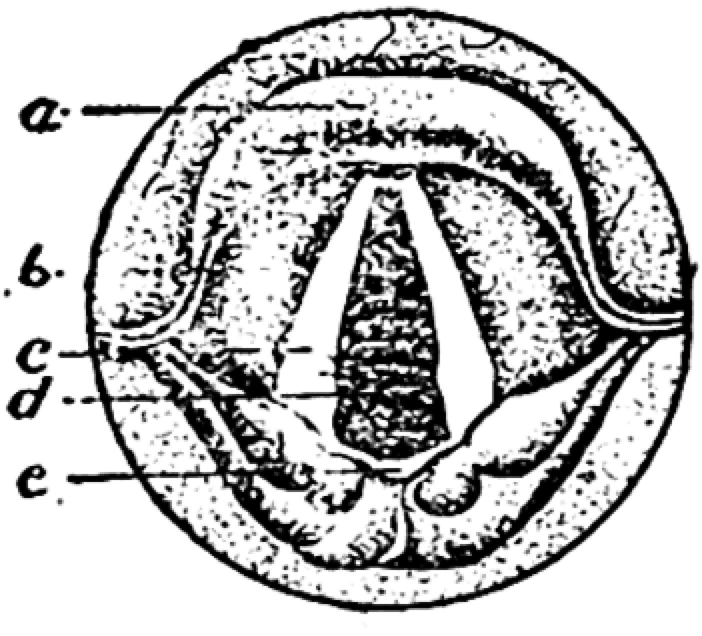


Fig. 110.
Diagram of Larynx.
(*a*) epiglottis
(*b*) false vocal chords
(*c*) true vocal chords
(d) trachea
(*e*) anterior wall of Larynx

The space left between them is called the *glottis*. If now the air is forced through the glottis, the edges of the chords are set in vibration, causing the air to issue in a series of puffs which produce a musical note. The pitch and quality of the note is changed by varying the tension and distribution of the mass of the chords with the help of muscles attached to the larynx. The so-called false vocal chords may slightly influence the vibrations of true vocal chords. In fact the vibrations of the vocal chords produce sound impulses travelling up the mouth, thereby producing a given vowel depending upon the shape of the mouth. Consonants may be either produced by sudden stoppage of the lips or by a sudden tongue movement. As mentioned already while singing a word *bat*, '*b*' and '*t*' are consonants produced by a sudden stoppage of the lips and tongue, while the vocal chords maintain the vowel sound '*a*' for a sufficient length of time. This sound passes to the outer air through the mouth and its adjoining cavities which act as resonators. The pitch of the mouth, however, can be altered at will by changing its volume or the size of its opening. The change in volume can be affected either by moving the tongue or by opening the jaws more or less widely, while size of the opening can be controlled by means of the lips. Thus the quality of the voice is greatly modulated by changing the shape and size of the mouth cavity, which go to produce different vowels.

141. Characteristic of a Vowel Tone.—The most important point in the quality of sounds is the resonance of the cavity of the mouth. In an ordinary musical note, the intensity of the upper partials decreases as the pitch increases, and when resonance chambers are employed, they are simply tuned to the fundamental tone and so reinforce that only. But when a vowel is uttered, the mouth cavity acts as a peculiar resonant chamber reinforcing *not* the fundamental but rather one or two partials of the vowel sound. Thus the singing of a vowel has a characteristic quality which differentiates it from that of a musical note. No musical instrument is such as can produce a vowel tone. The vowels, a, e, i, o, u are characteristic features of the mouth. It is mouth only which can sing any vowel at will. Prof. D. C. Miller has done a lot of work in this connection. He has concluded that if the frequencies of the *partials* of a given vowel be increased in a given ratio, the vowel characteristic is not preserved. In order to preserve it, their *amplitudes* must be increased in the same proportion. On analysis it was seen that to produce a given vowel *partials near a given pitch should have large amplitudes.* In short the pitch of a vowel is determined by the vocal chords but the vowel characteristic is only maintained by the vocal and other adjoining cavities which go to *reinforce the partials of the given pitch* and so preserve the vowel characteristic.

142. Vowel Theories.—There are at present two theories as to the nature of vowel sounds. They are called the 'Fixed pitch' and the 'Relative pitch' theories. Helmholtz believed in the fixed pitch theory, according to which each vowel is characterised by one or two restricted regions of *given pitch*, which are independent of the fundamental tone of the vowel. On the relative pitch theory, each vowel has a series of partials or overtones, the frequencies of which have a certain *definite relation* to the fundamental. Recent experiments of Wilis and Miller are in favour of fixed pitch theory. If this theory be true, it would be possible to change the vowels in a phonograph by running the instrument at a speed higher or lower than the original. If the vowels depend upon the *relative* frequencies (Relative pitch theory). quality should not change : while if they depend upon *absolute* frequency (Fixed) pitch theory), vowels should change. A number of investigators applied this test to the phonograghy and it was seen that *vowels are* changed, thus proving the results in favour of the fixed pitch theory.

143. Some Musical Instruments and Harmonic Constituents of their Notes.—We give below the harmonics of the notes of some musical instruments just to compare their quality.

(1) **A stopped organ pipe.**—It has only *odd* harmonics. A wide pipe gives practically a pure note. A narrow pipe gives the third harmonic strong and fifth is perceptible.

(2) **An open organ pipe.**—It has full series. A wide pipe gives a strong octave. A narrow pipe has harmonics as far as the sixth or seventh but of diminishing intensity.

(3) **The flute.**—It gives nearly a pure tone. Octave is faintly audible. No other harmonic is audible.

(4) **The clarinet.**—It has 3rd, 5th and 7th harmonics fairly strong. The 4th, 6th and 8th are faintly audible.

(5) **The Hautboy.**—It has full series ranging to the 16th or so, when they become inaudible.

(6) **The Human Voice.**—It has also full series of harmonics falling off gradually in intensity. They extend upto the sixteenth before being inaudible.

(7) **The Pianoforte.**—Its second harmonic is as strong as the fundamental. Third and fourth fall rapidly in intensity. seventh is hardly audible.

(8) **The Violin.**—It has a long series of harmonics falling gradually in intensity. Upto 4th they do so rapidly and then more slowly.

(9) **The Sonometer.**—Its overtones constitute harmonic series.

(10) **The Tabla (Indian).**—Its components form harmonic series. It is a percussion instrument but for its components, it is used in Indian Music as a musical instrument. Its tonal quality is much more superior to other drums.

For the sake of comparison it will be worth while to mention some other instruments or bodies which produce inharmonic overtones.

(1) **The Tuning-fork.**—Its overtones do not belong to *harmonic series.* They are much higher in pitch than the fundamental. The first is 2 to 3 octaves, and the second is 4 to 5 octaves above the fundamental. On striking they are audible for a few seconds. As compared with the fundamental, they die very quickly. If a resonance box is used, it not only strengthens the fundamental and so a tuning-fork on a resonance box gives almost a *pure tone,* hence its frequent use in the study of sound.

(2) **The Bell.**—Generally its overtones form a non-harmonic series. Here the lowest tone is called the "Hum Note". It is the second which mainly determines the pitch. See Art. 174 for the description of the bell.

(3) **Vibrations of rods and plates** also produce notes which contain inharmonic overtones.

144. The Human Ear.—The anatomical structure of the human ear is very complex and it may be briefly described as follows :—It consists of three principal parts, (Fig. 111) the *outer*, the *middle* and the *inner* ear. The outer ear (pinna) from which extends the ear passage, ends in a drum skin called *Tympanum*. Behind the tympanum there is the air cavity of the middle ear in which there is the chain of three bones or ossicles the *hammer m*, the *anvil i*, and the *stirrup s*, which connect the tympanum with the inner ear. The cavity of the middle ear is connected to the throat through the *Eustachian tube E*, whose lower end is closed by a valve which opens in the act of swallowing. It serves to equalize the pressure on each side of the Tympanum *T*. The inner ear in which the auditory nerve terminates and which is contained in a cavity in the massive part of the temporal bone consists of three parts, the *Semi-Circular canals S*, the *Vestibule V*, and a long chamber coiled up like a snail shell and hence known as the *Cochlea*. The inner ear communicates with the middle ear through two openings, (*i*) *Fenestra Rotundus R*, (round window) which is closed by a membrane and (*ii*) *Fenestra Ovalis O*, (oval window) closed by the foot of the stirrup bone. The interior of the inner ear is filled with a watery fluid, the *Endolymph*.

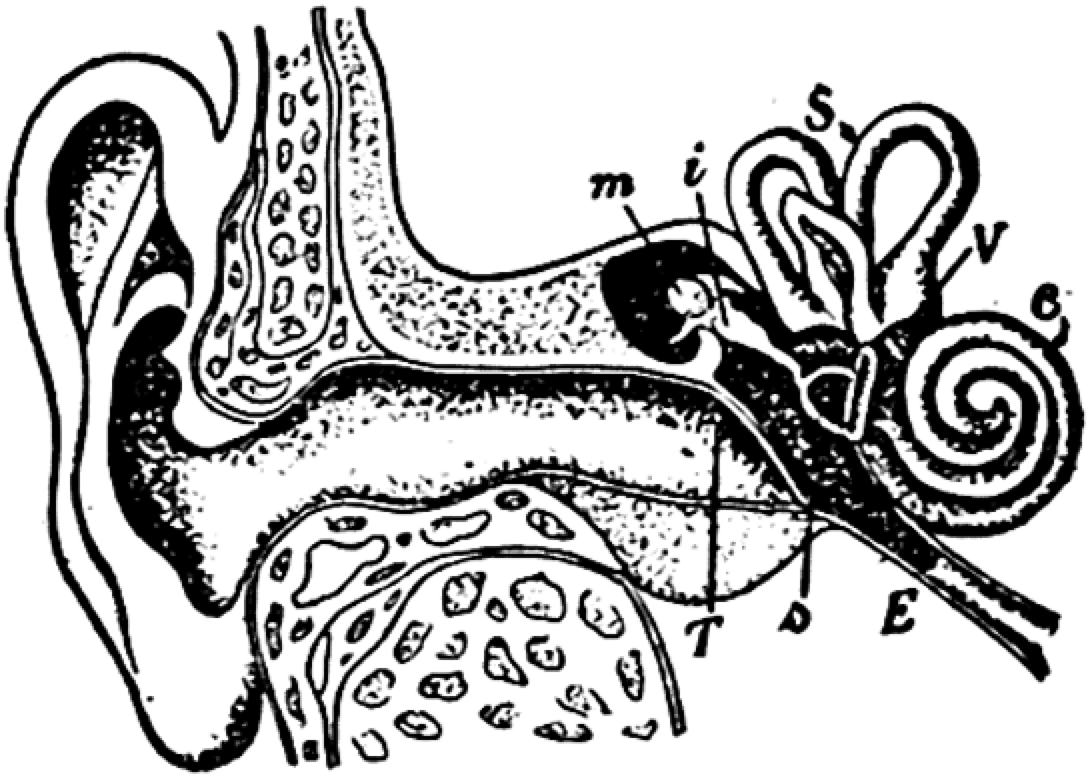


Fig. 111.

Fig. 112 shows a longitudinal section of the Cochela as if straight. It is believed to be the part of the ear by which musical sounds are distinguished in pitch and quality. It is divided into two chambers called *Scala Vestibuli* (*SV*) and *Scala Tympani* (*ST*) by a thin partition

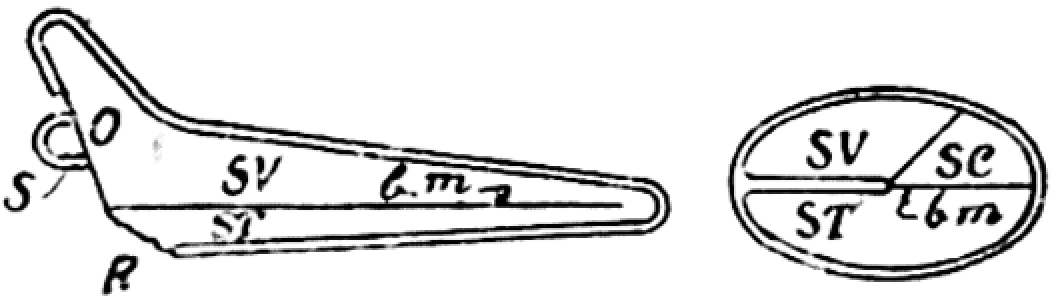


Fig. 112.

called *Basilar Membrane* which runs lengthwise through all its convulsions. The membrane is strongly fibrous in structure, the fibres running transversely across from one side of the tube of the Cochlea to the other. Forming an arch of triangular section over a portion of the basilar membrane is the *Organ of Corti* consisting of rods or pillars and hair cells. The fibres of the auditory nerve terminate into this basilar membrane so that a disturbance in any part of the membrane causes a stimulus to the corresponding filament of the auditory nerve. The membrane increases gradually in width as we pass inwards round the coils of the Cochlea, transverse fibres vary in length and may be compared to the string of a piano having their own periods of vibration and therefore resounding to different frequencies.

145. Action of the Ear.—The action of the ear is not fully understood and the explanation given below cannot be regarded as completely satisfactory. It must be remembered that both forced and resonant vibrations play a very important part in exciting the sensation of hearing. When sound waves, collected by the external ear and sent down the auditory canal, strike against the tympanum, they set up forced vibrations in it. These vibrations are transmitted through the ossicles *viz*, hammer, anvil and stirrup to the membrane O, called the oval window communicating with the labyrinth or the inner ear, the seat of hearing. On reaching the Cochlea the vibrations are handed on by incompressible fluid in its two chambers to the Organ of Corti and the basilar membrane. They will set into resonant vibrations those fibres of the membrane as are tuned to respond to the motions of that particular frequency and therefore the corresponding nerve filaments will be stimulated and transmit the excitation to the brain. The arrangement is that sounds of different pitches entering the ear simultaneously will produce sympathetic vibrations in different parts of the basilar membrane, stimulate different nerve filaments and so give rise to different sensations in pitch, loudness and duration. We now understand how the ear enables us to resolve complex sounds, hearing each simple harmonic component as a separate simple tone.

146. Sound Intensity and Limits of Audibility.—We have already defined Intensity of Sound as the average time rate of transfer of energy per unit area of a surface perpendicular to the direction of propagation. Equation (15) on page 63 gives the magnitude of this energy current. Simplifying this the intensity I of the sound is given by

$$I=2\pi^2n^2a^2v\rho \qquad \ldots \qquad \ldots \qquad (1)$$

where n stands for the frequency.

According to this result, the intensity of sound varies directly as the square of both the amplitude and the frequency of vibration.

Let us see how the intensity of sound varies with the excess pressure associated with the waves.

Now $$v=\sqrt{\frac{E}{\rho}}$$

where the elasticity E is equal to $-p\dfrac{V}{dV}$, dV being the change in the original volume V by excess pressure p

whence $$v=\sqrt{-\frac{pV}{\rho dV}}$$

For longitudinal waves we have seen in Art. 38 that the volumetric strain $\dfrac{dV}{V}$ is equal to $\dfrac{dy}{dx}$ *i.e.*, the slope of the displacement curve. Substituting $\dfrac{dy}{dx}$ for $\dfrac{dV}{V}$ in the equation for v and solving for p, we have

$$p=-v^2\rho\frac{dy}{dx}. \qquad \ldots \qquad \ldots \qquad \ldots \qquad (2)$$

For these simple harmonic waves,

$$y=a\sin\frac{2\pi}{\lambda}(vt-x)$$

hence $$\frac{dy}{dx}=-\frac{2\pi a}{\lambda}\cos\frac{2\pi}{\lambda}(vt-x)$$

Therefore equation (2) becomes

$$p=\frac{2\pi av^2\rho}{\lambda}\cos\frac{2\pi}{\lambda}(vt-x) \qquad \ldots \qquad \ldots \qquad (3)$$

$$=p_{max}\cos\frac{2\pi}{\lambda}(vt-x)$$

where $$p_{max}=\frac{2\pi av^2\rho}{\lambda}=2\pi an v\rho \qquad \ldots \qquad \ldots \qquad (4)$$

This may be taken to be the equation of a pressure wave and p_{max} is called the *pressure amplitude.* Note that this pressure wave is quarter period out of phase with our original displacement wave for it is a cosine function, whereas the displacement wave is denoted by a sine function. Combining equations (1) and (4) we get

$$I=\frac{p^2_{max}}{2\rho v} \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad (5)$$

It shows that the intensity varies as the square of the excess pressure. As the frequency does not occur in this formula it is clear that the intensity of sounds of different frequencies may be compared by just measuring the associated excess pressures.

In acoustical measurements we prefer to measure pressure variations rather than amplitudes. Experiments have shown that inorder that aerial waves entering the ear may produce the sensation of sound, they must have a certain intensity and certain frequency. The intensity as said above, is measured by the pressure variations and the values of p_{max} range from about 2×10^{-4} dyne/cm.^{2} for the faintest sound that can be detected by the ear up to about 200 dynes/cm.^{2} for a sound so loud that it begins to be painful. For ordinary conversation p_{max} may be about 1 dyne/cm.^{2}. Let us calculate the amplitude of the ordinary conversational waves at 20°C, at normal pressure and for $\lambda=100$ cm. taking the velocity of sound 344 m./sec. The density of air is ·00121 gm. per c.c.

From Equation (4)

$$a=\frac{p_{max}\times\lambda}{2\pi\ \rho v^2}$$

Substituting the values we obtain

$$a=\frac{1\text{ dyne/cm.}^2\times100\text{ cm.}}{2\pi\times\cdot00121\text{ gm./cm.}^3\times(34400\text{ cm./sec.})^2}$$
$$=1\cdot12\times10^{-5}\text{ cm.}$$

This shows that the amplitudes of sound waves in the air are very small.

To calculate the intensity of conversational sound waves use the relation (5)

$$I = \frac{p^2_{max}}{2\rho v}$$

Putting the values of p_{max}, ρ and v, we obtain

$$I = \frac{(1 \text{ dyne/cm}^2)^2}{2 \times 0{\cdot}00121 \text{ gm /cm}^2 \times 34400 \text{ cm./sec.}}$$
$$= 1{\cdot}2 \times 10^{-2} \text{ erg. sec.}^{-1} \text{ cm.}^{-2}$$
$$= 1{\cdot}2 \times 10^{-9} \text{ watt/cm.}^2$$

This sound output per sq. cm. of the wave-front is very small but the human ear is so sensitive that, it can detect sounds of very much lower intensity than this. This can be easily seen if we calculate the intensity corresponding to the value of p_{max} for the lower limit of audibility. Taking $p_{max} = 2 \times 10^{-4}$ dyne/cm.^{2} I works out to be 5×10^{-17} watt/cm^2. To get an idea of this sound output, it will not be out of place to mention that a buzzing mosquito develops an out-put of 10^{-4} watt.

The minimum intensity which a sound must possess in order to be audible is called *Threshold Intensity of Audibility.* If the intensity increases above a certain value called the *Threshold Intensity of Feeling* a sensation of pain is produced in the ear. This may be as high as 10^{13} times the minimum value. Further the vibrations must have a certain *upper and lower limit of frequency* in order to be perceived as sound. Fig. 113 shows the relationship indicating the *sensitivity* of the ear. The pressure amplitudes expressed in dynes per sq. cm are plotted as ordinates and frequencies as abscissae both logarithmically. The upper curve in the diagram represents the upper limit to the range of auditory sensation above which a sensation of pain is felt by the ear

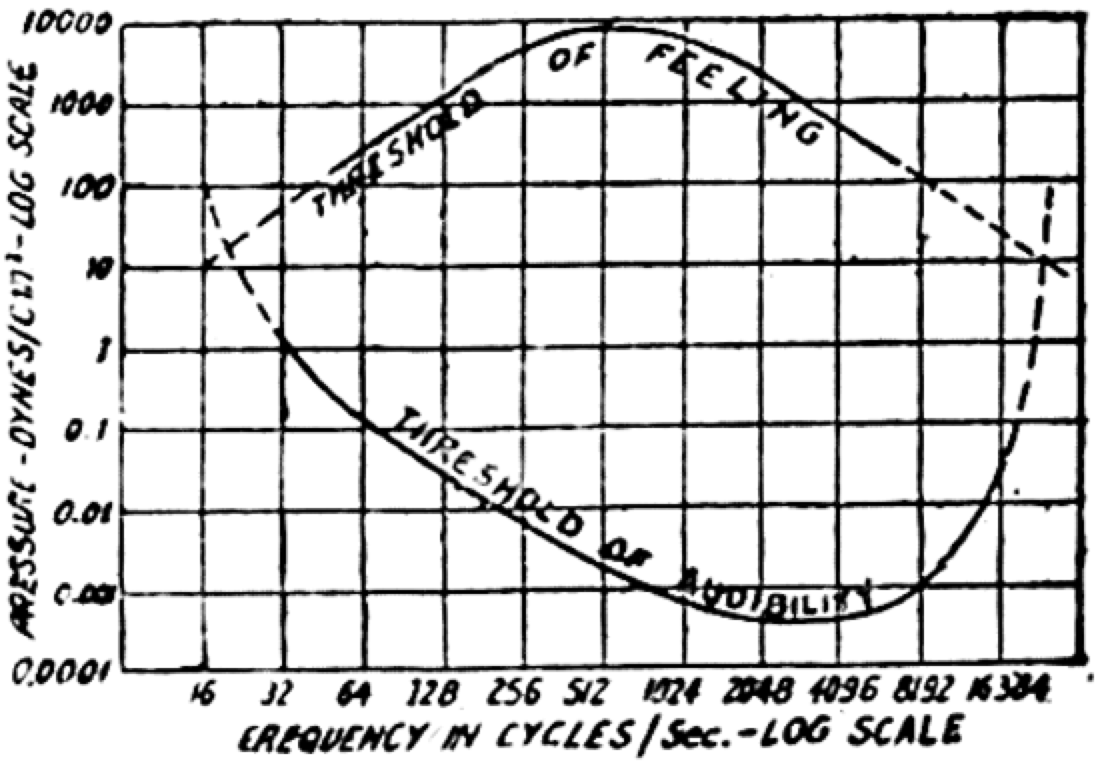


Fig. 113.

(threshold of feeling). The lower curve represents the lower limit of the amplitude at which a sound can be detected (threshold of audibility). The range of audibility lies within these two curves and the area enclosed is described as the **'Auditory Sensation area'**.

Helmholtz working with a double siren and long tuning forks found that for pure continuous tones the frequency of the lower limit is about 30 vibrations/sec. Below this the vibrations do not blend into a note but are perceived as separate impulses. The upper limit of frequency according to different experimenters varies within wide limits with advancing age ; at 20 it is about 200,00 at 35 about 160,00 and at 47 about 130,00 vibrations/sec. The elderly people cannot bear such sounds as chirping of crickets and squeals of bats. Despritz, and Appunn have concluded that they could hear frequencies as high as 40,000. The musical limits, however, are between 40 and 4000/sec. Thus unlike the eye which can perceive a range of barely an octave, the ear has an extreme range of obout 11 actaves, about seven octaves being musically available.

147. Resolving Power of the Ear, its Limitations and Ohm's Law—In Art. 145, we saw that the ear analyses the complex notes into their simple harmoino constituents, each constituent corresponding to a separate tone incapable of analysis, the basilar membrane serving as the acoustical equivalent of a spectroscope. This is in accordance with Ohm's Law which states that the only tones which give to the ear the sensation of a pure tone correspond to the pendular vibrations of the ear. Fourier's analysis of the complex vibrations of a stretched string confirms this view for the ear is found to recognize in them only those tones which agree in pitch with the calculated periods. Thus the ear is a practical Fourier analyser and the harmonic analysis in the case of musical notes derives its importance from this fact.

In this respect the ear differs from the unaided eye. If sunlight or light from an electric lamp or a discharge tube which is in fact a mixture of several colours consisting of ethereal waves of different wavelengths falls upon the retina, it is not resolved into its separate constituents ; thus eye sees only some one colour. On the other hand, when the attention is properly educated the ear can easily detect different constituents of a compound note disturbing the auditory nerve. On the theory of hearing, sketched above, it is expected that the relative phases of the components in a complex note should have no influence on the resulting sensation, for the same parts of the basilar membrane are excited whatever the phase of the constituents. There is, however, an exception to by made. In the case of beats, *i.e.*, whenever there are two constituent notes very close together in pitch, the pulsations of the tones are certainly due to changing relative phases of the two components which are alternately in and out of step. This pulsating effect may not be regarded as due to the failure of the general statement but due to the fact that *the two tones are so near in pitch that they affect closely adjacent portions of the basilar membrane.* So that the fibres midway between the responding portions will vibrate simultaneously in sympathy with both tones. They will, therefore, take the resultant motion and will vibrate alternately strongly and feebly, according as the two components are in the same and opposite phases. The ear is thus unable to distinguish them as separate tones. Like the optical instruments the resolving power of the ear is limited.

Further limitation is imposed on the resolving power of the ear by the masking effect of other sounds. When the masking tone is loud, it masks tones of a higher pitch better than those of a pitch lower than itself.

CHAPTER XI

VIBRATIONS OF AIR COLUMNS IN PIPES

148. Excitation of Vibrations in Air Columns.—Stationary longitudinal waves can be produced in a column of air by any periodic movement whose frequency coincides with one of the natural frequencies of the column. All wind instruments are provided with a column of air called a *resonator*, which may be in the form of a pipe or a rectangular air chamber. The periodic movement is caused by an important part of the musical instrument called the '*mouthpiece*' which is different in construction in different instruments. It is this mouthpiece which acts as a generator and supplies the energy necessary to maintain the vibrations in the column of air. We shall consider two modes of excitation which are of special importance and accordingly organ pipes are classified into two groups, (*i*) **Flute** or **Flue Pipes** and (*ii*) **Reed Pipes**. Each of these classes may have many varieties differing in shape and material.

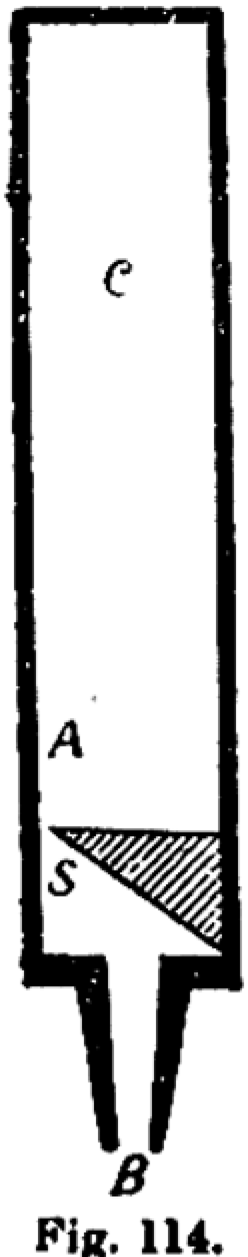


Fig. 114.

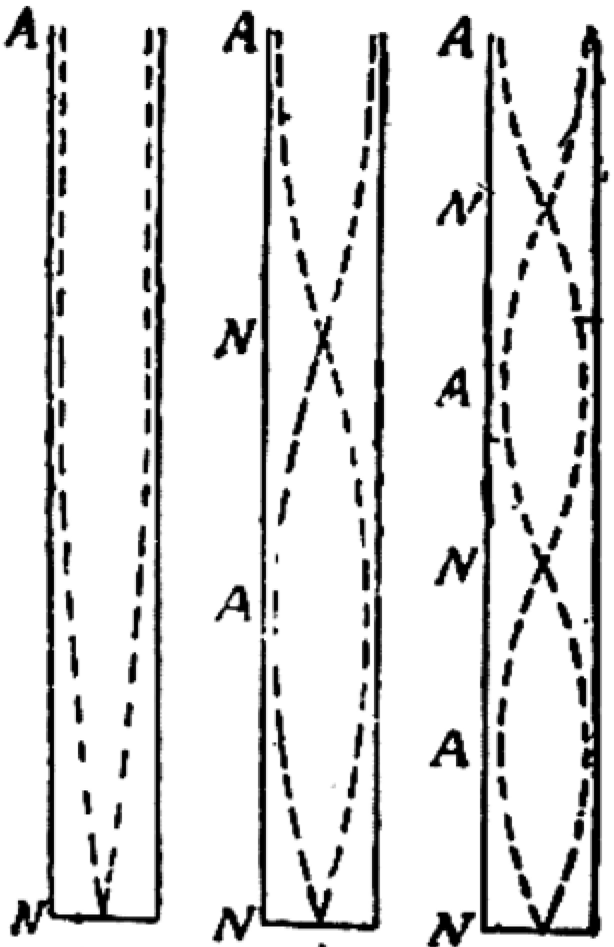
149. Action of a Flue Pipe.—Fig. 114 represents a section of a Flue Pipe. The end B is put on an acoustic bellows and wind is forced through it. It issues from the narrow slit at S in a thin sheet which strikes the sharp edge A and sets up vibrations in the body C. A sound is produced, the pitch of which depends upon the dimensions of the pipe and the velocity of the current of air. The pipe may be made of metal or wood circular or square in section. The upper end is closed in some pipes called *closed* organ pipes, and is open in others which are called *open* organ pipes. The blast at the mouth sends a train of waves along the pipe. These waves are reflected from the other end—open or closed, as the case may be. The direct and reflected waves will be superposed and produce stationary vibrations in the pipe. It is really a case of interference between two similar wave trains travelling with the same velocity in opposite directions previously considered in Chapter VII. The reflected wave will be different in nature in an open pipe to that in a closed one, because the open end of the pipe acts like a yielding wall, while the closed end acts as a rigid wall.

150. Closed Organ Pipe and its Period of Vibration.—The closed end acts as a rigid wall. *i.e.*, a wave of condensation or rarefaction is reflected as such. Thus at the closed end two condensations or two rarefactions will be superposed. There will be greatest change of density of the air in contact with the closed end. Velocity of the particles of air will be reversed after impact. There will be absolutely no motion of the particles over there and hence the closed end will act as a **node.** The open end of this pipe acts as a yielding wall. A wave of condensation arriving at the mouth deflects the sheet of air outwards and is reflected as a wave of rarefaction. The Bernoulli effect of the sheet of air which is constantly blowing across the outside of the mouth further increases the amount of this rarefaction and so at the open end these superpose with the result that the layer of air will have usual density. The displacement of the particles both in the direct as well as in the reflected wave will be in the same direction thus at the open end the resultant displacement will be great and this end consequently forms an **antinode.** The rarefaction produced at the mouth then travels towards the closed end where it is reflected as such. On reaching the mouth it sucks the sheet of air inwards and is thereby converted into a condensation and the whole process is repeated. Note that the energy for maintaining the vibrations is derived from the sheet of air. Fig. 115 (*a*) indicates the nature of the stationary waves when the fundamental tone is emitted. This amounts to saying that the period of vibration of a closed pipe is the time taken by a pulse in travelling twice up and down the pipe. A pulse in the form of a condensation makes two journeys to the top and back before starting again as a condensation, *i. e.*, to complete one cycle, the pulse moves four times the length of the pipe and so the wavelength $\lambda = 4 \times$ length of the pipe. This is the case for the fundamental tone. Other modes of vibration are described later.

151. Open Organ Pipe and its Period of Vibration.—Here both ends are open, and consequently they serve as antinodes. There must be a node in the middle in case of the fundamental. Fig. 116 (*a*) indicates the nature of the stationary wave in this case. This amounts to saying that the period of vibration of an open pipe is the time taken by a pulse in travelling *once* up and down the pipe. A condensation started at the mouth travels to the open top and reflects as a rarefaction. On reaching the mouth, it suffers a reversal in phase, and starts as a condensation up the pipe again. In other words to complete one cycle, the pulse moves twice the length of the pipe and so for the fundamental tone wavelength $\lambda = 2l$. Two other modes of vibration are shown in Fig. 116 (*b*) and (*c*). Really speaking an open organ pipe may be regarded as equivalent to two closed pipes of half its length.

152. Nomenclature of Modes of Vibration.—Sounding bodies are generally capable of vibrating in a variety of ways simultaneously giving rise to a complex note which consists of tones of different pitches or frequencies. The lowest tone* is called *Fundamental.* The rest are called *Overtones.* Overtones whose frequencies are in the ratio 1, 2, 3, etc., just as in the case of a string or an open pipe form a series called Harmonic series. Separate tones of the series may be called *Harmonic Oertones*, *Harmonics* or *Harmonic partials* or merely *partials.* The *first harmonic* will be called the first partial. Note that the term partial is restricted to the members of the harmonic series only. There are cases when the overtones are not harmonic such as the overtones of the note of a bell. Other examples are in the overtones of transverse vibrations of rods, drums, tuning forks etc. They are called *Inharmonic Overtones.*

153. Overtones of a Closed Pipe.—When a pipe is blown strongly, it emits a number of overtones along with the fundamental. For a closed pipe, the closed end is always a node and the open end an antinode. The fundamental and two other modes of vibration are shown in Fig. 115 (*a*), (*b*), and (*c*).



(*a*) (*b*) (*c*)

Fig. 115.

In the first case $l = \frac{\lambda}{4}$, in the second case $l = \frac{3\lambda}{4}$, and in the third case $l = \frac{5\lambda}{4}$,

i.e., $l = (2m-1)\frac{\lambda}{4}$ where m is any integer (1, 2, 3, etc.) and so $(2m-1)$ is an odd number.

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*A tone is the simplest constituent of a sound and is incapable of analysis.

$$\lambda=\frac{4l}{(2m-1)}.$$

But $$v=n\lambda \text{ or } n=\frac{v}{\lambda}.$$

$$\therefore \quad n=\frac{v(2m-1)}{4l}. \qquad \ldots \qquad \ldots \qquad (1)$$

This represents a series of tones whose fundamental is of pitch $\frac{v}{4l}$, and the overtones are of pitches 3, 5 and 7 times of the fundamental. Thus the fundamental and overtones of a closed pipe have frequencies proportional to the odd natural numbers 1 : 3 : 5 etc.

154. Overtones of an Open Pipe.—The fundamental mode of vibration may be diagrammatically represented by Fig. 116 (*a*). The other modes of vibration are shown in Fig. 116 (*b*) and (*c*). After every node there occurs an antinode and *vice versa*. The extreme ends must be antinodes in each case.

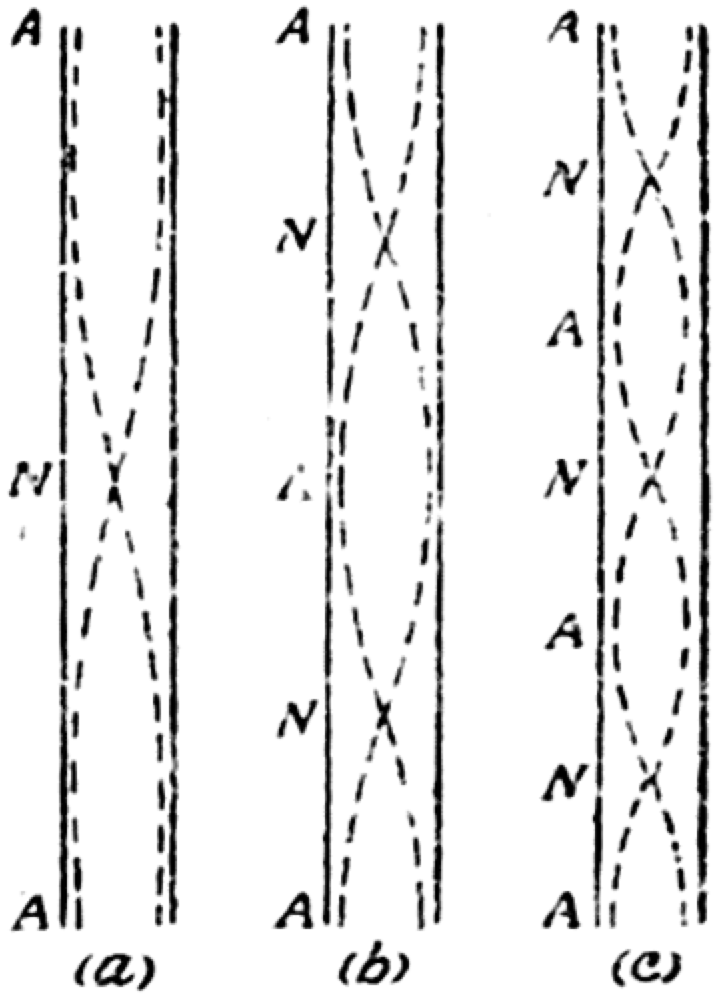


Fig. 116.

In mode (*a*), the length of the pipe, $l=2\frac{\lambda}{4}$; while in mode (*b*) $l=\frac{4\lambda}{4}$ and in mode (*c*) $l=\frac{6\lambda}{4}$.

i.e., $l=\frac{2\lambda}{4}, \frac{4\lambda}{4}, \frac{6\lambda}{4}$ etc.

$=2m\frac{\lambda}{4}$, where m is an integer (1, 2, 3, etc.)

or $$l=\frac{m\lambda}{2} \text{ or } \lambda=\frac{2l}{m}.$$

But $v=n\lambda$ $$\therefore \quad n=\frac{v}{\lambda}=\frac{v}{\frac{2l}{m}}=\frac{mv}{2l} \qquad \ldots \qquad (2)$$

This represents a series of tones whose fundamental has frequency $\frac{v}{2l}$, and the overtones have frequencies 2, 3, 4 times etc. of the fundamental. Thus the fundamental and overtones of an open pipe have frequencies proportional to the natural numbers 1 : 2 : 3 : 4 : etc.

From the above discussion we find that the frequency of the fundamental tone of a closed pipe is $\frac{v}{4l}$, and that of an open pipe of the same length is $\frac{v}{2l}$ *i.e.*, the tone emitted by an open pipe is an octave higher than that emitted by a closed one of the same length. Thus to obtain with a closed pipe the same fundamental tone as that given by an open one, the closed pipe must be half in length to that of the open one.

Example. *A closed pipe 4 ft. long full of a certain gas resounds to a given tuning-fork. If an open pipe resounding to the same fork and containing air be 5 ft. long, what would be the velocity of sound in the gas, taking velocity in air to be 1120 ft./sec.*

Taking into consideration their fundamental notes and denoting the wave length and velocity in gas by λ_g and v_g and in air by λ_a and v_a respectively we have

for the closed pipe, $l=\frac{\lambda_g}{4}=4$ ft. or $\lambda_g=16$ ft.

$$\therefore\ n=\frac{v_g}{\lambda_g}=\frac{v_g}{16} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (i)$$

for the open pipe $l'=\frac{\lambda_a}{2}=5$ ft. or $\lambda_a=10$ ft.

$$\therefore\ n'=\frac{v_a}{\lambda_a}=\frac{1120}{10}=112 \qquad \ldots \qquad \ldots \qquad (ii)$$

Since the pipes resound to the same fork we have $n=n'$, therefore from (i) and (ii)

$$\frac{v_g}{16}=112$$

or the required velocity$=16\times112=1792$ ft./sec.

155. Stationary vibrations in a closed end pipe.—Let us now see how stationary vibrations are set up in a flue pipe. We may regard the sheet of air at the mouth as sending a train of waves along the pipe. As explained in Art. 87 a compression incident at the closed end returns as a compression. The closed end must be situated at a node, and the particles must be at rest. Graphically the amplitude of the reflected wave at the closed end should be represented by a negative ordinate to nullify the displacement effect of the incident wave. Accordingly the thin and dotted curves in Fig. 117 are the *associated* displacement curves of the incident and the reflected waves respectively, while the thick continuous curve gives the displacement of the resultant wave The first stage gives the state of affairs at any instant $t=0$. The next stage represents the condition quarter period later. To get the incident and reflected waves in this case, we merely advance the thin curve by $\frac{\lambda}{4}$ towards the right and the dotted curve by $\frac{\lambda}{4}$ towards the left. Similarly the remaining stages after every quarter period are obtained. We see that the interaction between the direct and the reflected waves gives rise to stationary undulations in which N_1, N_2, N_3, etc. represent the *nodes* and A_1, A_2, A_3, etc., are the *antinodes*. The two sets of points are $\frac{\lambda}{2}$ apart, antinodes being situated midway between two nodes and *vice versa*. For properties of stationary waves see Chapter VII.

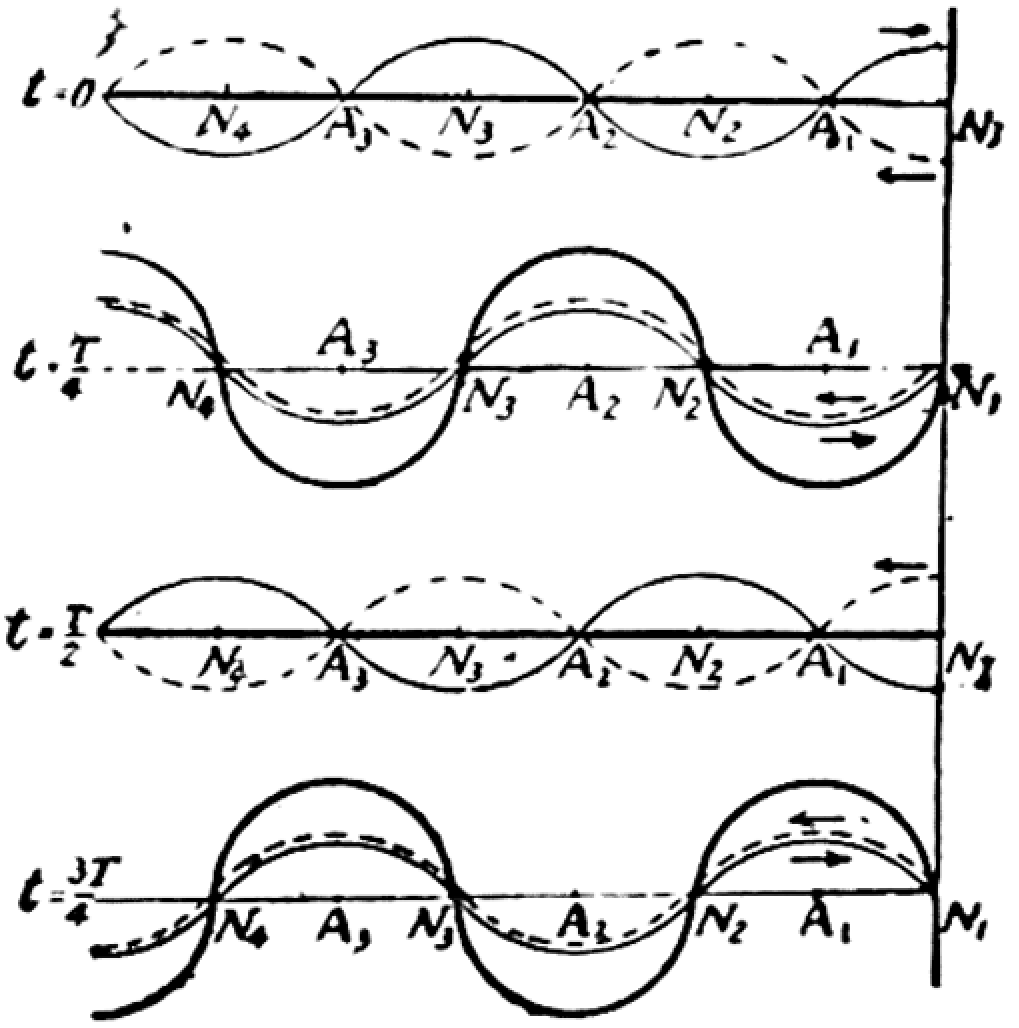


Fig. 117.

[*Note.*—Upward ordinates represent displacements to the right and *vice versa*]

Mathematical Treatment.—Let $y_1 = a \sin \frac{2\pi}{\lambda}(vt - x)$ be the equation of the *incident wave.* When reflection takes place, the closed end is to remain a node, so the amplitude represented by a in the incident wave will be represented by $-a$ in the reflected wave.

Hence $y_2 = -a \sin \frac{2\pi}{\lambda}(vt + x)$ is the equation of the *reflected wave.*

The displacement of the resultant wave is given by

$$y = a \sin \frac{2\pi}{\lambda}(vt - x) - a \sin \frac{2\pi}{\lambda}(vt + x)$$

$$= -2a \sin \frac{2\pi x}{\lambda} . \cos \frac{2\pi vt}{\lambda}$$

$$\left[\because \sin A - \sin B = 2 \sin \frac{A-B}{2} \cos \frac{A+B}{2}\right]$$

$$= A \cos \frac{2\pi vt}{\lambda}, \text{ where } A = -2a \sin \frac{2\pi x}{\lambda}.$$

That shows that each point of the wave executes S.H.M. of amplitude A, which varies from point to point along the wave,

when $t = 0$, $\cos \frac{2\pi vt}{\lambda} = 1$,

and $A = -2a \sin \frac{2\pi x}{\lambda}$ represents the equation giving the shape of the wave at that time.

At points $x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}$ etc. $A =$ zero, that is, there is no displacement. Such points called nodes are marked N_1, N_2, N_3, etc. in the figure.

while at points $x = \frac{\lambda}{4}, \frac{3\lambda}{4}$, etc. $A = 2a$ or $-2a$.

These points are antinodes and are shown in the figure by A_1, A_2, A_3, etc.

156. Stationary Vibrations in an open end Pipe.—We saw in Art. 87 that at the open end a compression returns as a rarefaction. The open end acts as an antinode *i.e.*, there is maximum displacement of air particles there. To obtain this condition graphically the amplitude of the reflected wave corresponding to the continuous thin line incident wave must be represented by the dotted line as shown in the various stages. The thick line curve will, as before, represent the resultant displacement curve. The stages representing the condition of the medium after every quarter period are obtained in the same manner as in the preceding section. Carefully note the positions of the *nodes* and *antinodes* and compare them with those obtained in Fig. 117. (Reflection at the closed end). Consecutive nodes or antinodes are $\frac{\lambda}{2}$ apart, but the first node N_1 is quarter wavelength from the end.

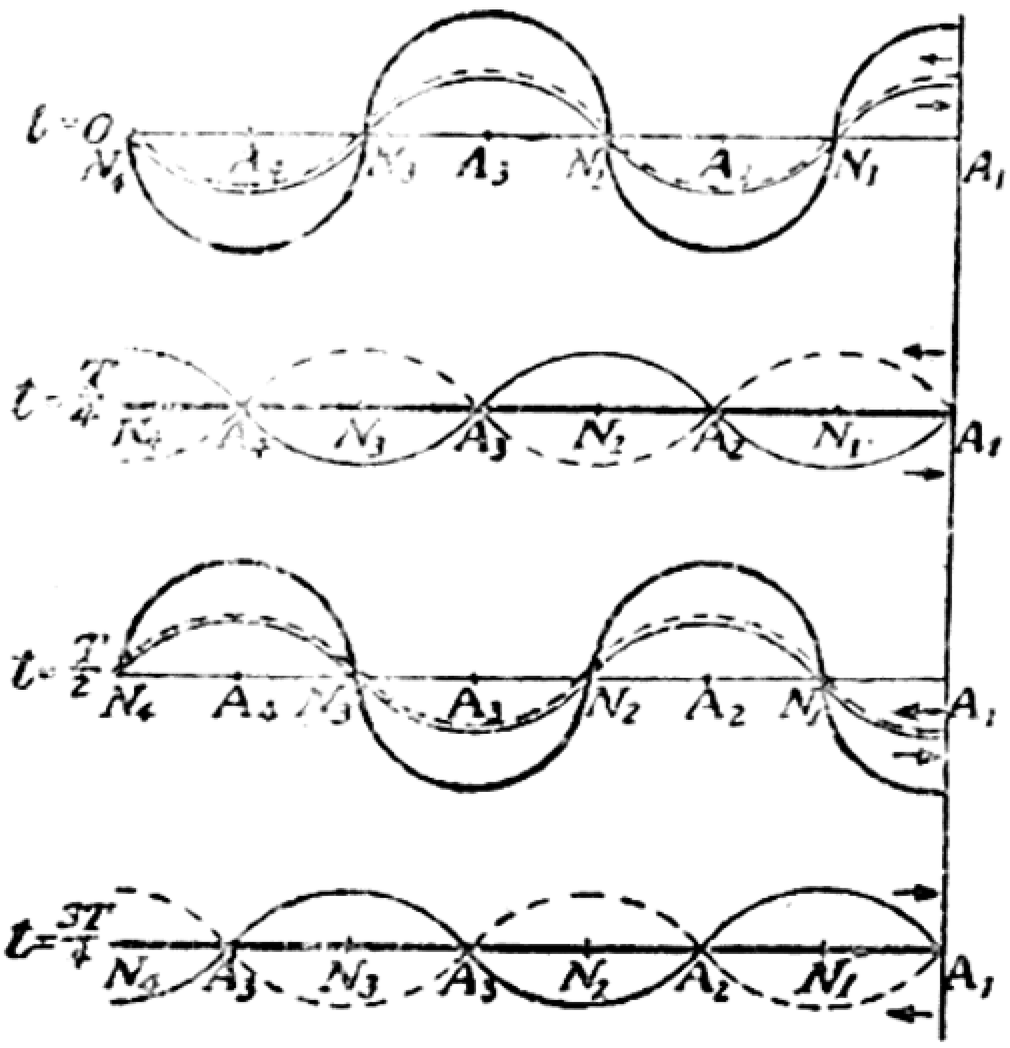

$t=0$
N_4 A_4 N_3 A_3 N_2 A_2 N_1 A_1
$t=\frac{T}{4}$
N_4 A_4 N_3 A_3 N_2 A_2 N_1 A_1
$t=\frac{T}{2}$
N_4 A_4 N_3 A_3 N_2 A_2 N_1 A_1
$t=\frac{3T}{4}$
N_4 A_3 N_3 A_3 N_2 A_2 N_1 A_1

Fig. 118.

Note.—Upward ordinates represent displacement to the right and *vice versa*.

Mathematical Treatment.—As before,

$y_1=a\sin\frac{2\pi}{\lambda}(vt-x)$ is the direct (incident) wave.

From the condition of reflection at the open end it is clear that

$y_2=a\sin\frac{2\pi}{\lambda}(vt+x)$ is the equation of reflected wave, in order that the free end may be situated at an antinode.

The displacement of the standing waves formed by the superposition of the two is given by

$$y=a\sin\frac{2\pi}{\lambda}(vt-x)+a\sin\frac{2\pi}{\lambda}(vt+x).$$

$$=2a\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi vt}{\lambda}=A\sin\frac{2\pi vt}{\lambda},$$

where $$A=2a\cos\frac{2\pi x}{\lambda}$$

This shows that each point executes simple harmonic motion of amplitude A, which varies from point to point along the wave.

$A=2a\cos\frac{2\pi x}{\lambda}$, represents the equation giving the shape of the wave at a particular time.

At points $x=0, \frac{\lambda}{2}, \lambda$, etc. $A=2a$ or $-2a$.

These points are antinodes and are shown by A_1, A_2, A_3 etc., in the figure, while at points $x=\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$ etc, $A=$zero. *i.e.*, there is no displacement, Such points, called nodes, are marked N_1, N_2, N_3 etc. in the figure : It is also clear from above that the first node N_1 is a quarter wavelength from the free end.

157. Reflecton of Sound Waves at the Open End more in details.—When reflection take place at the fixed end of a string or the closed end of a pipe, it is complete *i.e.*, the reflected wave is of the same intensity as the incident wave and so the amplitudes of both the incident and the reflected waves are denoted by equal ordinates.

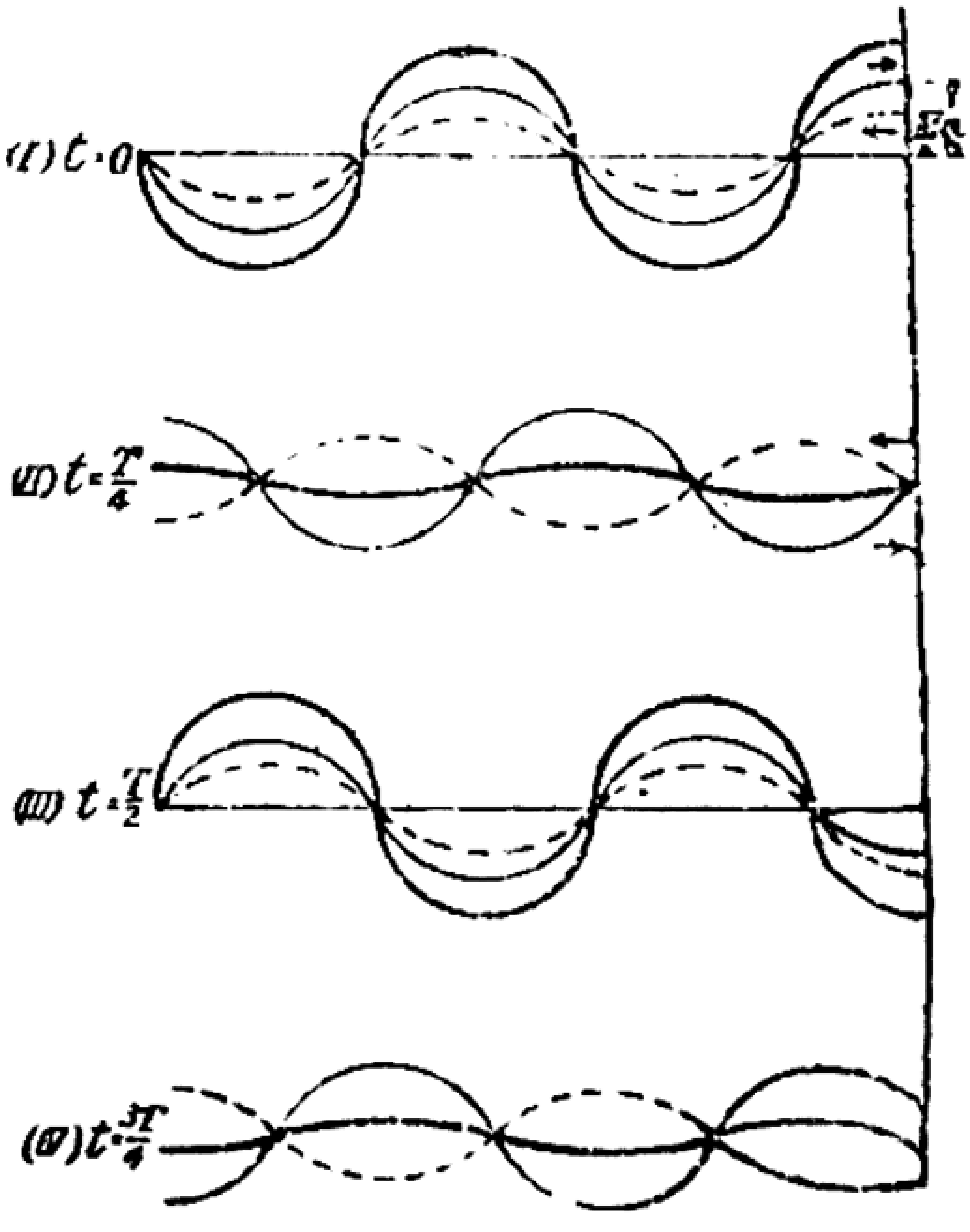


Fig. 119.

But when a wave arrives at an open end, it meets a discontinuity and so the reflected wave is not of the same intensity as the incident wave and hence the interference within the tube is not complete. Here the amplitude of the reflected wave is represented by an ordinate $a' < a$, the ordinate representing the amplitude of the incident wave, and hence its intensity being proportional to the square of the amplitude is also less. Thus, within the tube, there is a progressive wave travelling down it and superimposed upon the steady vibration. This progressive wave comes out of the open end. In other words the energy of the incident wave is partly reflected and partly transmitted. Stages (I) to (IV) o Fig. 119 are drawn in the same manner as in the preceding section. The incident wave is denoted by the thin continuous line, the reflected wave of less ordinate by the dotted line and the compounded wave by thick line. It may be noted that the resultant displacements in stages (II) and (IV) are no longer represented by a straight line as before when 'a'' was assumed equal to 'a'.

Analytically we proceed as follows :—

As before $y_1 = a \sin \frac{2\pi}{\lambda}(vt - x)$ ... (1) (incident wave).

$y_2 = a' \sin \frac{2\pi}{\lambda}(vt + x)$ ... (2) (reflected wave).

where $a' < a$.

Equation (1) may be put as

$$y_1 = (a - a') \sin \frac{2\pi}{\lambda}(vt - x) + a' \sin \frac{2\pi}{\lambda}(vt - x) \qquad \dots (3)$$

Combining (2) and (3), we have

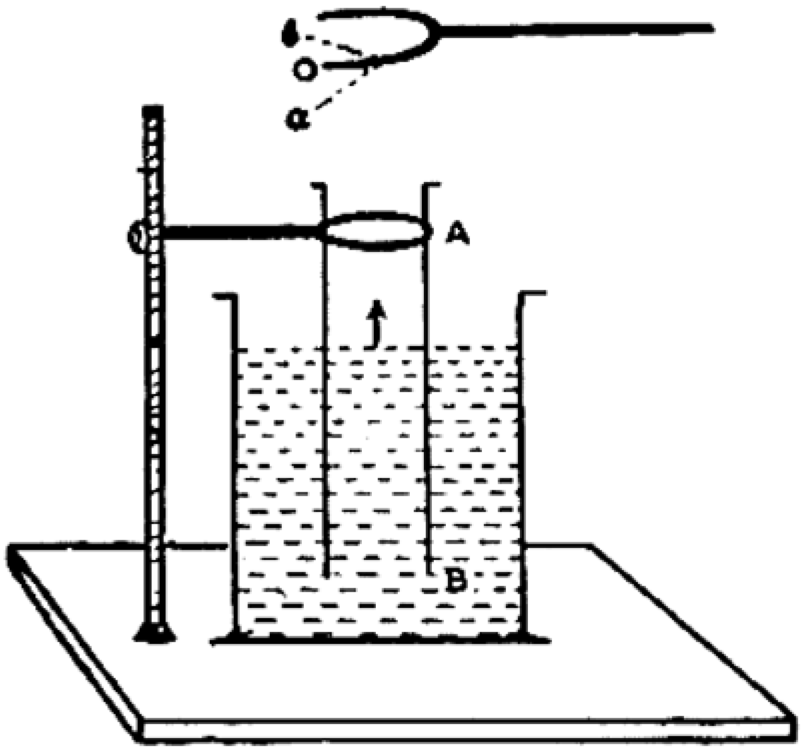
$$y = (a - a') \sin \frac{2\pi}{\lambda}(vt - x) + a' \sin \frac{2\pi}{\lambda}(vt - x) + a' \sin \frac{2\pi}{\lambda}(vt + x)$$

$$= (a - a') \sin \frac{2\pi}{\lambda}(vt - x) + a' \left[\sin \frac{2\pi}{\lambda}(vt - x) + \sin \frac{2\pi}{\lambda}(vt + x) \right]$$

$$= (a - a') \sin \frac{2\pi}{\lambda}(vt - x) + 2a' \cos \frac{2\pi}{\lambda} x \sin \frac{2\pi}{\lambda} vt,$$

i.e., y = a progressive wave that emerges
+ a stationary or steady vibration.

158. Resonance in Air Column.—Clamp a glass tubing AB open at both ends as shown in Fig. 120. End B dips into water contained in a glass jar. This enables the length of the air column to be adjusted. Now when a vibrating tuning



Fig, 120.

fork is held near the open end *A* of the tube, the prong of the fork gives the air column, a sequence of taps causing in it stationary vibrations, provided the length of the tube is such that the frequency of the taps coincides with the natural frequency of vibration of air column. It may be suggested here that these vibrations closely resemble those of a spiral of wire fixed at the upper end, when properly timed taps are given to the weight attached to its lower end. To do this experimentally, loosen the clamp and holding the tuning fork over the open end, raise or lower the tube till a reinforcement of sound takes place *i.e.*, **resonance** occurs. Fix the clamp, and measure the length of air column Now the period of air column is the same as that of the fork. The air at the open end must be at an 'antinode', and that at the closed end at a 'node'. The distance between a node and the nearest antinode is $\frac{\lambda}{4}$, hence the length of the wave *i. e.*, λ is equal to four times the length of the air column ($\lambda = 4l$).

Let us see how the resonance is brought about.

Suppose the lower prong of the fork vibrates between limits *b* and *a*. Starting with the prong at *b* about to move dowwards. a wave of compression is sent down the tube. When the prong arrives at O its mean position, the compressional wave reflects from the surface of water (denser medium) as a wave of compression and just reaches the open end when the prong comes to *a*. Now the compressional wave on reflection from the open end (rarer medium) goes downwards as a wave of rarefaction while the prong also tends to go from *a* to *b* giving rise to another wave of rarefaction going downwards. *These two reinforce each other* and after suffering reflection at the closed end come up to *A*, just when the prong reaches *b*. This combined wave of rarefaction on reflection at the open end travels downwards as a wave of compression which is further amplified by another wave of compression set up by the movement of the prong from b to a. Thus the air column is thrown into a state of vigorous vibration. With this the amplitude of the sound wave becomes appreciable, and we get *Resonance.*

It is seen from above that when the prong makes a half vibration *i. e.*, from b to a, the wave travels twice the length of the air column (first downwards and then upwards) Thus when the wave travels four times the length of the air column, the prong makes one complete vibration *i. e.*, wave length of the note is equal to four times the length of the air column.

159. Correction for the Open End and its Determination.- It may be further mentioned that there is one more point of difference between reflection at the closed and the open ends of pipes. Whereas, in the former case the reflection takes place just at the end, in the latter the circumstances are slightly different. At the open end the reflection is due to the openness of the outside medium, and so particles are free to move in all possible directions. Just at the open end the air particles are not perfectly free and therefore it is at a little distance farther from the end that complete reflection is caused. The distance from the end of the pipe to the point of complete reflection is called '*End correction.*' Open pipes are of two kinds, flanged or unflanged. In the former case there is more restriction as to the freedom of motion of all particles to cause complete reflection, hence the end correction is a little greater than that in an unflanged pipe. The unflanged pipe acts as a resonator whose mouth is enlarged, and consequently the pitch of a pipe is higher without the flange than with it. According to Rayleigh's determination the correction for an unflanged pipe is $\cdot 62R$ where R is the radius of the pipe. For a flanged end the correction is $\cdot 82R$. Blaikley determined the correction for an *Unflanged Pipe* to be $\cdot 58R$ as follows:—

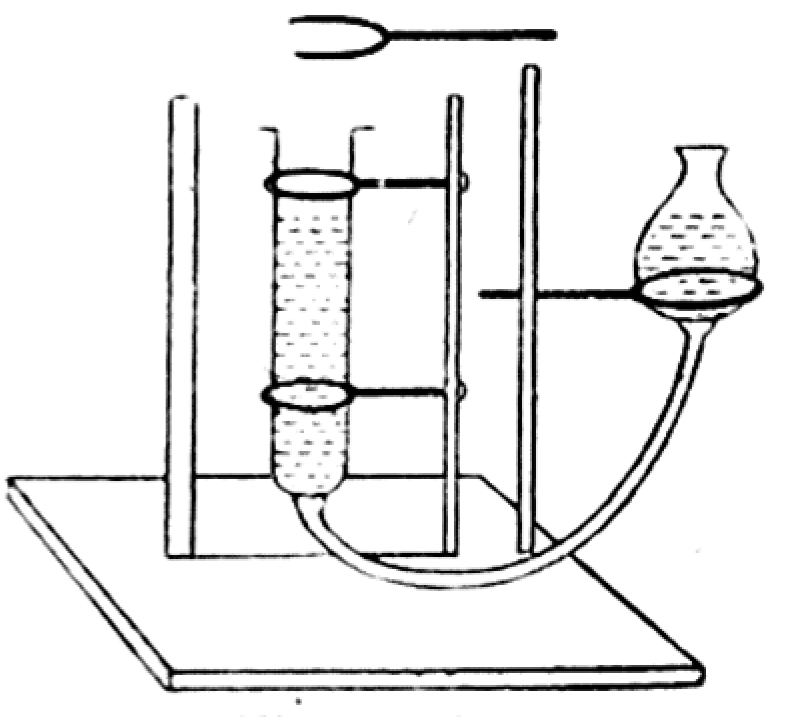


Fig. 120. (a)

A wide glass tube open at both ends is mounted vertically. The lower end which is drawn out into a narrow tube is connected by an India-rubber tube to a reservoir which can be slided along a vertical stand. A metre rod is fixed to the upright stand to measure the length of the resonance column, [Fig. 120 (*a*).]

To begin with, water is poured into the resevoir so that it fills the rubber tube and a part of the glass tube. The reservoir is raised till water level inside the glass tube is near its mouth, and a tuning-fork of known frequency is made to vibrate near it. The water level is gradually lowered till resonance occurs, the water surface forming the closed end of the pipe. Let the observation be repeated three times and the mean length of the resonance column be l_1. Water level is lowered still further (fork vibrating) till a second resonance position is obtained and let the mean length be l_2. See Fig. 120 (*b*) Thus the tube gives maximum resonance when its length with the end correction added to it is $\frac{\lambda}{4}$, $\frac{3\lambda}{4}$, $\frac{5\lambda}{4}$ etc.

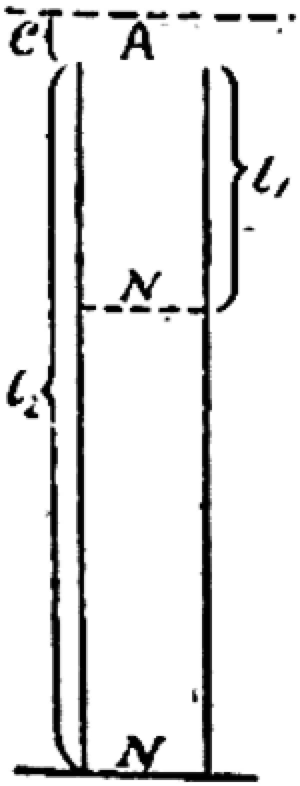


Fig. 120 (*b*).

For the first resonance position, $l_1+c=\frac{\lambda}{4}$ ... (1)

For the second resonance position. $l_2+c=\frac{3\lambda}{4}$ ... (2)

Multiplying equation (1) by 3, we get $3l_1+3c=\frac{3\lambda}{4}$ ... (3)

Equating (2) and (3), gives

$$3l_1+3c=l_2+c$$

or

$$2c=l_2-3l_1$$

$$\therefore\quad c=\frac{l_2-3l_1}{2}.$$

160. Effect of End Correction on the Overtones and the Quality of the Note :—When the correction is applied to a closed organ pipe, its length alters to $l+c$. Equation (1) Art. 153 would become

$$n=\frac{v(2m-1)}{4(l+c)}$$

For an open pipe the length changes to $l+2c$, since both ends are open,

$\therefore$ Equation (2) Art. 154 becomes

$$n=\frac{mv}{2(l+2c)}$$

This would mean that the frequencies of the fundamentals and their overtones in both cases are slightly lowered; but their relative pitches remain unaffected.

Frequencies of the fundamentals of the closed and open pipes with end corrections taken into consideration will thus become

$$n_1=\frac{v}{4(l+c)} \text{ and } n_1'=\frac{v}{2(l+2c)}$$

$$\therefore \quad \frac{n_1'}{n_1}=\frac{2(l+c)}{(l+2c)}$$

i.e., n_1' is not exactly twice of n_1 as already deduced without the correction.

The corrections explained above are true in the case of a *Narrow Cylindrical* pipe *i.e.*, when the radius of the pipe is small as compared with the wavelength. If the pipe is *wide*, the correction not only depends upon its width but varies with the wavelength as well. In case of a narrow pipe, the pipe would be merely regarded as longer by an amount c, and so the *overtones* in such pipes will be a *little flatter* than what they would be if no correction were needed. While in the case of pipes of wide bore, overtones are all sharper than the harmonics of the fundamental, and this divergence increases with the overtones of higher order. In case of narrow tubes this effect is less marked and lower proper tones (overtones) fairly agree with the harmonic series (1, 2, 3, etc.).

'**End Correction**' also affects the *quality* of the note. A narrow tube when blown strongly gives *proper tones* which agree closely with the *harmonic series* and therefore with the vibrations of the sheet of air blowing across its mouth as they can also be expressed by the harmonic series. As the energy for maintaining the vibrations in a pipe is derived from the air sheet, in this case the resonance will extend to a considerable number of the lower members of its modes of vibration, and the note will contain many harmonics. The nearer the proper tones are to the harmonics, the more the resonance effect, and the greater will be the intensity of the corresponding harmonic constituents of the note. In case of wide pipes proper tones differ sufficiently from the harmonic series, so the resonance effect is less. Such pipes give notes containing few harmonic constituents with the result that they are *smooth* and *dull.* On the other hand a narrow pipe is rich in harmonic constituents and the sound emitted is full of *volume* and *sweetness*.

161. Velocity of Sound in Air by Resonance.—The result $\lambda=4l$, where l is the length of air column enables us to determine the velocity of sound in air, for $v=n\lambda=4nl$. Knowing l from the resonance experiment and n from the frequency of tuning fork, we can find v.

The formula $v=4nl$ is not accurate as the correction for the open end is not taken into account. If the first resonance length is to be considered then tuning length $=l_1+c=l_1+\cdot58R$.

and
$$\frac{\lambda}{4}=(l_1+\cdot58R),\ i.e.,\ \lambda=4\ (l_1+\cdot58R)$$

$$\therefore\ v=n\lambda=4n\ (l_1+\cdot58R).$$

R is the internal radius of the tube being half of the internal diameter measured by an inside vernier calipers.

The 'end correction' can be eliminated if two positions of resonance are taken as explained in Art. 159.

Subtracting equation (1) from (2) on page 225, we get

$$l_2-l_1=\frac{\lambda}{2},\ \text{or}\ \lambda=2(l_2-l_1)$$

$\therefore\ v=n\lambda=2n(l_2-l_1)$. This value is then to be corrected for temperature and humidity to find the velocity at 0°C.

162. Methods of Tuning Flue Pipes.—The following are the several methods of tuning the open pipes :—

(1) A hole is made in the side of the pipe near the open end. This is covered with a flap. By pulling the flap the length of the pipe is virtually shortened and the pitch is raised.

(2) Upper part of the tube is made to slide telescopically over the lower part, so that the length of the pipe can be increased or decreased at will.

(3) Many pipes have a number of cylindrical tubes of soft metal, which are tuned with the size of the opening at the top of the pipe. By putting a hollow cone at the end, the opening is slightly closed thus lowering the pitch. If a solid cone is pressed, the opening is enlarged and raises the pitch. The pipe here acts as a resonator, with enlarged mouth.

(4) Two flaps are used on each side of the mouth of the pipe. By bending them inwards, the size of the mouth diminishes, and the pitch is accordingly lowered. The opposite effect is obtained by bending them outwards against each other.

In the case of closed pipes, the upper end is tightly closed by a movable fitting plug. To alter the pitch for tuning, the length is varied by pulling it out or pushing it in.

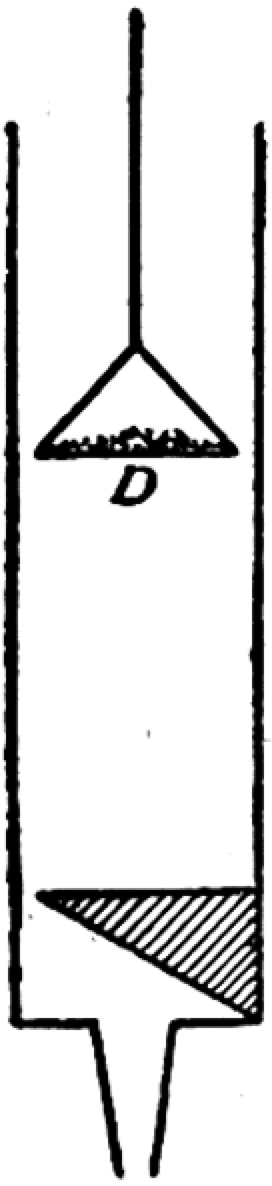


Fig. 121.

163. Experiments to Demonstrate Nodes and Antinodes in Organ Pipes.—(*i*) An open pipe with one side made of glass is held in a vertical position as shown in Fig. 121. A small paper drum *D* with some fine dry sand sprinkled on it is lowered into the pipe by means of a thread. When the pipe is sounded, stationary vibrations are set up into it with nodes and antinodes at fixed positions. When the drum is at an antinode, the sand particles dance about on account of the motion of air, whilst at nodes no movement of the sand is noticed. For the fundamental tone, a node in the middle and antinodes at the ends are located. When the pipe is blown a bit stronger to produce its first overtone, two nodes at one quarter of the length of the pipe from either end (which is an antinode), with an additional antinode in the middle are observed.

(*ii*) **Koenig's Manometric Capsule** [Fig. 122 (*a*)] is also used for finding the position of nodes and antinodes. It consists of a small chamber divided into two compartments by a thin diaphragm *D* made of Indian-rubber. Coal gas enters one of the compartments by the pipe *A*, and leaves by the jet *J*, where it burns into a tall thin flame. Attached to the other compartment is a short tube *T* to be inserted in the wall of the pipe. When the tube *T* is near a node, the variation in pressure drives the membrane in and out, and the flame, in turn will jump up and down in tune with the vibration of the membrane. The fluctuations in the height of the flame being very rapid are not generally seen with the naked eye and to do so the image of the burning jet is viewed in a revolving mirror, which is a hollow box having its four side faces made of plane mirrors and mounted so as to be capable of rotation about a vertical axis passing through its centre. When the tube is at an antinode the flame is at rest, and the image produced by the rotating mirror on a wall or a screen is a continuous band. When it is at a node, the flame dances and the upper edge of the band shows a series of teeth.

This capsule can also be used to determine the character of the musical note as well. Note that with the increase in the frequency of the note, the upper edge of the band will show a larger number of teeth.

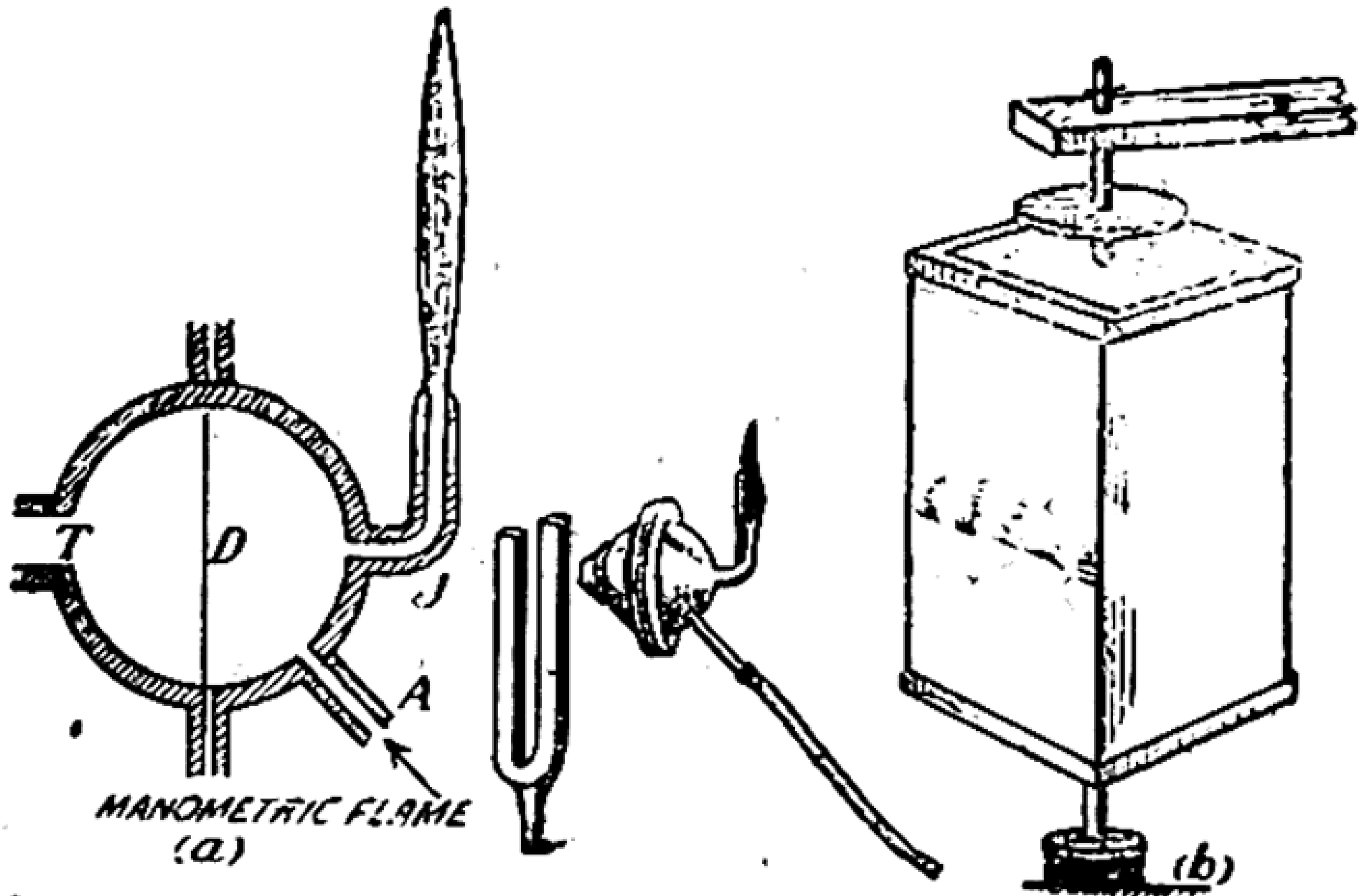


Fig. 122. Revolving mirror for viewing manometric flame.

164. Reed Pipes—Here the vibration is caused by a tongue of metal called the reed, which replaces the sheet of air at the bevelled surface of a flue pipe. When air is blown into the box *B* through the tube *A*, the tongue *R* vibrates before an opening *O*, and sends a series of puffs through it. If the tongue is a little larger than the opening, it beats at each vibration against it and so called a *beating* or *striking* reed. The beating reed is curved outwards, so that the closing is not sudden due to air pressure but is effected gradually from the fixed to its free end. This avoids the harsh tones produced by it. If the tongue is slightly smaller than the opening, it is called a *free* reed, which does not cut off the full supply of air as is done by a beating reed. In order to improve the quality of a note, a cylindrical pipe (as in a clarinet) or a conical pipe (as in hautboy or bassoon) is used. Beating reed is generally associated with such a pipe which really mellows down the harsh tone produced without it. If pipes are to be dispensed with, free reeds (as in a harmonium) may be adopted

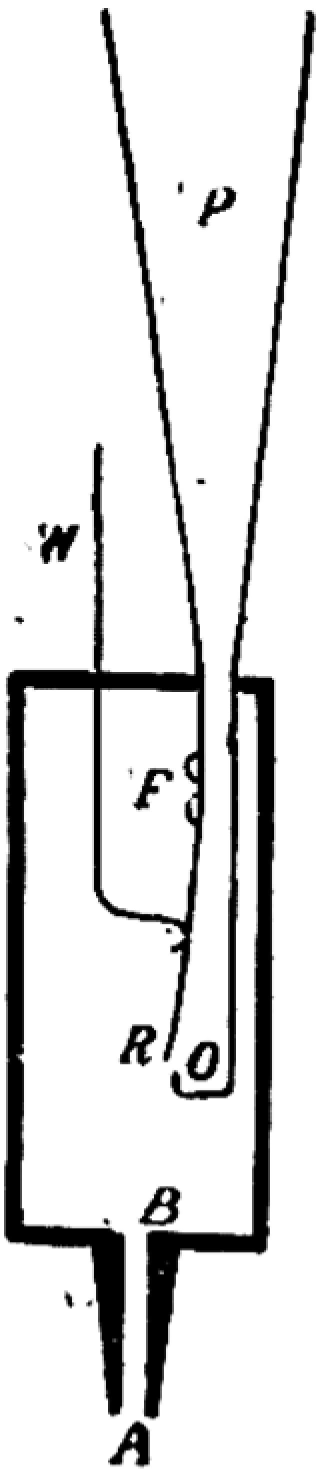


Fig. 123.

for the purpose. But presence of higher harmonics to a greater or less degree is the main cause of the characteristic penetrating quality of reed pipes.

165. Action of the Reed.—On blowing a reed pipe, air passing the sides of the reed into the opening, starts a condensation in the pipe which reflects back as a rarefaction at the top free end. In the meantime the reed closes the opening and the coming rarefaction again has to go back as a rarefaction which returns from the free end at the reed as a condensation. This causes the reed to open and allows more air to enter, thus further increasing the compression. So the process is repeated. Thus a pulse of condensation, to complete its full vibration, has to travel four times the length of the pipe, *i.e.*, the wavelength is four times the length of the pipe. So a reed pipe is to be regarded as closed at the reed end unlike the mouth of a flue pipe which is an open end. In short in spite of the fact that air and energy enter the pipe at the reed end, it acts as it were a node.

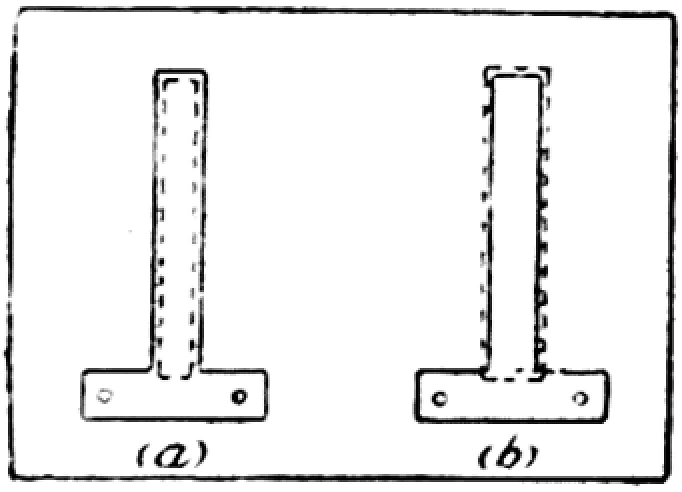


Fig. 124.

166. Effect of the Reed on the Pitch of the Pipe.—If the reed is very flexible, the length of the pipe really controls the pitch of the note produced. On the other hand, if it be stiff and stout, it would force the vibrations of the pipe and the note produced would be mainly due to its natural frequency. In case of metal reeds generally used in organ pipes, there is a reciprocal action, the reed forcing the vibrations of the pipe and the pipe trying to change the natural period of the reed. Hence we must have the reed and the pipe of nearly the same pitch so as to improve the quality and enable the pipe to speak just when blown into. A *cylindrical reed pipe* (clarinet) will only give the *odd* harmonics, but by making the tube *conical* (hautboy and bassoon), this would be avoided, and proper tones in the ratio 1 : 2 . 3 : 4 etc. could be produced.

167. How to tune a Reed Pipe.—A *beating* reed is tuned with the help of the wire *w* shown in the figure, held against one side of the reed near its root. By pulling it up, the length of the vibrating reed is increased and the pitch lowered and *vice versa*. In case of a *free* reed, tuning is effected by a wire clip grasping it on both sides near its root. The less the vibrating part left free to vibrate, the sharper the pitch. In case no such arrangement exists and we want to tune a particular reed once for all (as in the case of harmonium reeds), we raise the pitch by scraping at the tip of the reed. To flatten it, it is scraped at its base.

168. Effect of Change in Temperature on the Pitch of the Pipes.—A change in temperature affects a *flue* pipe in two ways. If the temperature rises, the pipe gets a little longer due to expansion and also the density of air within it is diminished. The alteration in the size of the pipe is so slight that it produces an inappreciable effect on wavelength and hence on the pitch, while the density of air is sufficiently diminished to alter the pitch by an appreciable amount. We have seen *vide* Art. 62 (*iv*) that the velocity of sound $v \propto \sqrt{T}$, where T is the absolute temperature. But $v = n\lambda$, hence $n \propto \sqrt{T}$, λ remaining almost unchanged.

In case of a *reed* pipe, the rise in temperature softens the reed and so it becomes a bit flatter, but the pipe on account of the rise in velocity becomes sharper as shown above. The change in the pitch of the reed is less than that of the pipe and so on the whole with rise of temperature, the pitch of the reed pipe increases. Of course the change in pitch is less than that of a flue pipe. Thus it is only at one temperature that the different sections of the organ can be accurately in tune with each other.

CHAPTER XII

VIBRATIONS OF RODS, PLATES AND BELLS

169. We shall now give a brief account of a few miscellaneous cases of vibration which are of not much consequence in the theory of music *viz*,, the vibrations of rods, plates and bells.

A rod may be made to vibrate longitudinally or transversely, the exact mode of vibration will depend both on the stroking and on the manner in which the rod is clamped. The rod need not be stretched like a wire and it is the elasticity of the material, not the tension which restores the successive portions of the rod to their original relative positions.

169a. Longitudinal Vibrations of Rods :—

Case 1. *Free at both ends.*—The vibrations are quite similar to those of an air column in an open organ pipe. Such a rod is clamped in the middle and fundamental tone is produced by rubbing it lengthwise with a piece of wet flannel or resined cloth. The middle point acts as a node, and the two ends as antinodes. The wavelength is therefore twice the length of the rod. Waves of compression and rarefraction run along the rod. After reflection at the ends they produce stationary vibrations similar to that in a column of air. No end correction is needed here. Both even and odd harmonics may be produced.

To demonstrate the existence of longitudinal vibrations in a rod suspend a light pith ball pendulum in front of and in contact with one end of the rod. When the latter is stroked with the resined cloth, the ball will fly out because of the impacts with the rod.

For longitudinal waves in a rod the velocity is given by the formula $\sqrt{\frac{Y}{\rho}}$, where Y is Young's modulus, and ρ is the density of the material of the rod. Overtones can be produced by clamping it at suitable places along its length and rubbing the shorter section. Velocity of sound V in a rod$=\frac{\lambda}{T}$ where λ is the wavelength in the rod and T is the period of vibration of the rod. Since the period of vibration of the rod is the same as the period of the vibrations produced in the air, $V'=\frac{\lambda'}{T}$ where V' is the velocity and λ', the wavelength of sound in the air. Therefore $\frac{V}{V'}=\frac{\lambda}{\lambda'}$ *i.e.*, the velocity of sound in the rod and the air are proportional to their corresponding wavelengths. We shall use this relation in Kundt's apparatus to be described later on.

Case 2. *Fixed at one end.*—Vibrations are similar to those of air in a closed pipe. The fixed end is a node and the free end an antinode so that $\lambda=4l$. For the first overtone in which the additional node comes at one third l from the free end, we have $\lambda=4l/3$. To generalize only the odd harmonics are produced by a rod clamped at one end.

Case 3. *Fixed at both ends.*—To produce its fundamental, it is rubbed in the middle with a resined piece of leather. The fixed ends are nodes, and the middle point an antinode.

170. Transverse Vibrations of Rods.—Transverse vibrations are also possible in a rod. There are many ways in which a rod can vibrate transversely, the mode of vibration being different varying with the manner in which it is supported. Two important cases are worth considering firstly when one end is fixed and the other free, and secondly when it is supported at two intermediate points acting as nodes. The first case is termed as fixed-free vibration. Vibrations to some extent are similar to those of a closed pipe. The overtones are, however, not in the frequency ratio of 1 : 3 : 5 : etc., but are inharmonic and approximately in the

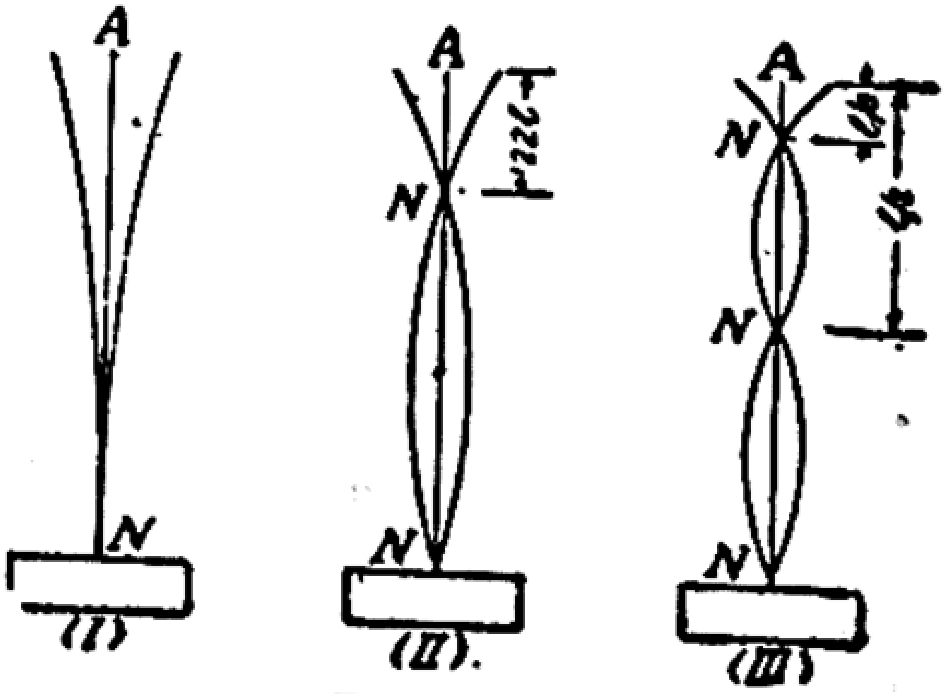


Fig. 125.

ratio 1 : $6\frac{1}{4}$: $17\frac{1}{2}$. These modes of vibration are shown in stages (I) (II) and (III) of Fig. 125.

In the second case the rod takes the form shown in Fig. 126 so as to produce its fundamental. The points of support will be positions of no vibration and the ends, those of maximum vibration. Between the supports there will be an antinode. Other modes may be produced by having additional supports between these two and will have frequencies different from those of harmonic series, for example, the first overtone is 6·5 times that of the fundamental.

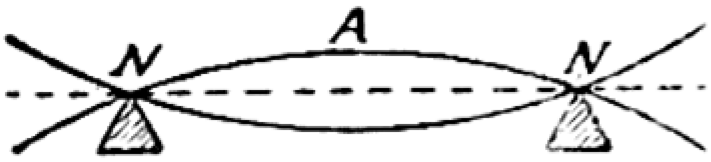


Fig. 126,

171. Method of tuning a Fixed-Free Rod.—If the free end is scraped off the mass and hence inertia is diminished and as practically no change is made in the elastic forces, the pitch is raised. If the rod is filed near the fixed end, elastic forces are weakened without any appreciable change in its inertia, and the pitch is therefore lowered. The free reeds of a harmonium are fixed free rods, and as described above they are tuned by scraping off the metal at the free end or at the base, according as it is desired to raise or lower the pitch.

172. Tuning-fork.—The tuning-fork is a very valuable instrument used in acoustical measurements. If handled properly its pitch remains constant for a number of years. Thus it is generally taken as a standard of pitch.

It may be regarded as two fixed-free rods on the same base. The free ends when struck gently execute transverse vibrations due to its inertia and rigidity. The note produced is practically a pure tone. The overtones if produced at all are very few and belong to inharmonic series which die away much more quickly than its fundamental. On mounting it on a resonance box, the tone is made purer. The box may have its overtones but as its fundamental agrees with that of the fork, it is strengthened. It is this purity which makes the fork a valuable instrument in the scientific study of sound, because sounds produced by all other instruments are wanting in this respect. The frequency of the fork can be accurately determined with the help of Helmholtz's vibration microscope. Tuning can be done by filing the prongs at the ends to raise its pitch; while scraping it near the base lowers the pitch. Electrically maintained tuning-forks give a sound of the same frequency for any length of time.

The tuning-fork is also regarded as a single free-free rod curved at its middle. In other words it is a bar bent into the form of U. Fig. 127 shows in three stages how a tuning fork may be built up from a straight bar. As the rod is bent gradually, the consecutive nodes on either side of the centre come nearer and nearer and the transverse motion of the portion between them becomes less and less. When the prongs vibrate outwards and inwards, the portion NN moves up and down. This motion is communicated to the top of the resonance box, and then to the air. The frequency of a fork varies directly the thickness of the prongs and the velocity of sound in the material of the fork, and inversely as the square of the length of the prongs,

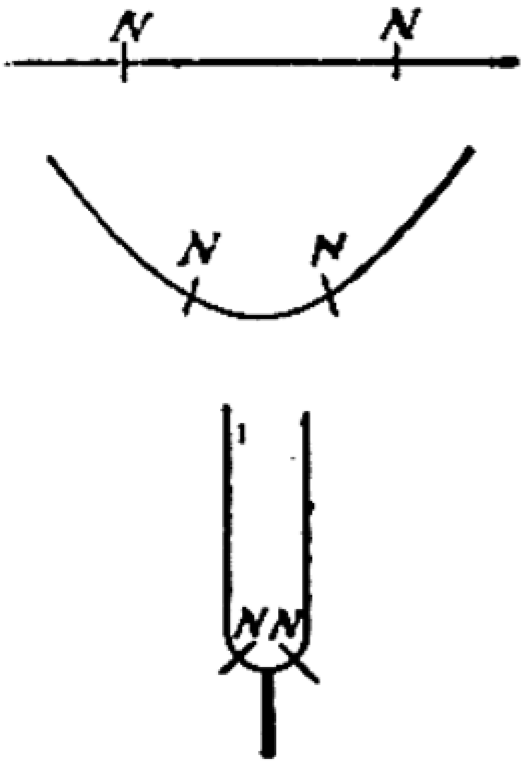


Fig. 127.

i.e., $N=\dfrac{m^2k}{2\pi l^2}\sqrt{\dfrac{Y}{\rho}}$, where $k=\dfrac{a\text{ (thickness)}}{\sqrt{12}}$,

l=length of the prongs, Y is Young's modulus, ρ the density of the material of the fork and m is a constant having different values for different tones. For the fundamental tone $m=1{\cdot}875$. There may be a very slight change in pitch when the temperature is varied. With rise in temperature, size of the prongs increases, and the rigidity of the material of the fork decreases. Both these factors tend to lower the pitch. This fall in pitch is about ·01% for 1°C. rise in temperature.

Now $$N=\frac{m^2k}{2\pi l^2}\sqrt{\frac{Y}{\rho}}.$$

Simplifying this relation dimensionally, we get $N\propto\sqrt{Yl}$,

In case of steel, coefficient of linear expansion$=+\dfrac{1}{10^5}$, while temperature coefficient of Young's modulus of steel

$$=-\frac{2}{10^4}$$

The latter has by far the greater effect, thus the frequency is slightly lowered. In case of steel it is expressed as

$$N_t = N_o(1 - 0{\cdot}00011t)$$

where N_o and N_t are the frequencies at 0° and t°C respectively and t represents the rise in temperature.

Nickel-steel (an alloy) has much smaller temperature coefficient and so tuning forks made of this alloy are being commonly used now-a-days.

173. Vibrations of Plates. Chladni's Figures.—The vibrations of plates are not of much practical importance but they afford very interesting illustrations of the phenomenon of **Permanent Interference**. The complete nature of their vibration can be found mathematically with great difficulty but they are rendered interesting by a simple experimental method devised by Chladni for showing the nodal lines in the various modes of vibration. Let us consider a rectangular plate which may be looked upon as a broad rod For example, the square plate *ABCD* (Fig. 128) may be considered as a rod of ledgth *AB* and width *BC*, or as a rod of length *AD* and width *AB*. In the first case, it may be considered to vibrate about *QS* as a nodal line, and in the second case about *PR* as a nodal line. If the plate is firmly clamped at its centre *O*, and bowed at one corner, *QS* and *PR* are nodal lines and the corners are points of greatest vibration. To observe them a little fine sand is evenly sprinkled over the plate. It is held at a point *R* at the centre of one side and bowed near the corner *C*, the sand is thrown off the parts of greatest vibration and gathers along the two diameters *PR* and *QS* parallel to the sides of the square. These diameters are nodal lines and the corners are the antinodal regions. Any two nearest corners on opposite sides of a nodal line are always in opposite phase *i. e.*, if 1 is coming up, 2 is going down, 4 is coming up and 3 is going down. The plates used for obtaining these figures may be square or round fixed at some point. Having fixed the plate at the centre as in Fig. (129), hold at *N* with the finger and the thumb, and bow it at *A*, different patterns will be obtained.

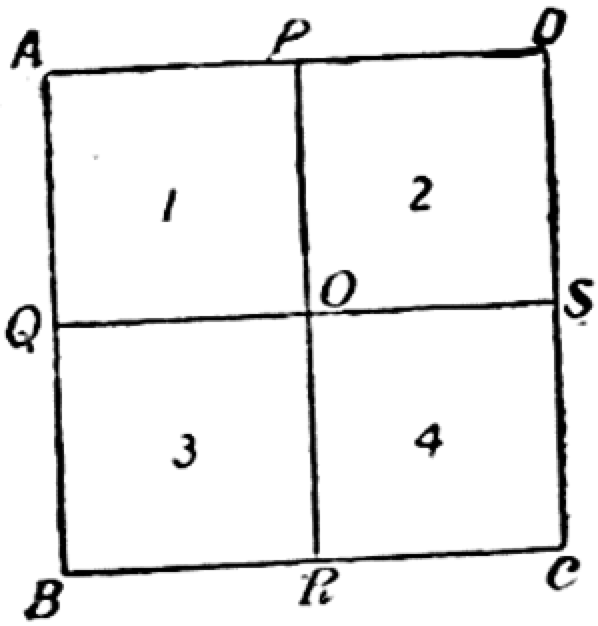


Fig. 128.

In the case of round plates fixed at the centre nodal lines are radial, dividing the plate into an even number of sectors.

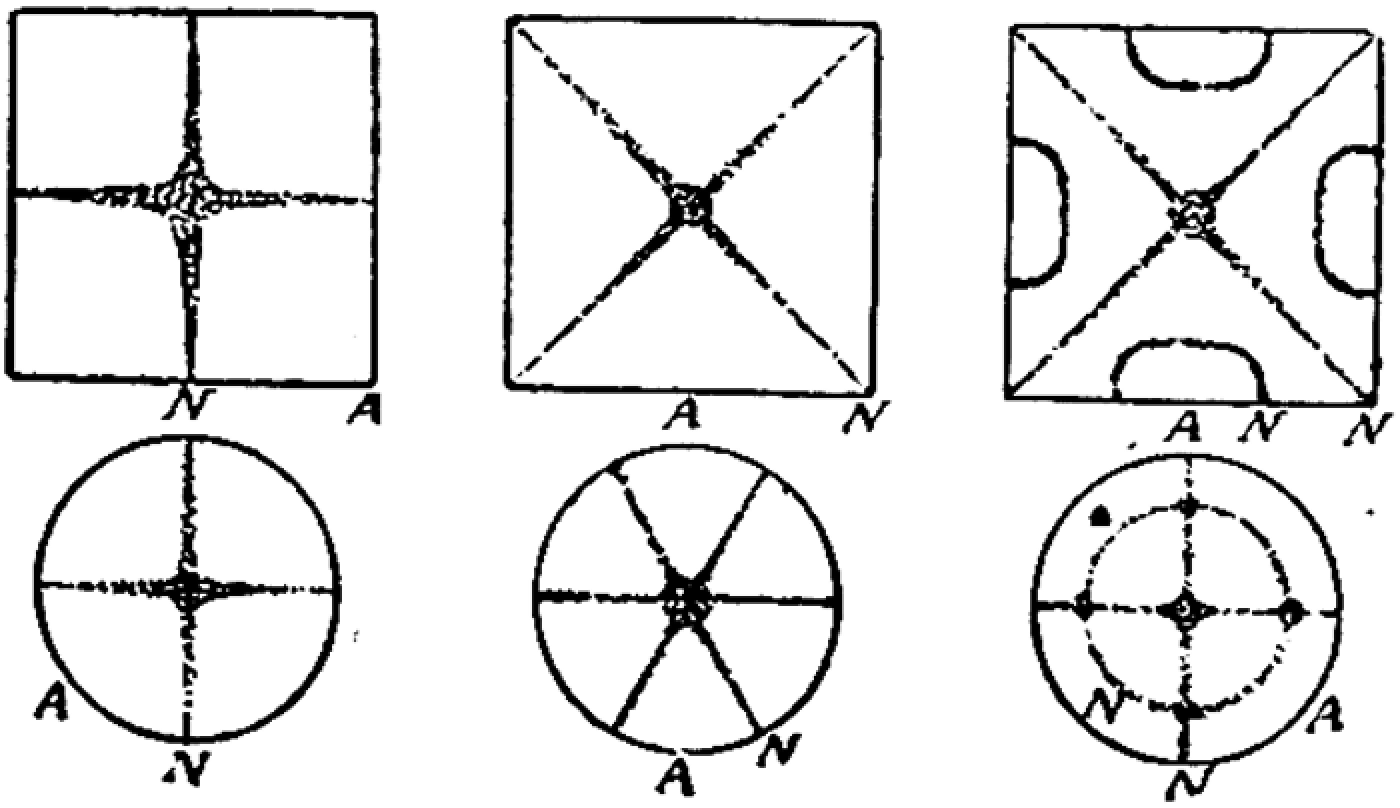


Fig. 129.

To get concentric nodal circles in them, the plate is not fixed at the centre but placed on three small wooden cones lying on a nodal circle, and then hammered gently at the centre.

174. Vibrations of Bells.—Bells are only round plates. In them nodal lines are of two classes. In the first class the nodal lines run radially from the central support of the bell to its edge and in the second class they run round the bell at various heights above the mouth. The former class includes the case when for the fundamental note the bell vibrates in four equal segments separated by nodal lines. To produce the overtones they can also vibrate in 6, 8, 10 or 12 *i.e.*, an even number of segments. In fact the overtones are not harmonic overtones as given by a vibrating string or an organ pipe. The fundamental tone which determines the tone of the bell is not the lowest tone, but the lowest but one. The first note is called a "hum note", the fundamental is the second, and the next three notes are called the "tierce", the "fifth" and the "nominal". These form an inharmonic series but in spite of this on combination, produce a delightful effect. Sometimes the various parts of the bell due to unequal loading produce beats which are distinctly heard. The pitch of the fundamental is directly proportional to the thickness and inversely as the area of the surface of the bell. The existence of nodes and antinodes can be demonstrated by suspending pith-balls by light strings into a vibrating bell with its mouth upwards. At the nodes the balls will be at rest, whilst at the antinodes they will be agitated. Fig. 130 represents the mouth of the bell when it is emitting its fundamental. Points N_1, N_2. N_3 and N_4 are on the nodal lines. The segments of the rim between them vibrate in and out. Any two consecutive ones vibrate in opposite phases. For example at a certain moment the arc $N_1A'N_2$ will be $< N_1AN_2$ in the normal position, and arc $N_1B'N_4 >$ arc N_1BN_4. A little later $N_1A'N_2$ will be $>$ arc N_1AN_2 in the normal position, and arc $N_1B'N_4$ will be $<$ arc N_1BN_4. Thus it is clear that during each vibration a little of the rim is transferred through N_1 from the segment N_1AN_2 to the segment N_1BN_4 and back again. In other words there is a small tangential vibration of the rim at N_1, N_2, N_3, and N_4 as well as a radial vibration at A, B, C and D. This explains why passing a finger round the moistened rim of a wine glass can set it in vibration. The motion of the finger causes tangential vibration which also sets the radial vibration and makes the wine glass sing.

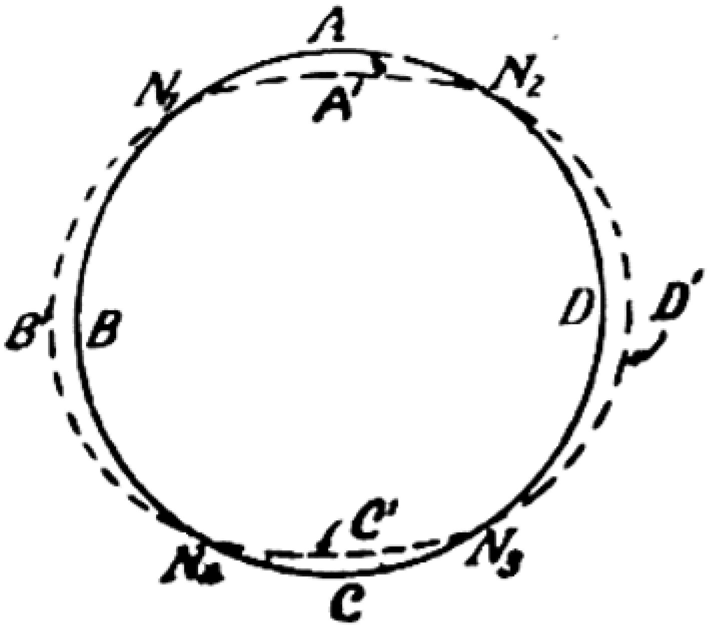


Fig. 130.

Bells are not generally accurately in tune when first cast. An experienced maker knows how to tune them and to modify the overtones suitably by putting them on a lathe and removing the metal from the inside.

It will not be out of place to mention here that a bell may be looked upon as a bent circular disc loaded near the middle just as a tuning fork is a bent rod. A tuning-fork, as said above, almost gives a pure tune, an organ pipe when blown gently may also produce the same, but a bell when vibrating would never do it. The note produced by it is generally a complex one.

175. Vibrations of Stretched Membranes.—The modes of vibration in case of stretched membranes as kettle-drum, Indian Tabla etc., are somewhat similar to those of a plate. We do not propose to enter into the details of these modes and the corresponding nodal lines as they are of no practical importance. Such instruments are not tuned to any particular notes and are used only to accentuate the rhythm of music. The frequencies of the different modes form a non-harmonic series such as 1 : 1·594 : 2·136. By putting a hollow chamber underneath as provided in the Indian Tabla, their frequencies are largely modified. In addition to this, the centre of the membrane is loaded with a paste of iron filings and gum. which greatly improves its mode of vibration and the components now practically form a harmonic series. For this reason tabla is used in Indian music not only as an accompaniment but as a musical instrument. In quality it is much more superior to other drums.

176. Musical Sound from a Diaphragm.—It might be understood from above that a diaphragm gives out a non-musical sound. But the gramophone sound box (Fig. 146) or the modern form of loudspeakers Art. 211 (*c*) used in Radio or Talkie machines are provided with some sort of diaphragms the vibrations of which produce music. In all these cases a thin diaphragm is mounted in a casing such that its natural frequency of vibration though non-musical, is either too high or too low as compared with the pitch of the musical scale used. It is always due to the forced vibrations either mechanical or electrical in origin, that the diaphragm oscillates with the particular pitch which it produces.

177. Period of Vibration of an Elastic Body deduced by the Method of Dimensions.—If we have two elastic bodies, say tuning-forks similar in shape and made of the same material, but one being double the size of the other, the smaller fork will be an octave higher than the larger one. The way in which the period of vibration (t) depends upon any linear dimension l, the density ρ, and the elastic coefficient E of the material, can be determined from the dimensions of the various quantities.

Let $t=Kl^x \rho^y E^z$ where K is a numerical constant that does not depend on the units of length, mass and time ; the values of x, y, and z are determined from the *Theory of Dimensions* as follows. The dimension of periodic time (t) is T^1, l is length and has dimension L^1, ρ denotes density *i.e.*, the mass per unit volume and has dimensions $\frac{M}{L^3}$. Coefficient of elasticity, say, volume elasticity is force per unit area divided by change in volume per unit volume, *i.e.*, $\delta p \div \frac{\delta V}{V}$. Force is the product of mass and acceleration and has dimensions $\frac{ML}{T^2}$. Therefore the elastic coefficient $\left(\delta p \div \frac{\delta V}{V}\right)$ has dimensions

$$\left\{\left(\frac{ML}{T^2}\right) \Big| \; L^2\right\} \Big| \; \frac{L^3}{L^3} = \frac{M}{LT^2}.$$

The dimensional equation of the above relation is

$$T^1 = KL^x\left(\frac{M}{L^3}\right)^y\left(\frac{M}{LT^2}\right)^z$$

or

$$[T^1] = K.\ L^{x-3y-z}\ M^{y+z}.\ T^{-2z}$$

From the principle of *Homogeneity of Dimensions* in a physical equation we know that the indices of the fundamental units on both sides must be equal,

$\therefore$ considering the power of L, $x-3y-z=0$
,, ,, M, $y+z=0$
,, ,, T, $-2z=1$

whence $x=1,\ y=\frac{1}{2},\ z=-\frac{1}{2}$

$$\therefore \quad t = Kl\sqrt{\frac{\rho}{E}}.$$

This shows that t is directly proportional to the square root of the density of the body, and inversely as the square root of the coefficient of elasticity. If the two bodies, as said before, are made of the same material, and are similar in shape, the period of vibration is directly proportional to the linear dimension of the body or the frequency will be inversely proportional to it. Thus a Helmholtz resonator on having its mouth and body doubled in diameter, will have its pitch lowered to half its original value.

CHAPTER XIII

CONCORD AND DISCORD

178. When two notes of different frequencies are sounded simultaneously, some combinations produce an agreeable or pleasant effect while others a disagreeable or unpleasant effect on the ear. The former is a case of **concord** or **consonance**, and the latter that of **discord** or **dissonance**. The concord and the discord mainly hinge upon the ratio of the frequencies of the two notes called the *Interval*. Simpler the ratio *i.e.*, smaller the integers representing the ratio, the smoother and more pleasing is the corresponding interval. Table below gives some *consonant* intervals in the order of their pleasing effect. As we go from intervals standing higher up in the table to those standing lower, *i.e.*, from unison to a semitone, dissonance increases

Table

Unison	1 : 1
Octave	2 : 1
Fifth	3 : 2
Fourth	4 : 3
Major third	5 : 4
Minor third	6 : 5
Major tone	9 : 8
Minor tone	10 : 9
Semi-tone	16 : 15

On reaching the interval $\frac{9}{8}$ the effect becomes quite unpleasant. It was Helmholtz who was the first to give physical explanation of the various degrees of consonance of different intervals in case of two pure tones and also in the case of complex notes having a number of harmonics. The following sections are devoted to the outlines of his theory.

179. Helmholtz's Theory of Consonance and Dissonance.— According to Helmholtz's theory the physical basis of all dissonance is the roughness caused by the presence of beats either between the fundamentals or between their overtones. Slow beats are not very disagreeable, while rapid beats are not perceived by the ear as separate due to the persistence of aural impression. Between these two limits, there exists a varying degree of dissonance. For example, suppose we have two tuning-forks (which almost give pure tones when struck gently) of the same frequency, say 512, mounted on resonance boxes. If the pitch of one is lowered by sticking a small quantity of wax or a sliding weight to one of its prongs, on sounding them together beats are heard. Upto about 4 beats per second no unpleasant effect is noticed. On further loading the lower fork, interval is gradually widened till the beats merge into a rattle and produce a jarring effect on the ear. There occurs maximum dissonance, when about 32 beats per second are produced. This number correspounds to about a semitone interval (16/15) between the forks. If the lower fork is further weighted, beats become rapid, jarring decreases, and ultimately it ceases at 85 beats/sec. This corresponds to the interval of minor third (6/5). Slowly and slowly as we proceed from unison to major third (5/4) a varying amount of dissonance is noticed, the maximum of course being at the interval of a semitone (16/15). It is in this way that we get the order of the consonant intervals mentioned in the Table on the previous page.

It should, however, be remembered that the result is not the same for all parts of the musical scale. As said above for frequencies of notes in the neighbourhood of 512, the dissonance reaches a maximum at about 32 beasts per second, that is, when the notes are a semitone apart. For higher frequencies the number of beats that gives maximum dissonance increases though not in proportion to the frequency and the interval that gives the worst effect diminishes *i.e.*, it becomes less than a semitone. Similarly, for low frequencies the worst number of beats is less than 32 and the worst interval greater than a semitone. Mayer's table given below indicates the number of beats per second when the discord is maximum and when it vanishes respectively.

Mayer's Table.

Frequency of lower tone	Number of beats per second	
	When dissonance is maximum	When dissonance vanishes
64	6·4	16
128	10·4	26
256	18·8	47
384	24·0	60
512	31·2	78
640	36·0	90
768	43·6	109
1024	54·0	135

But in discussing below the consonance of the important musical intervals, we shall assume that the most dissonant interval is not far from a semitone remembering, however, that there is some roughness upto an interval of a tone or more.

We have considered above the case of combination of two pure tones only. But when a chord is formed from two or more complex notes, in addition to the beats between their fundamentals we have to take into account the beats arising from the combination of the harmonics of the various notes with each other. Rather a single complex note may have itself the elements of dissonance In this case as we go higher in the series, they come within beating distance of each other. It is why a complex note of long range of powerful harmonics will be more or less harsh. In those cases where harmonics fall in intensity as we rise in the series, we may ignore, for our purpose all harmonics above the sixth. We discuss below the consonance of some important musical intervals.

180. Consonance of the Important Musical Intervals.— In discussing the consonance of the various musical intervals the following assumptions are made :—

(1) each of the two notes has full series of harmonics,

(2) harmonics above the sixth are weak and so they are ignored, and

(3) the effect of the combination tones is not taken into consideration.

(*a*) **Unison, 1 : 1.**

Each component of the first note superposes over that of the second and is therefore strengthened without the formation of beats. The interval is most consonant.

(*b*) **Octave, 2 : 1.**

First note (lower note) 1 2 3 4 5 6
Second note (upper note) 2 4 6 8 10 12.

The second note introduces no harmonics which are not present in the first. No roughness is produced. The consonance is perfect.

(*c*) **Fifth, 3 : 2.**

First note (lower note) 2 4 6 8 10 12
↖ ↙
Second note (upper note) 3 6 9 12 15 18

Here 3rd harmonic of the second note is beating within the tone of 4th and 5th harmonics of the first note. The tone $\frac{9}{8}$, $\frac{10}{9}$ though producing some roughness is greater than the interval of a semitone ($\frac{16}{15}$) which produces maximum roughness. So fifth, though not perfect is a good concord.

(*d*) Fourth, 4 : 3.

First note (lower note) 3 6 9 12 15 18

↙ ↙ ↖

Second note (upper note) 4 8 12 16 20 24

This is clearly a worse case. In addition to the slightly dissonant intervals of $\frac{9}{8}$ (major tone), ($\frac{20}{18}=\frac{10}{9}$ minor tone). ($\frac{18}{16}=\frac{9}{8}$ major tone), we have the 4th harmonic of the second note making bad interval of a semitone ($\frac{16}{15}$), with the fifth of the first note.

(*e*) Major third, 5 : 4.

First note (lower note) 4 8 12 16 20 24

↙ ↖

Second note (upper note) 5 10 15 20 25 30

This is still worse because we have one semitone ($\frac{16}{15}$), and the other ($\frac{25}{24}$) almost a semitone included within the range of first six harmonics.

(*f*) Semitone, 16 : 15.

First note (lower tone) 15 30 45 60 75 90

↖ ↖ ↖ ↖ ↖ ↖

Second note (upper note) 16 32 48 64 80 96

Each harmonic of the second note is a semitone above the corresponding harmonic of the first note. Thus there are six semitones within our range of 6 harmonics. This produces the maximum dissonance effect.

In a similar manner we can proceed to find the amount of discord for other intervals.

Fig. 131 illustrates the relative amounts of discord for various intervals on three different Musical Scales (*See Chapter XIV*).

Helmholtz obtained the above curve showing the relative amounts of discord for various intervals throughout the octave. It is clear from this curve that the discord is greatest just below the semitone ($^{16}/_{15}$) and above the seventh ($^{15}/_{8}$) and least at the octave and fifth ($^{3}/_{2}$). We also find that the octave and

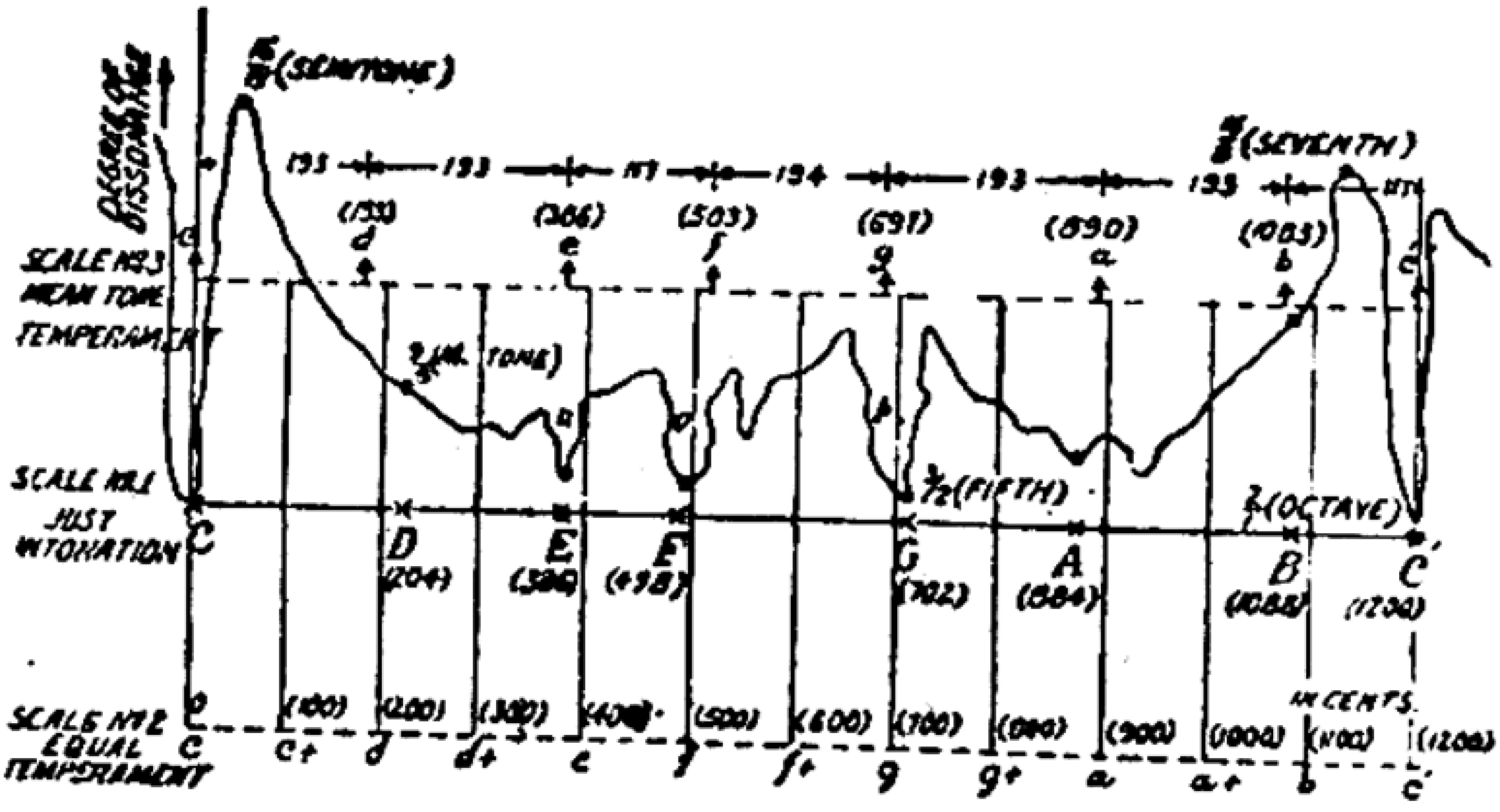


Fig. 131.

the fifth are bounded by very steep parts of the curve. This explains why a slight amount of mistuning in these intervals produces a large amount of dissonance which is readily detected. Consequently these are the easiest intervals to be tuned accurately.

181. Beats caused by Difference Tones and its Effect on Music.—In the last section we have shown that an exact octave gives a perfect consonance, while an exact fifth though not perfect is a good concord. Below we consider two instances *of two pairs of tuning-forks nearly an octave and nearly a fifth apart* and *free from overtones* yet producing beats.

(1) *Forks of frequencies nearly an octave apart* (*i*) *100 and 198 or* (*ii*) *400 and 795.* When two such forks are sounded together, their notes differ too much in pitch, and so cannot beat directly. Evidently the cause lies in the difference tones beating with the primaries. For the first pair of forks the first difference tone has a frequency 98. This will give two beats per second with the note of frequency 100. Similarly, in the second case first difference tone has a frequency $795-400=395$ which gives 5 beats per second with the note of frequency 400. These beats are actually heard when the notes are sounded loudly.

(2) *Forks of frequencies 400 and 598.* This is a case of

mistuned fifth. The higher note is two vibrations a second too low to form a perfect fifth. Here the first difference tone is $(598-400)=198$. There are two second difference tones $(598-198)=400$ and $(400-198)=202$. The frequency of the former second difference tone agrees with the lower primary but that of the latter (202) makes four beats per second with the First Difference Tone (198). As the Second Difference Tones are always very faint, beats formed in this case are very feeble.

We conclude, therefore, from the above discussion that although in the above two cases the interval between two notes is more than a major third when according to Helmholtz all dissonance should disappear yet the appearance of beats caused by the difference tones in the neighbourhood of a mistuned fifth or a mistuned octave introduces a small element of dissonance in music, which is faint near the fifth and somewhat more perceptible near the octave.

182. Consonance of an interval formed by two notes which have not both the full series of harmonics.—If both notes have odd harmonics, it can be seen that the consonance is generally better. In case one of the notes is played, say by the hautboy having full series of harmonics, and the other played by the clarinet having odd members, the results produced will be different. The smoothness will vary according to which of the notes is higher in frequency. We consider this case below :—

1st Case. *Clarinet above Hautboy* (Interval Major third $\frac{5}{4}$).

Hautboy of full series (lower note) 4 8 12 16 20 24
↙ ↖
Clarinet of odd members (upper note) 5 15 25 35

Here there are almost two pairs of semitones.

2nd Case. *Hautboy above Clarinet* (Interval $\frac{5}{4}$).

Hautboy of full series (upper note) 5 10 15 20 25 30
Clarinet of odd series (lower note) 4 12 20 28

No clash within the six harmonics is noticed. Hence we conclude that in case of these two musical instruments to play major third (5/4), it is better to have hautboy with its upper note as its fundamental.

Similarly, it can be shown that to play the fourth (4/3) with good effect, the clarinet should play the upper note.

Of course, we have proceeded above on the assumption that harmonics above the sixth are weak and so shown as absent. This is why we are getting such a striking difference. Otherwise really speaking in the actual working of these instruments the higher harmonics of these musical instruments are so strong that we cannot neglect them.

183. Effect of Combination Tones formed by the Harmonics.—Theoretically a number of 'combination tones' are possible from the harmonics of the two notes. These, if powerful enough, might introduce new tones and so modify the conclusions arrived at previously, where no combination tones were taken into account. It has been noticed that the harmonics are themselves very weak as not to be able to generate combination tones except the First Difference tone. This also being not very strong, does not practically affect the consonance. It has been, however, shown by Helmholtz that the First Difference tones produced in the case of full series of harmonics will not cause beats. They might slightly increase a beating effect of the same frequency produced as a result of the interaction of harmonics themselves. If both notes have odd harmonics, the difference tones might introduce even terms, and might slightly affect the consonance.

Effect of the difference tones produced by the fundamentals has not been considered. They may have some effect in changing the character of consonance, though they may not cause beats.

CHAPTER XIV

MUSICAL SCALES

184. Musical Scale (Diatonic).—A series of notes separated by definite and simple musical intervals is said to constitute a **Musical Scale**. The Diatonic Scale is the most common musical scale. It consists of a series of eight notes the interval between the first and the last being an octave. The series of notes is denoted as :

$$\begin{array}{cccccccc} C & D & E & F & G & A & B & C' \\ \textit{sa} & \textit{re} & \textit{ga} & \textit{ma} & \textit{pa} & \textit{dha} & \textit{ni} & (\textit{sa})' \end{array}$$

All these notes are arranged in the order of increasing frequencies which may be represented by

$$\begin{array}{lcccccccc} & 1 & \frac{9}{8} & \frac{5}{4} & \frac{4}{3} & \frac{3}{2} & \frac{5}{3} & \frac{15}{8} & 2 \\ \text{or} & 1 & 1{\cdot}125 & 1{\cdot}250 & 1{\cdot}333 & 1{\cdot}500 & 1{\cdot}667 & 1{\cdot}875 & 2 \end{array}$$

i.e., if the frequency of the first note be 24, the relative frequencies of the other notes of the diatonic scale will be

$$\begin{array}{ccccccccccccccc} (24) & & (27) & & (30) & & (32) & & (36) & & (40) & & (45) & & (48) \\ & \underbrace{\qquad}_{\frac{9}{8}} & & \underbrace{\qquad}_{\frac{10}{9}} & & \underbrace{\qquad}_{\frac{16}{15}} & & \underbrace{\qquad}_{\frac{9}{8}} & & \underbrace{\qquad}_{\frac{10}{9}} & & \underbrace{\qquad}_{\frac{9}{8}} & & \underbrace{\qquad}_{\frac{16}{15}} & \\ & \text{(Major} & & \text{(Minor} & & \text{(Semi} & & \text{(Major} & & \text{(Minor} & & \text{(Major} & & \text{(Semi} & \\ & \text{tone)} & & \text{tone)} & & \text{tone)} & & \text{tone)} & & \text{tone)} & & \text{tone} & & \text{tone)} & \end{array}$$

If the frequency of the note C be taken as 264, the Scale will be denoted by

264 297 330 352 396 440 495 528.

The above scale consists of three main intervals $\frac{9}{8}$, $\frac{10}{9}$, $\frac{16}{15}$ known as major tone, minor tone, and semi-tone so that the sequence of intervals in the Major Diatonic scale is major tone, minor tone, semi-tone, major tone, minor tone, major tone, semi-tone or, neglecting the difference between major and minor tones, tone, tone, semi-tone, tone, tone, tone, semi-tone. It could be extended both above C' and below C. *The names of the consonant intervals discussed in the previous chapter are derived from the positions of the notes in this scale.*

This scale has been in use in the West and the East for many centuries, and its origin is unknown. But intervals mentioned above can be obtained by the sub-division of a string, and it is possible that the scale might have taken its rise from this fact, because the ancients had the knowledge of the natural tones of vibrating strings. By suitably combining the most consonant intervals, the above scale beginning with C could be built up as follows :—

Let the most consonant intervals $\frac{4}{3}$, $\frac{3}{2}$ and 2 (the Fourth, the Fifth, and the Octave) arranged in the ascending order of pitch, and named F, G, and C' respectively be added to C. Thus if we take the frequency of the lowest note as unity, we can represent the four notes as :

C	F	G	C'
1	$\frac{4}{3}$	$\frac{3}{2}$	2

Now the best combination of three notes called a Major Triad, with frequencies in the ratio 4 : 5 : 6 is placed on the first three intervals. By placing the Triad on C, we get the frequencies 1, $1\times\frac{5}{4}=\frac{5}{4}$, and $1\times\frac{6}{4}=\frac{3}{2}$. For the Triad on F we get

$$\tfrac{4}{3},\ \tfrac{4}{3}\times\tfrac{5}{4}=\tfrac{5}{3},\ \text{and}\ \tfrac{4}{3}\times\tfrac{6}{4}=2\,;$$

and for the Triad on G, we get $\frac{3}{2}, \frac{3}{2}\times\frac{5}{4}=\frac{15}{8}$, and $\frac{3}{2}\times\frac{6}{4}=\frac{9}{4}$. When we bring the note $\frac{9}{4}$ down by an octave, we get $\frac{9}{4}\div 2=\frac{9}{8}$. Thus arranging all these notes falling in the compass of an octave in the ascending order of frequencies we have the required diatonic scale with C as tonic :—

C	D	E	F	G	A	B	C'
1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2

It will not be out of place to suggest here that the above derived scale obtained by combining the most consonant intervals has its advantage from the point of view of harmony, but it cannot be said that it took its rise from any such considerations.

The student should note that in music the sum of two intervals is obtained by multiplying them and not by adding them ; and the difference between them is obtained by dividing them. Suppose the frequencies of some musical notes A, B, C, D etc., are n_1, n_2, n_3, n_4, etc. The interval between B and $A=\dfrac{n_2}{n_1}$; between C and $B=\dfrac{n_3}{n_2}$; between D and $C=\dfrac{n_4}{n_3}$. Hence the interval between D and $A=\dfrac{n_2}{n_1}\times\dfrac{n_3}{n_2}\times$

$\frac{n_4}{n_3}=\frac{n_4}{n_1}$. The difference between the intervals between *B* and *A* and that between *C* and $B=\frac{n_2}{n_1}\div\frac{n_3}{n_2}$.

185. The Scientific Basis of the Diatonic Scale.—It was Helmholtz, as mentioned in the last chapter, who studied the consonance of musical intervals by widening the interval unison upto an octave. Graph drawn on page 245 illustrates the relative amounts of discord for various intervals within an octave. From this we find that at *C* (unison) harshness is zero. By increasing the interval slightly to $\frac{16}{15}$. we at once reach maximum harshness. For the interval $\frac{9}{8}$. it becomes less. At *E* it is still less ; at intervals represented by *F* and *G* harshness is almost zero. These notes are bounded by very steep parts of the curve, and any deviation from the just intonation in the case of these notes will give us a large amount of harshness as represented by points *o* and *p*. This is not so with any other note within an octave, and so musicians will never tolerate any deviation of *F* and *G* notes from the right frequencies. Harshness or roughness according to Helmholtz's theory of consonance is a beat phenomenon occurring between two notes or between some of their harmonics. For example, consider the interval $\frac{3}{2}$ and its three harmonics written below :

Fundamental	Harmonics		
B	B_1	B_2	B_3
300	600	900	1200
A	A_1	A_2	A_3
200	400	600	800

We find there is no beating effect anywhere, the number of beats between any two harmonics being 100 or more which are too rapid to be perceived. In case B's frequency is slightly raised to 304, and the true fifth thus mistuned ; writing as before :

B	B_1	B_2	B_3
304	608	912	1216
A	A_1	A_2	A_3
200	400	600	800

we see that B_1 beats with A_2 giving 8 beats which produce a certain amount of harshness. Besides this, combination tones will also cause additional confusion. This clearly explains that any deviation of a note on this scale from its just frequency, specially as seen in the case of mistuned fifth, will cause roughness. The reason why this scale has survived to the present day is that no other scale is so well-fitted to provide harmonious combinations.

***186. A New System of Indicating Intervals.**—It is sometimes convenient to express intervals in a new system of measuring them in which the unit is called a *Cent*. The name 'cent' is used because 100 cents make the semitone of those instruments in which twelve equal semitones are the intervals occurring in an octave. The number of cents in a given interval are taken to be proportional to the logarithm of the interval.

An octave is made equal to 1200 logarithmic cents.

$\therefore\ 1200 = K \log \frac{2}{1}$ or $K = \frac{1200}{\log 2}$ and to express any interval $\frac{n_2}{n_1}$ in cents, we multiply its logarithm by K as defined above.

$$\therefore \text{ Interval } \frac{n_2}{n_1} = \frac{1200 \log \frac{n_2}{n_1}}{\log 2} \text{ cents.}$$

For example,

$$\text{interval } \frac{9}{8} = \frac{1200 \log \frac{9}{8}}{\log 2} = 204 \text{ cents.}$$

The intervals of the Equal Temperament Scale (see Art. 189) expressed in cents are 100, 200, 300, 400, 500, 600, 700, 800, 900, 1000, 1100, 1200 for $\frac{1200 \log 2^{\frac{1}{12}}}{\log 2} = 100$;

and $\frac{1200 \log 2^{\frac{2}{12}}}{\log 2} = 200$, etc.

We saw in Art. 184 that the resultant interval was obtained not only by the sum of the intermediate intervals but by a process of continued multiplication, a method which is somewhat troublesome.

On this system the arithmetic sum of the measures of intermediate intervals shall be the measure of the resultant interval. To illustrate this the intervals of the Diatonic Scale expressed in cents are shown in the table below :—

Interval		*On Diatonic Scale*	*In Cents.*
Unison	$\frac{C}{C}$	1 : 1	0
Major tone	$\frac{D}{C}$	9 : 8	204
Major third	$\frac{E}{C}$	5 : 4	386
Fourth	$\frac{F}{C}$	4 : 3	498
Fifth	$\frac{G}{C}$	3 : 2	702
Major Sixth	$\frac{A}{C}$	5 : 3	884
Seventh	$\frac{B}{C}$	15 : 8	1088
Octave	$\frac{C'}{C}$	2 : 1	1200

For tuning purposes it is convenient to make use of a very small interval so far not mentioned. This is called *Comma*, and it is the difference between the major tone $\frac{9}{8}$ and the minor tone $\frac{10}{9}$. Its vibration ratio $=\frac{9}{8}\div\frac{10}{9}=\frac{81}{80}=1{\cdot}0125$. It is equal to 2·5 cents. This is about one-fifth par of a semitone[*] being aproximately equal to $\frac{111{\cdot}6}{5}=22{\cdot}3$ cents.

187. Defects of the Diatonic Scale : Temperament—A necessity.—The first note from which the other notes of a scale are derived is called the Key-note or Tonic. It is the tonic about which the whole music hinges. The demand of the Modern music is that it frequently modulates or changes its tonic to provide variety. A composition may begin with *C* as tonic, and soon change its key-note to *D*. This would mean that another scale with D as tonic is required. If this is done, we shall find that all the notes of the two scales will not be exactly the same, but some new notes designated as '*sharp*' and '*flat*' have to be introduced, according as the new note is higher or lower in pitch than the corresponding one of the Diatonic Scale. For example for a certain musician having C (256) as the tonic we have the scale as

---

[*] **Intervel semitone=111·6 cents.**

$$\begin{array}{cccccccc} C & D & E & F & G & A & B. & C' \\ 256 & 288 & 320 & 341 & 384 & 427 & 480 & 512 \end{array}$$

$$\diagdown \tfrac{9}{8} \wedge \tfrac{10}{9} \wedge \tfrac{16}{15} \wedge \tfrac{9}{8} \wedge \tfrac{10}{9} \wedge \tfrac{9}{8} \wedge \tfrac{16}{15} \diagup$$

Now if he finds that this C(256) is too low for his voice, and he needs the next note D(288) as the tonic, then the modulated scale will become

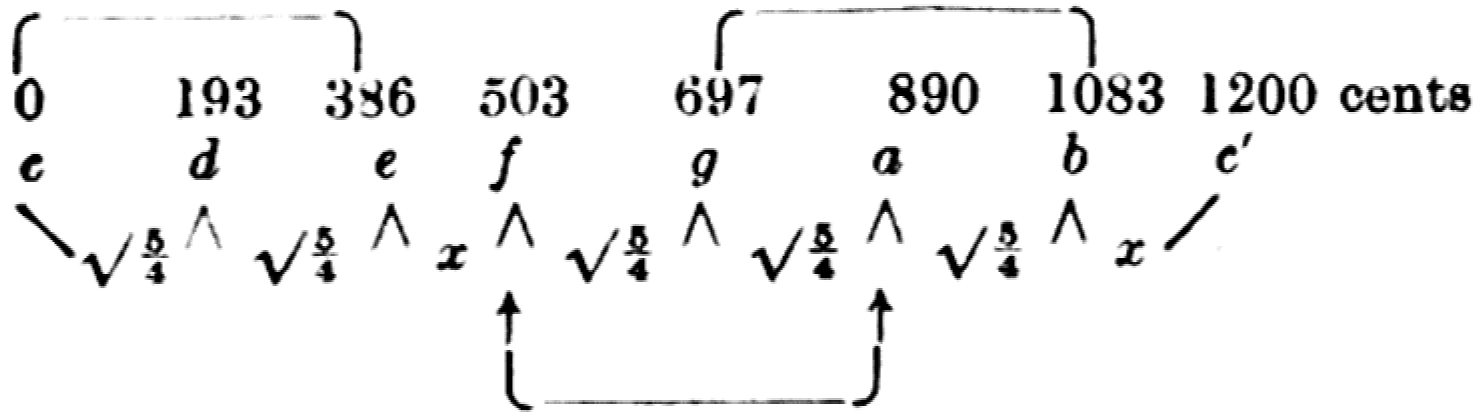
$$\begin{array}{cccccccc} 288 & \underline{324} & \underline{360} & 384 & \underline{432} & 480 & \underline{540} & 576 \end{array}$$

$$\diagdown \tfrac{9}{8} \wedge \tfrac{10}{9} \wedge \tfrac{16}{15} \wedge \tfrac{9}{8} \wedge \tfrac{10}{9} \wedge \tfrac{9}{8} \wedge \tfrac{16}{15} \diagup$$

If we compare the frequencies of these notes with those in the scale of C, we find that four new notes which are *underlined* have to be added. if any other note is used as the keynote, some more new keys will have to be introduced. If we proceed in the same way to take all of the notes in the Scale of C as tonic, we shall have to introduce other new notes. It is found that within the compass of an octave eleven such notes have to be added, that is, we shall have nineteen notes in all including the octave. Moreover, the modern music requires some additional "accidental" notes, a semitone above or below the notes of the diatonic scale and it should be possible to treat any of these accidentals as the tonic of the diatonic scale. This would mean the addition of still more notes in an octave. In instruments such as violin and sitar, which have no fixed keys, this causes no difficulty, as the pitch of each note is determined by the performer. In instruments such as painoforte and harmonium which have fixed keys, the pitch cannot be altered by the performer. Once the pitch of a key is adjusted by the instrument maker or tuner, it cannot be altered. Hence modulation is prevented by mechanical difficulties in the instruments of the key-board type. Consequently a compromise has to be effected so as (*i*) to limit the number of notes as is found practicable and (*ii*) to alter their frequencies a little from the true diatonic scale to provide modulation. Such a compromise is called **Temperament** and the scale thus derived, is styled as a **Tempered Scale.**

Let us now discuss the two kinds of temperaments (*i*) **Mean tone temperament** and (*ii*) **Equal tone temperament.**

188. Mean-Tone Temperament.—This temperament receives its name from the fact that for the majority of notes, geometric mean of the major and the minor tones ($\sqrt{\frac{9}{8}\times\frac{10}{9}}=\sqrt{\frac{5}{4}}=193$ cents) is taken as the interval between successive notes. In this system, the major thirds are kept pure, and the note which lies between the notes of each major third is put half way between them.



For example in the key *c*, each of the thirds *c—e*, *f—a*, *g—b* is adjusted to have frequency ratio 4 : 5. *d*, *g* and *a* are put half way between them respectively, so that each interval *c—d*, *d—e*, *f—g*, *g—a*, and *a—b*=mean tone$=\sqrt{\frac{9}{8}\times\frac{10}{9}}=\sqrt{\frac{5}{4}}$. Again intervals *e—f* and *b—c'* are made equal and of such size as to fill up an octave

i.e., $\sqrt{\frac{5}{4}}.\sqrt{\frac{5}{4}}.x\sqrt{\frac{5}{4}}.\sqrt{\frac{5}{4}}.\sqrt{\frac{5}{4}}.x.=2$

or $\frac{25}{16}\sqrt{\frac{5}{4}}x^2=2$ or $x=1{\cdot}07$

'*g*' on this temperament is flattened by quarter of a comma; *i.e.* by $\frac{21{\cdot}5}{4}=5{\cdot}3$ cents approx. This flattened fifth is the basis of the Mean tone System.

Thus the mean tone temperament makes *c—d*, *f—g*, and *a—b* half a comma too small, *d—e* and *g—a* half a comma too large, and *e—f* and *b—c'* quarter of a comma (about 5 cents) too large, [refer to Fig. 131]. In this system we have two intervals $\sqrt{\frac{5}{4}}$ and $x=1{\cdot}07$ of different sizes which mainly play the same part as the 3 intervals $\frac{9}{8}$, $\frac{10}{9}$ and $\frac{16}{15}$ of different sizes in the just diatonic scale. Hence the number of additional notes when the tonic is changed in this scale is less. In this respect it is superior to the diatonic scale but the modulated notes in no way improve the consonance. The total number of notes cluding the additional ones being still quite large it is almost in disuse. The **Equal Temperament** scale nowadays universally adopted is described in the following section.

189. Equal Tone Temperament.—We saw in Art. 187 that eleven new notes have to be introduced in order to be able to use any of the notes in the scale of *C* as tonics. On calculating the frequencies of these notes we find that they occur in 6 groups, three notes between *C* and *D*, one between *D* and *E*, one between *E* and *F*, two between *F* and *G*, one between *G* and *E* and three between *A* and *B*. The frequency of the new note between *E* and *F* is so close to *E* that it may be neglected and each of the remaining five groups may be replaced by a single intermediate note thus giving us only five new notes. On account of the position which they occupy in the diatonic scale, they are called *C* sharp, *D* sharp, *F* sharp, *G* sharp and *A* sharp. The addition of these five new notes gives us twelve notes in all, excluding the octave and enables the singer to play with any of the notes in the scale of *C* as tonic. This led to the division of the octave into twelve intervals. But even this compromise does not enable the singer to use any one of the new notes as the key note. In order to overcome such difficulties the octave is divided into twelve *equal* intervals called *Equal Temperament Semitones* and the scale is known as **Equally Tempered Scale.** This is one that is now in general use.

Since the equal temperament semitone is such that it gives an octave when added to itself twelve times, it follows that if x denotes the ratio which measures the interval

$$x^{12}=2 \text{ i.e., } x^{*}=2^{\frac{1}{12}}=1\cdot0595$$

The ratios of the series of notes in the equal temperament scale are given below† :—

c	*c+*	*d*	*d+*	*e*	*f*	*f+*	*g*	*g+*	*a*	*a+*	*b*	*c′*
1	$2^{\frac{1}{12}}$	$2^{\frac{2}{12}}$	$2^{\frac{3}{12}}$	$2^{\frac{4}{12}}$	$2^{\frac{5}{12}}$	$2^{\frac{6}{12}}$	$2^{\frac{7}{12}}$	$2^{\frac{8}{12}}$	$2^{\frac{9}{12}}$	$2^{\frac{10}{12}}$	$2^{\frac{11}{12}}$	2
1	1·059	1·122	1·189	1·260	1·335	1·414	1·498	1·587	1·682	1·782	1·888	2

c+, d+, f+, g+ and *a+* represent the five additional notes.

---

* This interval $=\dfrac{1200\log 2^{\frac{1}{12}}}{\log 2}=100$ cents.

† '*c+*' is read as *c sharp*.

The ratios of the *chief* notes of the tempered scale are

c	*d*	*e*	*f*	*g*	*a*	*b*	*c′*
1	1·122	1·260	1·335	1·498	1·682	1·888	2

Neglecting the difference between major and minor tones, tempered diatonic scale is of the sequence, tone, tone, semitone, tone, tone, tone, semitone as in the just diatonic scale but the tones are all equal and exactly twice the semitone.

The ratios of the notes of the *just* diatonic scale are

1	1·125	1·250	1·333	1·500	1·667	1·875	2

If we compare these ratios with the tempered scale ratios we see that none of them differs by as much as 1%, the fourth and the fifth are within $\frac{1}{8}$%. [For dissonance of the intervals in this scale refer to Fig. 131.]

At the present time in all instruments in common use there are twelve equal intervals in an octave.

A single note between *c* and *d* is used both for *c* '*sharp*' or *d* '*flat*'

(*c* #) (*d* ♭)

Eight white keys on the harmonium give a scale not much different from the diatonic scale, while 5 black keys are added which give notes dividing the interval of a tone into two semitones, differing a little from the true diatonic semitones. All the keys together form an equally tempered scale. Beginning with *c*, we play on the approximate diatonic scale; and the black keys are not used at all. Starting with any other key, black keys have to be used. The question of introducing additional notes for changing over to any note as a tonic does not arise here. Of course all the notes on this scale are made equally bad, instead of having some better or worse as in the case of mean-tone-temperament (see curve Fig. 131). In

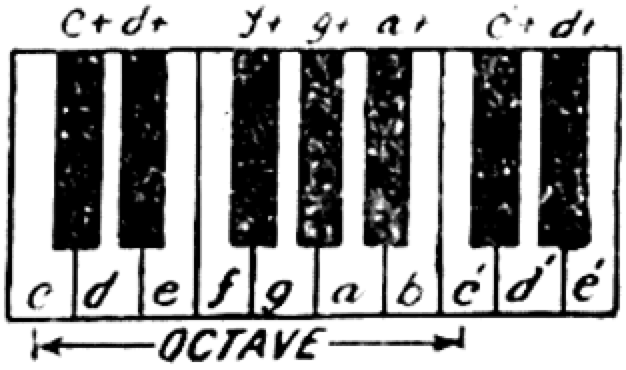


Fig. 132. Reeds of the Harmonium.

spite of this fact, "It should be observed," as Prof. Poynting and Sir J .J. Thomson justly remarked, "that compromise or temperament is not a question of merely instrument and mechanism but is a necessity in music from incommensurability of different intervals". While Mr. Donkin says, "The whole structure of modern music is founded on the possibility of educating the ear not merely to tolerate or ignore, but in some degree to take pleasure in slight deviations from the perfection of the diatonic scale.

CHAPTER XV

MUSICAL INSTRUMENTS

189a. Musical instruments :—In this chapter we propose to give a brief account of a few typical musical instruments of common use both in the East and the West merely with a view to illustrate points of definite scientific interest developed in the Text rather than a musical stand-point. They are divided into three main classes : (1) Stringed instruments (2) Wind instruments and (3) Percussion instruments. For a detailed study of their design and construction the interested reader is advised to read 'Specialized Treatises'. It is herein we find that the practical achievements have more often preceded the theoretical treatment.

189b. The Stringed Instruments :—They all depend for the production of the musical notes on the transverse vibrations of strings and the laws of vibrating strings already formulated are applied consistently in their construction. The vibrations are produced in the strings by (*i*) striking such as in the *Pianoforte* (*ii*) plucking such as in the *Sitar*, the *Veena* etc., and (*iii*) bowing, such as in the *Violin* and *Sarangi* and other allied instruments.

189c. The Pianoforte.—It is an instrument of the key board type like the Harmonium. The strings are thin steel wires stretched on a metal frame with a sound box of large surface to reinforce the vibrations. In order to secure sufficient loudness each note has one to three strings whose pitches can be altered by changing their mass, length or tension. The bass strings are thicker made so by a covering of fine wire wound specially around them without impairing their flexibility and are several feet long while those at the treble end are thinner and only two or three inches long. The strings are made longer and heavier in the bass in order to equalise the tension. In a pianoforte there are about 7 octaves and it could not be possible to have such a wide range of frequencies by merely changing the tensions of strings of equal lengths. The instrument is played with both hands and the vibrations are excited by hammers covered with felt and actuated by levers from the piano keys the object of the felt being to reduce the suddenness of the impact.

As harmonics above the sixth are considered undesirable in a pianoforte string, they are suppressed by so adjusting the hammers as to give blows at points from one-seventh to one-ninth the length of the string from the end. No harmonic would therefore be produced that requires a node at the point struck.

Experimental work by Helmholtz and others indicates that the shape and the hardness of the hammer also affects the quality of the note. Blow with a hard hammer striking at a single point would encourage the formation of long and too obstrusive series of harmonics, consequently hammers with rounded end are used for bass strings and narrower ones for the treble to compensate for quality differences. In the designing of pianoforte, however, the technical experience of the instrument makers and the musicians through a long period counts for so much.

189d. The Sitar.—**The Sitar** and the **Veena** are played by the hand and the strings are plucked either by the finger nail or by the apex of a conical wire frame known as plectrum and worn on the first finger. Fig. 132 (*a*) shows a sketch of the *Sitar*. There are seven strings of copper and steel of different diameters mounted on a semi-cylindrical finger board about 3 ft. long ending on one side in a big hemi-spherical resonance box. The strings are tied at one end to three nails on the side of the hemi-spherical resonator and passing over an ivory bridge fixed at its top are fastened at the other end to tightening pegs of wood, two of which are at the top of the other end of the finger board and the rest at its sides. They enable the player to tune

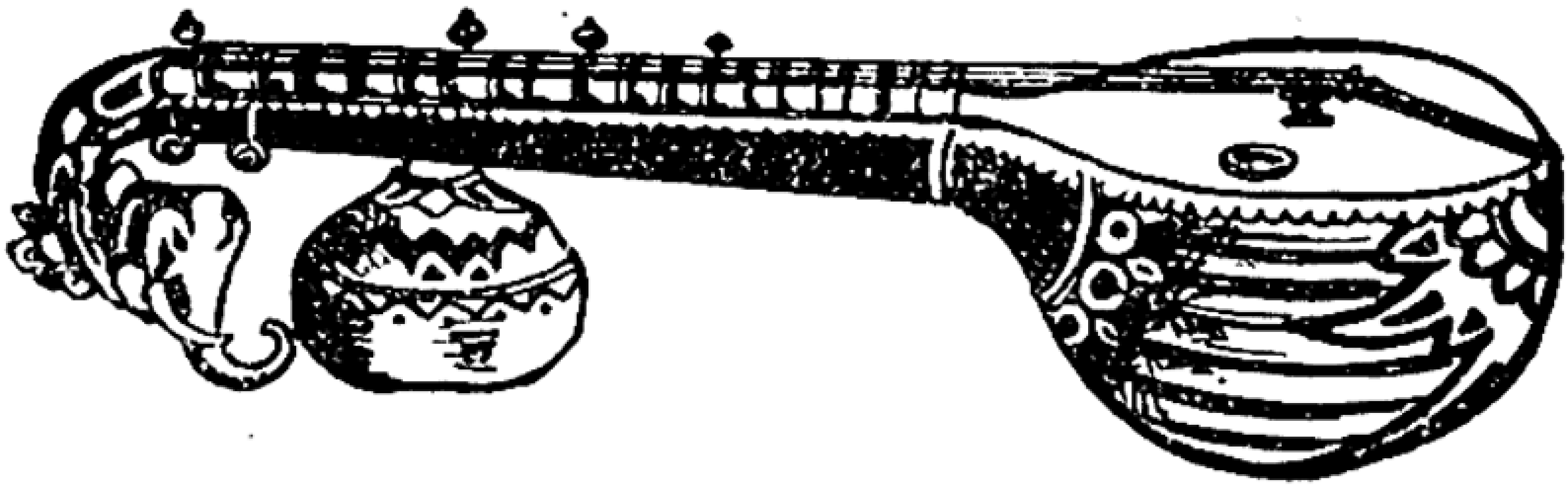


Fig. 132 (*a*).

the strings to different notes by altering the stretching force. The player plays on the strings mainly by the three fingers of the right hand and the notes other than those to which the strings are tuned are obtained by pressing them down at appropriate lengths by the fingers of the left hand against the adjustable curved metal rods called 'Frets' which number about 19 and cross the finger board. For instance if the string is pressed in the middle, it produces its octave and by pressing it at one fourth, it produces a double octave and so on.

Besides the seven main strings, there are stretched underneath them fine metallic wires generally eleven in number tuned to different frequencies by adjusting their lengths by means of tightening pegs also fixed on the side of the finger board. They are not plucked but are set in forced vibrations thus increasing the intensity of sound. They are called '*tarban*'.

The *Veena* is built on the same principle as the *Sitar* and differs from it in having two resonance boxes one on either end of the finger board. It also has seven main strings of the same length but much stouter and thicker. The number of five resonating wires called '*tarban*' is more than eleven to have still greater intensity of sound. The tone is not feeble as in sitar and blends well with other instruments.

The *Tanpura* has four strings of equal length with no '*frets*'. It is used with vocal music to mark the rhythm and to give a greater resonance effect. The wires are tuned to different frequencies and are plucked with two fingers of the right hand.

The *Guitar* has six strings and is otherwise just a *Sitar*. It is plucked by the thumb and the fingers of the right hand.

It may be noted that in all these instruments, the thumb or the fingers of the right hand or both are the exciters, the strings the vibrators, the sound box the resonator, while the fingers of the left hand and the frets form the manipulative arrangement for the production of the scale.

189e. The Harp.—In the Harp the strings are of fixed lengths and there is a separate string for every note of the musical scale. In order to play the instrument in different keys the strings can be shortened or lengthened by a small fraction to raise or lower the fundamental by a tone or a semitone at the option of the player by means of pedals. The strings are plucked by the thumb and the fingers of both hands as in a pianoforte. The octaves are produced on the longer strings of the harp by touching in the middle with the fleshy part of the palm of the hand. The sound board is smaller than that of a pianoforte and the higher tones are generally weaker. The note is therefore soft and dull mingling extremely well with the horns, the trombones and the brass instruments.

Mandolin and **Banjo** belong to the family of the Harp.

189f. The Violin.—The most important of the family of instruments in which the strings are *bowed* are the *Violin* and *Sarangi*. The Violin has generally four strings of the same length made of catgut or steel of different diameters mounted on a special type of sound box which acts as a resonator Fig. 132 (*b*). At the upper ends the strings an fastened to the tightening pegs which enable the player to tune the strings by altering the stretching force. The strings are tuned to the notes g, d^1, a^1, e^2 making fifth between each consecutive pair that is, the thin string produces a note of $\frac{3}{2}$ times the frequency of the note produced by the neighbouring thicker string. In playing the instrument, the notes intermediate to those to which the strings are tuned, are obtained by pressing them against the finger board with the fingers of the left hand at various places and, when necessary, the harmonics are produced by bowing and damping them at correct places as in a piano.

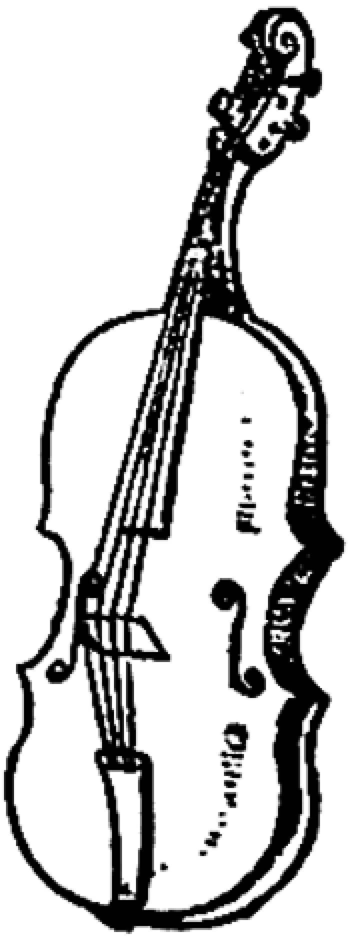


Fig. 132 (*b*).

The finger board is smooth and without frets which are fixed in the *Sitar* and *Veena* at points to which the strings are to be shortened for various notes of the scale. Note the absence of frets on the finger board makes it difficult for the beginner to play but in the hands of a skilful performer it permits the use of the grace, that is, the gliding of a note continuously from one pitch to another.

The Action of the Bow.—The bow with its resined hairs is used as the exciter. It produces a sustained sound and the note of the string is continuously under the control of the performer. The action of the bow is briefly as follows :—

The string adheres to the bow and the point in contact with t moves with uniform velocity until the tension overcomes the static friction, when it slips past the bow and returns with a uniform velocity as shown by Helmholtz's experiment described in Art. 54 (*a*). When it reaches the end of the swing, it comes to rest relative to the bow, another part of the bow grips it and draws it aside and so on. That it moves exactly with the bow has been demonstrated by Sir C. V. Raman and others. On account of the difference in static end dynamic friction more work is done on the string by the bow during its displacement than is done by the string on the bow during the subsequent slip and accounts for the energy emitted as sound and used in overcoming frictional losses. The intermittent dragging action of the bow serves to maintain the natural vibrations of the strings in the short intervening periods. Although the energy for maintenance comes from the bow, the note is determined by the string.

The motion of a bowed string is very complex and the vibrations are not at all simple harmonic vibrations. Applying Fourier's Theorem to the vibration curve of a point on a violin string shown in Fig. 45, we find that it requires a large number of the series to express it and the note of violin therefore consists of a large number of harmonics irrespective of the position of the point that is bowed. The *quality* of the resulting note, however, is not determined by the motion of the string alone for the sound received by the ear does not come chiefly from the string direct. The belly, the sides and the back of the sound box and the air inside it all pulsate in response and modify the *tonal* quality. The distinctive character of the note of a violin according to Helmholtz, is due to the strength of 6th to 8th harmonics as compared to those of other instruments.

189g. The Wind instruments.—They are classified into two groups (*i*) Flutes *i.e.*, pipes without reeds, (*ii*) Reed instruments (*a*) with pipes such as Clarinet, Bassoon etc. (*b*) without pipes such as the Harmonium.

189h. The flute..—It is a modified form of the open organ pipe comprising the orchestral flute pipe and that smallest high pitched piccolo. In all these instruments we have a cylindrical pipe open at one end and pierced with a hole near the other end, across which the player sends a blast of air from his lips and so produces the sound, the vibrations being set up in the same way as in the flue pipes of an organ.

In the simplest type for the production of the various notes of the scale, six holes are drilled along the side of the tube to be covered by the fingres. But generally the number of holes is large. They are covered by padded keys which are pressed down with fingers. When all the holes are closed and the performer blows into the tube gently, the lowest note corresponding to the full length of the pipe is produced. Both ends are antinodes and the middle point is a node. If the holes were as large as the bore of the tube, the pressing down of the key over any one of the holes would reduce the length of the pipe to that between the mouth hole and this hole and thereby the pitch is raised.

The distances of the holes from the mouth hole are inversely proportional to the frequencies of the diatonic scale. When the holes are small they can be easily covered by fingers but the notes emitted are weak and it is for this reason that in modern flutes the holes are covered with keys and therefore can be made larger.

The action of a flute is like a Helmholtz resonater whose pitch can be raised by enlarging the opening. So in a flute the size and the position of the hole both matter in altering the pitch. It is why the holes may within limits be made in any position convenient for the fingers. Any deviations from the correct pitches can be set right by adjusting the size of the hole.

The compass of a flute extends over three octaves. The three notes d^1, d^2, d^3 are indicated as open notes. They are the fundamental, its octave and double octave respectively and are obtained with only slight modifications of fingering. The full chromatic scale may be regarded to be built upon three of the tones natural to the whole tube or parts of it. For producing say d^2, we have the same length of the tube as for d^1, but the octave or the second natural tone is now produced instead of the fundamental. The utterance of this higher note is favoured by opening the first hole of the six (that nearest the mouth hole) whereas for the fundamental all the six holes are closed. No adequate theory of the use of the holes and the resulting notes can be given.

189i. The Clarinet.—It is a reed instrument with a single cane reed beating on a rectangular opening in the side of the mouth piece and a cylindrical pipe with a small bell mouth. The reed end as already explained in Art. 164 acts as a closed end, the tones possible to the instrument form the odd harmonic series. By overblowing we produce a twelfth above the fundamental and not the octave as in a flute. To bridge this large gap a sufficient number of holes and keys are provided in the sides of the tube. The characteristic quality of the note produced is·due to the weakening or extinction of the evenly numbered harmonics. It has been shown that some of the even harmonics are feebly present as the air blast contains the whole series. The reed and the air column in forming the vibrating system give a note whose pitch almost entirely depends upon the length of the column and considerably lower than that proper to the reed for the latter on account of its small mass is constrained to vibrate with the pitch proper to the air column.

There are many clarinets in use similar to each other but of different pitches. The *Sexaphone* belongs to this family and differs in having a conical tube and therefore a full series of harmonic overtones.

189l. The Hautboy and the Bassoon.—Both have double cane reeds and conical tubes terminated by bells. The Bassoon from the scientific point of view is a bass Hautboy. The cane reeds are slightly curved and bound together with their concave faces towards each other so that between their edges there is a narrow aperture. When the player blows into the mouth, this narrow aperture is alternately opened and closed thus admitting puffs of air into the tube which serve as the exciter. The reeds vibrate transversely as well as longitudinally. The pitch is, however, settled by the length and the shape of the tube. On account of their conical bores they give the full harmonic series of overtunes and therefore the arrangement of the holes and the keys are like a flute.

The quality of the note obtained from the Hautboy is some what penetrating consisting as it does of an extended retinue of harmonics.

The Bassoon has a much greater length of the tube and is for convenience, doubled on itself. The reed is fitted on a tube fixed side ways and the holes are bored obliquely so that their outer ends are close together to suit the fingering and their inner ends are sufficiently wide apart to produce the required pitches.

The Hautboy and the Bassoon are therefore essentially similar, the changes in form of the Bassoon are merely to adapt it to the convenience of the performer.

189k. The Harmonium.—It is a wind instrument of the key board type like the piano but in this the air is set into vibrations by means of thin oblong metal reeds each one of which is tuned to its appropriate pitch. A reed is fastened at one end to a block in which there is a hole behind the reed of the same shape but slightly bigger in size so that the reed can vibrate freely.

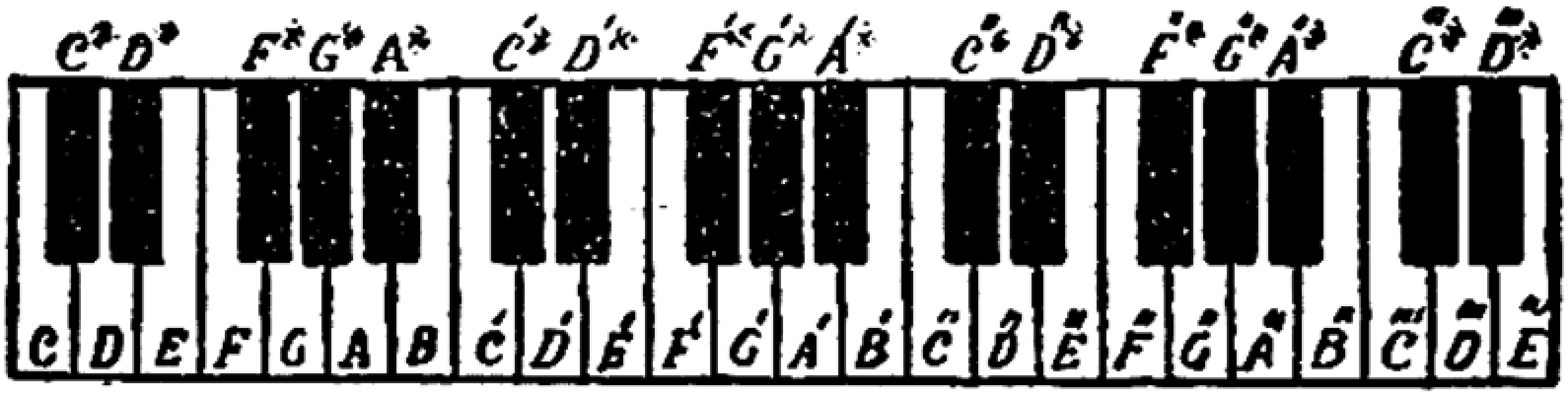

C* D* F* G* A* C'* D'* F'* G'* A'* C''* D''* F''* G''* A''* C'''* D'''*
C D E F G A B C' D' E' F' G' A' B' C'' D'' E'' F'' G'' A'' B'' C''' D''' E'''

Fig. 132 (c)

Fig. 132 (c) shows the key board of a harmonium. The series of notes provided in it is on the equally tempered tempered scale *viz.*, 13 notes in an octave each a semitone apart. The major notes of the diatonic scale sa, re, ga, ma, pa, dha, ni, sa are indicated by the white keys and the five additional notes in each octave *re* flat, *ga* flat, etc., by black keys. In all there are generally three and a half octaves with about 41 reeds each one of which is controlled by a key.

When the wind is forced from the bellows through the aperture underneath a reed and a key is pressed down thus opening the passage for the air, the blast of air sets the reed into vibrations and produces a fairly loud sound. The quality of the notes produced is not as sweet as that produced by an organ or a piano but it is rather cutting on account of the following two reasons (*i*) the absence of pipes whose resonance modifies the quality and (*ii*) the presence of non-harmonic overtones in the vibration of reeds. It is for this reason that it has come into disuse as a component of the orchestra.

189l. The Brass Instruments.—There is a great variety of brass instruments which form the essential accompaniments of bands such as the bugle, the trombone and the horn. We shall only describe briefly the several features common to all.

The tube is quasi-conical or hyperbolical in shape with a bell mouth at the bigger end and a cup-shaped mouth piece at the narrow end to which the player's lips are applied. The lips constituting a double membranous reed are alternately open and closed and along with the air column form a vibrating system. Which of the notes natural to the instrument will be produced is settled by the adjustment of the lips to the mouth piece, the tension of the lips and the pressure of the air used. But the exact pitch of the note depends chiefly upon the tube and the temperature of the air in it at the moment. The shape is so designed by the makers as to approximate closely to full series of harmonic tones. But the closeness of that approximation differs in different types of instruments. The intensity or feebleness of a tone will depend upon whether the tones natural to the instrument are very closely in tune or out of tune with the corresponding tones of the harmonic series and this affects the quality of the note.

Much, therefore depends upon the diameter of the tube, its length, the proportions of the bell and the shape of the mouthpiece itself. If the diameter of the tube and bell are small in comparison with the length as in the Trumpet and the Trombone., the higher tones natural to the instrument are brought into prominence. . Such instruments therefore, give a large number of harmonics and a brilliant tone. The shallow cupshaped mouth-pieces are also helpful. With deeper mouthpieces and either much wider tubing or larger bell as in Euphonium and French horn respectively we have the higher tones less prominent and a softer quality results.

There are various mechanisms for providing the scale in some of the brass instruments such as by the use of holes, slides and valves, the details of which concern the designer.

189m. Percussion Instruments.—The drums, the kettle-drum and the Indian Tabla belong to this class of instruments. They are all practical illustrations of the uses of stretched membranes in music already discussed in Art. 175 which the students are advised to read before proceeding further. The tension is applied to the membrane by the stretching frame which consists of a number of strings tied to the circumference of the membrane. The notes are produced either by hammers as in drums and kettle-drum or by hands as in *Tabla*. In *Tabla* it is usual to have two drums tuned to the fundamental and its fifth. Like all percussion instruments it is used as a rhythm marker.

CHAPTER XVI

ACOUSTICAL MEASUREMENTS

190. Determination of Velocity of Sound in open Air.—The earlier experiments for the determination of the velocity of sound in open air were performed by the direct method of observing the time interval between the seeing of the flash of light due to the firing of a distant gun and the arrival of sound. This method is simple, but liable to a number of errors :—

(1) **Effect of Wind.**—The velocity of sound is affected by wind. When the wind is blowing the velocity relatively to the ground is not the same as its rate of propagation through still air. The wind may be blowing in the direction of sound propagation or against it. In the former case it assists the sound velocity while in the latter case it chokes it. This effect of wind may be eliminated by taking observations from either of the two distant stations. This is the method of 'reciprocal firing'. In this case one observer fires a gun and another at the distant station measures the time interval referred to above. Afterwards the second observer fires a gun and the first observes the time interval. Mean of these two values gives the correct time interval. Velocity is then calculated by dividing the distance between the observers by the mean time interval. In case the line joining the observers is oblique to the direction of wind, calculations are not simple, but when the velocity of the wind is not very great, mean value of velocity of sound obtained by the above method almost approaches the correct value.

(2) **Temperature and Humidity of the Air.**—Corrections due to these have been discussed in Chapter V, and should be applied as stated therein.

(3) **Personal Equation.**—It is not possible to record the time at the exact moment when the flash is seen or the sound is heard and hence some error always creeps in. Again, for the same observer the error for events of different classes is different *i.e.*, the visual impression of the flash of firing and the aural impression made by the report of the gun take different times to react upon the observer. If errors, causing delay in the two cases would have been the same, the time interval would have

been correct, and also it would have been independent of the observer. But this error known as 'personal equation' is different with different persons and also changes with different classes of events.

Regnault in 1864 tried to eliminate the 'personal equation error' of an observer by making an *automatic* record of the flash and the arrival of the sound. The gun shot broke an electrical circuit stretching across the muzzle of the gun, and caused a mark on the revolving drum of a chronograph. At the receiving station, the waves pressed a stretched membrane forward which broke a second electrical circuit producing another mark upon the drum of the same chronograph. The interval between the two marks on the drum which gives the time taken by the sound to travel the distance from the gun to the membrane can be calculated provided the speed of rotation of the drum is known. This did not completely eliminate the 'personal equation' as the membrane also had some 'personal equation'; this, however, would be constant, and its value could be found. Regnault observed that velocity of sound in still air at 0°C. was equal to 332 metres per second. A feeble sound was found to travel slower and a loud sound faster. He also observed that as the distance increased and the sound grew weaker, the velocity reached a limiting value of 330·6 metres/sec. The experiments with pipes showed that the velocity was still less. This is mainly due to the friction between the air and the pipe as some of the sound energy is converted into heat. The narrower the pipe, the greater is the reduction. According to theoretical investigations of Helmholtz and Kirchhoff, $V_a - V_p \propto \frac{1}{d\sqrt{N}}$, where V_a is the velocity of sound of frequency N in open air and V_p is the velocity of the same sound in a pipe of diameter d. For wider pipes of diameter about a metre, the velocity is the same as in the open air.

The velocity of sound in air does not depend upon the kind of sound, nor upon its frequency. When a band is playing at a considerable distance, the harmony of the music is not disturbed. If velocity *varied* with the frequency, music as we know it would be impossible, for the different notes of a chord would not reach a distant ear in the same *sequence* in which they are emitted. If, however, the sound is very intense, as in the neighbourhood of an explosion Regnualt observed that it travelled with an abnormally high velocity.

191. Modern Outfit for the Determination of Velocity of Sound in open Air.—Two microphones (Fig. 133) M_1 and M_2 about 8 killometres apart, are separately connected to two separate primary coils, P_1 and P_2 through separate batteries B_1 and B_2. The two primary coils are surrounded by a single secondary coil S connected to a sensitive mirror galvanometer. When the gun is fired a slight change in the current in the microphone M_1 takes place, which in turn induces a current in S and

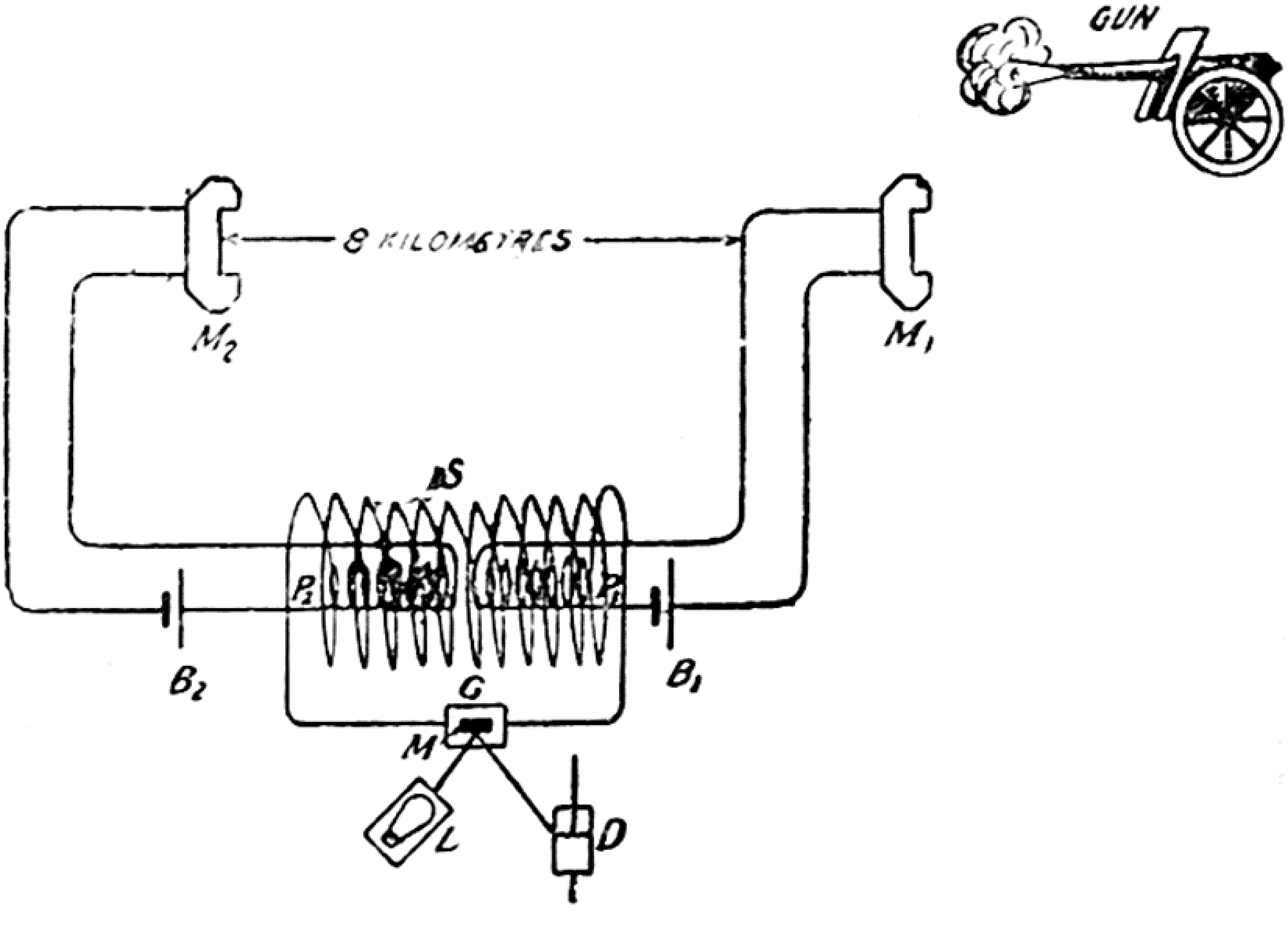


Fig. 133.

a ray of light reflected from the mirror M of the galvanometer is deflected across a photographic film wrapped round a revolving drum D which is rotated at constant speed. The film also receives time signals from a standard tuning-fork. On the arrival of the sound at the second microphone M_2, again an induced current is produced in S and the ray of light is deflected once more. The interval between two deflections being read accurately from the film, the velocity of sound in air is calculated.

192. Velocity of Sound in Water.—The velocity of sound in water is more than four times its velocity in air. It was first accurately determined by Colladon and Sturm in 1823 in the water of Lake Geneva. Two boats were moored on the lake at a distance of about 9 miles. At one boat a bell *B* below the surface of water served as a source of sound. The moment the bell was struck, a flash of light was produced by the gun-powder *P*. The arrival of the sound was detected by a large ear trumpet *T* immersed in water, the small end *E* of which was in the ear of the observer on the other boat. The time interval was about 9 seconds and the velocity of sound in water was calculated to be 1435 metres/sec. at 8·1°C. **Threlfall** and **Adair** showed that the velocity of sound in water increased with the intensity of sound. For a loud sound the velocity was about 35% greater than that for a feeble sound. Experiments to determine the velocity of sound in water were also performed by Regnault in water pipes. His methods resembled those used in air pipes. As the pipe decreased in diameter, velocity of sound in water also decreased correspondingly. For wider pipes of about 1 metre diameter, or more the walls of the pipes ceased to exist as it were, and the velocity remained unaffected.

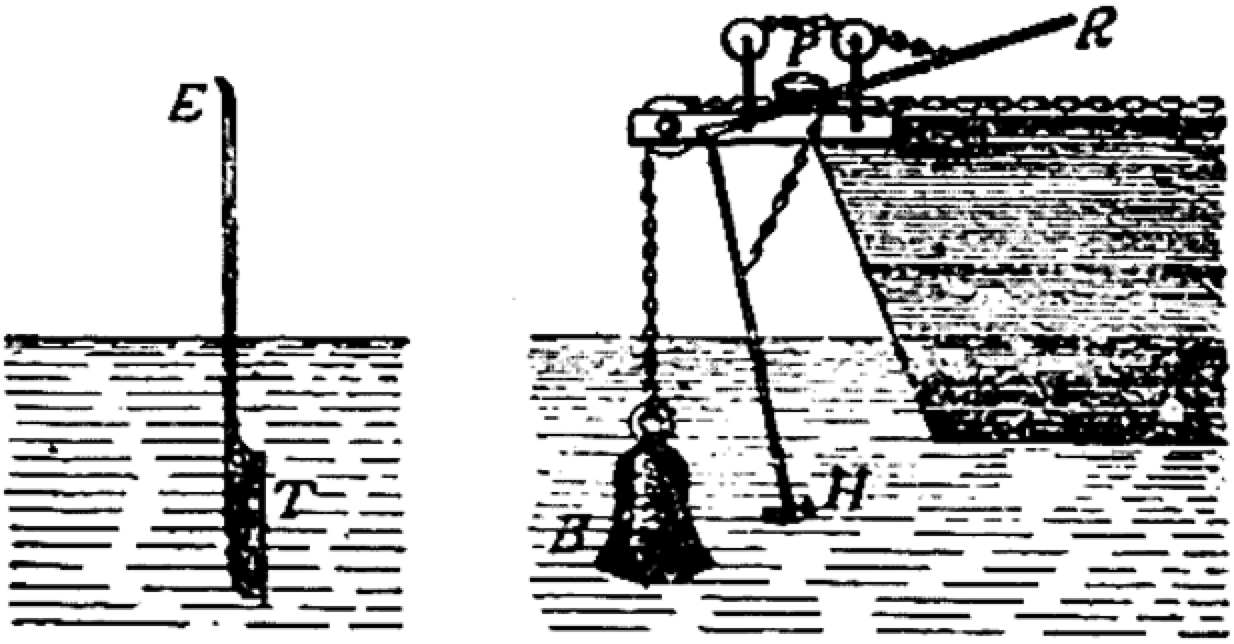


Fig. 134.

193. Velocity of Sound in a Gas by Seebeck's Tube.—As already described in the chapter on Interference, (Art. 73), the points of silence are observed with the given gas inside the Seebeck's tube.

For the first minimum $BC=\frac{\lambda}{4}$ or $\lambda=4.BC$; note that the length is to be measured from the point *C* which is the centre of the side tube L (Fig. 57). No correction for the open end is to be considered and so $V=n\lambda=4n.BC$.

This method has the advantage that a small volume of gas is used, and so its temperature and degree of humidity can be accurately determined.

For finding the velocity of sound in a heavier gas, say CO_2 the tube is kept vertical with the open end A upwards. If the gas is lighter (hydrogen or coal gas), the end A should be downwards.

194. Measurement of Velocity of Sound by Organ Pipes. For this purpose organ pipes of adjustable length are needed. Such a pipe is tuned to a fork of known frequency. This can be easily done by the observation of beats. Having done this ; the length of the pipe is noted. The pipe is then increased in length till it gives the same note in the next higher mode of vibration. The length of the pipe is again noted. The difference between the two lengths is $\lambda/2$. Finding λ in this way and knowing n, the frequency of the fork, velocity of sound in air can be calculated.

195. Hebb's Laboratory Method of finding the Velocity of Sound in Air (1905).—A whistle of known frequency n was

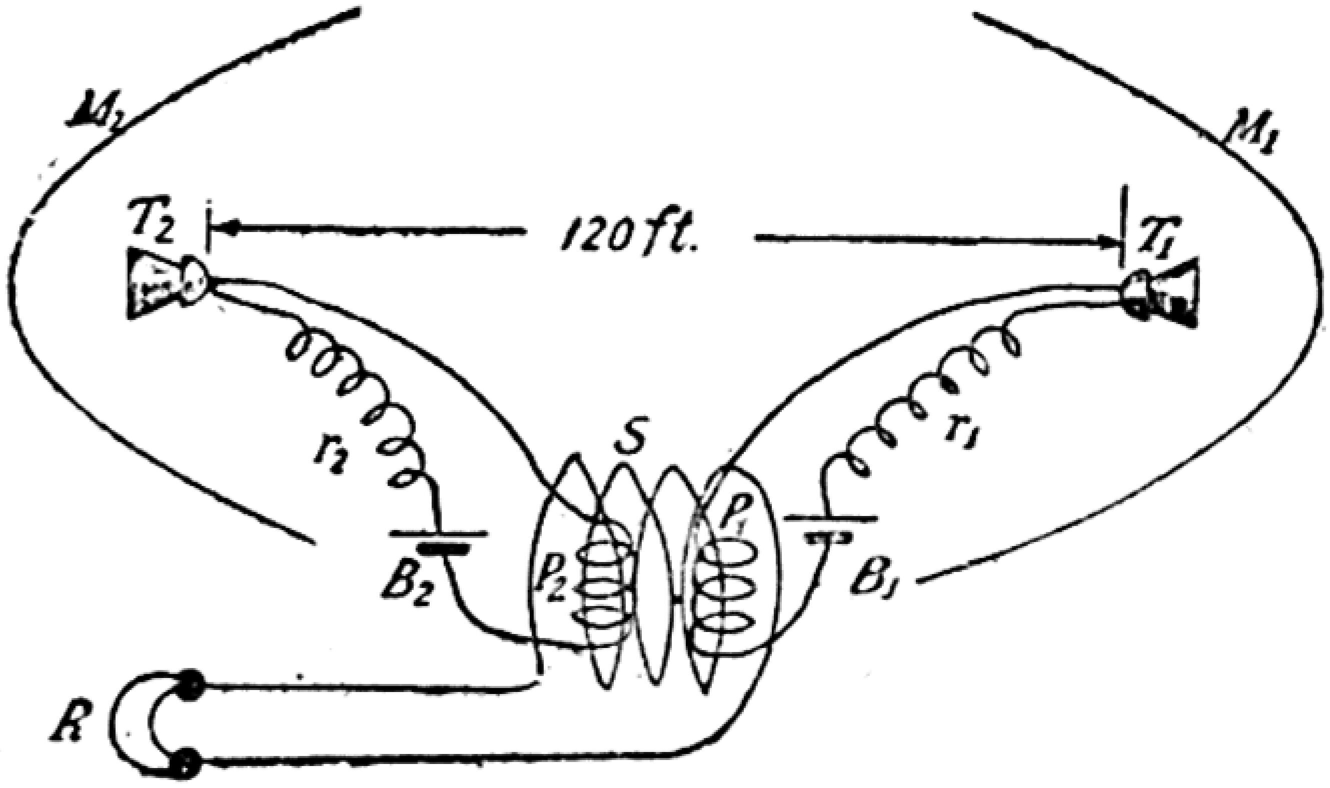


Fig. 135.

blown steadily at the focus of a parabolic mirror M_1 made of plaster of Paris, 5 ft. in aperture and 15″ in focal length. A microphone T_1 was placed near it. Sound waves after reflection from it travelled parallel to its axis and after reflection at a second parabolic mirror M_2, were received by a second microphone T_2 placed at its focus. T_1 and T_2 were connected in electrical circuits as shown in the figure. P_1 and P_2 were the primaries of a special induction coil. S is its secondary to which is connected a telephone receiver R. When the whistle is blown, alternating currents are generated in both the primaries but with a phase difference depending upon the distance between the microphones T_1 and T_2. For example, a compression produced at T_1 gives an inward displacement to its diaphragm and produces an alternating current in P_1 in a particular phase. Suppose the distance between T_1 and T_2 is such that there is reaching a rarefaction at the second microphone T_2, this will move the diaphragm outwards and produce an alternating current in P_2 in the opposite phase. The resultant current produced in the receiver R connected to the secondary surrounding the primaries P_1 and P_2, will be zero and no sound will be heard in the telephone. Now shift the microphone T_2 along with the mirror to the left such that distance T_1T_2 increases by $\frac{\lambda}{2}$. That would mean that when a compression is received at T_1, there would also arrive a compression at T_2 making both the diaphragms move inwards simultaneously, and thus send currents in the primaries in the same phase. Then the telephone will produce maximum sound. On shifting T_2 by $\frac{\lambda}{2}$ again, no sound will be heard as the displacements of the diaphragms will be in opposite phase. Thus the distance between two successive maxima or minima is one wavelength. To have the minima as sharp as possible the effects of the two microphones were adjusted to equality by resistances r_1 and r_2 in their respective circuits. Having measured the wavelength accurately, the velocity is calculated by the formula $V=n\lambda$. This method is free from all objections to long-distance methods stated above, such as errors due to wind, temperature, humidity difference of speed at the source due to very intense sounds, and 'personal equation' of an observer or of a recording device.

196. Kundt's Tube and Dust Figures.—The earlier form of the Kundt's apparatus was a single glass tube A about 5 cm. in diameter and 2 metres long forming what is called a 'wave tube' with an adjustable piston P at one end. The other end is closed by a cap through which passes a wooden rod BD called the sounding rod clamped in the middle at C. The end D is provided with a circular disc, a little smaller than the bore of the tube which must be supported so that its axis and that of the rod are in the same line. Lycopodium powder or fine dust is spread along the dried tube. The free end of the rod is rubbed with a piece of wet flannel to set it into longitudinal vibrations. The ends B and D of the rod will thus be antinodes and the middle point, a node, *i.e.* length of the rod, $l_r = \frac{\lambda_r}{2}$.

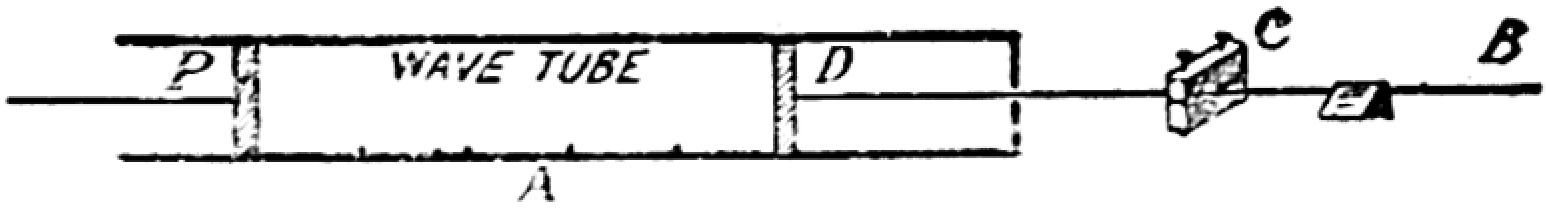


Fig. 136.

or $$\lambda_r = 2l_r$$

where λ_r is the wavelength in the rod.

The disc communicates its vibrations to the air contained in the tube. By changing the position of the adjustable piston the air column is set into stationary vibrations. If the vibrations of the rod are maintained for a short time the powder gathers in small heaps at nodes. The distance between any two consecutive heaps, $l_a = \frac{\lambda_a}{2}$, where λ_a is the wavelength of the note in the air. Thus the wavelength λ_a is determined. If n is the frequency of the note produced by the rod, u_r and u_a are the velocities of sound in the rod and the air respectively,

then $$u_r = n\lambda_r \quad \ldots \quad \ldots \quad (1)$$
and $$u_a = n\lambda_a \quad \ldots \quad \ldots \quad (2)$$

From equations (1) and (2), it is clear that if n, the frequency of the note is known, the absolute velocity of sound in a rod or in air or a gas can be calculated. But we do not generally use the Kundt's apparatus for this purpose, because the glass rod gives a note of such a high pitch that its frequency is not easily determined with great accuracy. Hence the apparatus is mostly used for comparing the velocities in the material of the rod and the air or in different gases

Now dividing (1) by (2), we get

$$\frac{u_r}{u_a}=\frac{\lambda_r}{\lambda_a}=\frac{2\times\text{length of rod}}{2\times\text{dist. between any two consecutive heaps}}$$

$$=\frac{\text{length of the rod } (l_r)}{\text{Dist. between any two consecutive heaps } (l_a)}\ \cdots\quad (3)$$

if u_a is given, u_r the velocity of sound in the rod can be calculated.

When the tube is filled with any other gas (say CO_2) we have as before for the velocity of sound in the gas

$$u_g=n\lambda_g\quad \cdots\quad \cdots\quad \cdots(4)$$

where λ_g is twice the distance between two consecutive heaps

Dividing (4) by (2), we have

$$\frac{u_g}{u_a}=\frac{\lambda_g}{\lambda_a}=\frac{2l_g}{2l_a}=\frac{l_g}{l_a}\quad \cdots\quad \cdots\quad \cdots(5)$$

Thus the velocity of sound in different gases can be compared. If u_a is known, absolute value of u_g, the velocity of sound in CO_2 can be determined. The following precautions must be observed to obtain good results :—

(1) Clean the wave tube with a piece of cotton wool and dry it thoroughly.

(2) A strip of dry lycopodium powder about 2 mm. wide should be used to cover the entire length of the tube.

(3) Rod should be clamped exactly at the centre.

(4) The disc should fit the wave tube as perfectly as possible without touching the tube.

A better way of obtaining dust figures is firstly to strike it gently to bring all the lycopodium into a line along the bottom of the tube. Then rotate the tube about its axis till the lycopodium is just on the point of slipping down. Under these circumstances when the air column is set into stationary vibrations, the dust will fall to the bottom of the tube everywhere except at the nodes. The distance between the consecutive heaps at the bottom or the distance between the centres of the clear spaces near the sides from where the powder fell, could be accurately determined. The objection to single wave tube apparatus is that it is not certain whether the pitch of the note in the case of two gases introduced in the tube one after the other is the same, as a slight change in the temperature will alter the pitch of the rod appreciably. Moreover, it is not easy enough to work with the air or gas at the same temperature. For this reason the later form of Kundt's apparatus with two tubes shown in Fig. 137 is more commonly employed for comparing the velocity of sound in different gases accurately.

197. Kundt's Apparatus with two Tubes.—A and B are two glass tubes of the same cross-section fitted with adjustable pistons P_1 and P_2. The sounding rod D_1D_2 is adjusted with its ends inside the tubes. It is clamped at E_1 and E_2 one quarter of its length from the ends so that D_1, D_2 act as antinodes. E_1, E_2 being fixed act as nodes, and C the middle point acts as anti-node. In other words when rubbed lengthwise, λ_r is equal to the length of the rod (l_r). Discs D_1 and D_2 communicate the vibrations to the air and the gas contained in the tubes A and B respectively, which are set into a state of stationary vibration. Proceeding in a way similar to that adopted in case of a single tube, we measure the wavelengths in the two tubes by noting the distances between the consecutive heaps of powder and using the relation

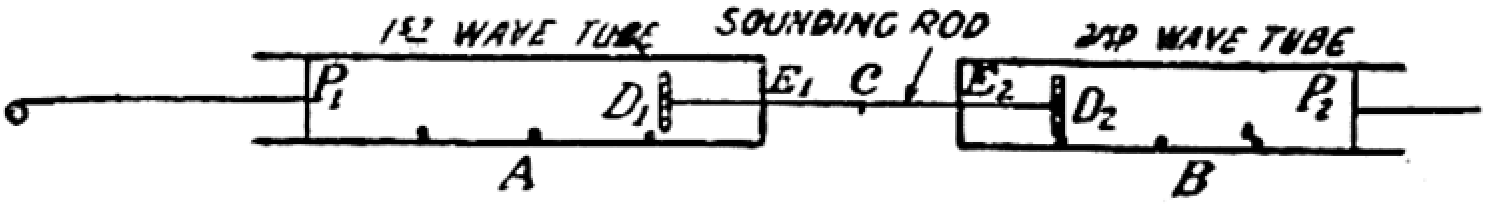


Fig. 137.

$$\frac{u_g}{u_a}=\frac{\lambda_g}{\lambda_a}=\frac{l_g}{l_a}=\frac{\text{Dist. between any two consecutive heaps in the gas,}}{\text{Dist. between any two consecutive heaps in air}}$$

we can compare the velocities of sound in different gases. We can also use the above apparatus for comparing the velocities in the same gas filled in tubes A and B of different diameters.

198. Ratio of the Specific Heats of a Gas by Kundt's Tube.— The velocity of sound in a gas is $\sqrt{\frac{\gamma P}{\rho}}$, where γ is the ratio between its two main specific heats, and ρ is the density of the gas at pressure P. Working at the same temperature and pressure for gas and air we have,

$$u_g=\sqrt{\frac{\gamma_g P}{\rho_g}} \text{ and } u_a=\sqrt{\frac{\gamma_a P}{\rho_a}}$$

i.e., $$\frac{u_g}{u_a}=\sqrt{\frac{\gamma_g}{\rho_g}\times\frac{\rho_a}{\gamma_a}} \quad \text{or} \quad \left(\frac{u_g}{u_a}\right)^2=\frac{\gamma_g\times\rho_a}{\rho_g\times\gamma_a}$$

But $$\left(\frac{u_g}{u_a}\right)^2=\left(\frac{\lambda_g}{\lambda_a}\right)^2 \qquad \text{[Equation (5) on page 275]}$$

Hence $$\left(\frac{\lambda_g}{\lambda_a}\right)^2=\frac{\gamma_g}{\gamma_a}\times\frac{\rho_a}{\rho_g}$$

$$\therefore \qquad \frac{\gamma_g}{\gamma_a}=\frac{\rho_g}{\rho_a}\left(\frac{\lambda_g}{\lambda_a}\right)^2$$

or $$\gamma_g=\gamma_a\times\frac{\rho_g}{\rho_a}\times\left(\frac{\lambda_g}{\lambda_a}\right)^2.$$

199. Velocity of Sound in Liquids by Kundt's Tube.—Kundt's dust tube method can also be employed for the determination of the velocity of sound in liquids. The liquid used in the tube should be air free. In place of lycopodium powder very fine iron filings are used. Figures can be obtained in the same way as in the case of gases.

200. Young's modulus for a solid by Kundt's tube.—Having determined u_r the velocity of sound in the material of the rod from relation (3) as stated above, we can calculate the Young's modulus Y from the relation $u_r=\sqrt{\frac{Y}{\rho}}$, where ρ is the known density of the solid rod.

201. Determination of the temperature coefficient of the velocity of sound in a gas.—Using a single tube, and surrounding it by a steam jacket, we can determine the velocity of sound in a gas at steam temperature as before. Knowing u_{100} and u_0, temperature coefficient $\frac{u_{100}-u_0}{100}$ is calculated.

202. Measurement of Pitch by Cagniard de la Tour's Siren.—Early methods of finding the frequency of a note employed **Savart's Toothed wheel** and the **Disc Siren** (Chapter X). In the case of a **Savart's wheel** when a card piece is held against the teeth, and the wheel is turned, firstly separate taps are heard, but as the speed of the wheel is increased, these blend into a single note. This note is brought in unison with that of the given sounding body. Then the number of revolutions n in a known time t is determined by means of the counting mechanism. If m be the number of teeth in the wheel, the frequency of the note emitted by it will be $\frac{mn}{t}$. This will be the same as that of the given note. In the case of **Disc Siren**, the speed of the disc is so adjusted that the musical note produced by it is brought in unison with the unknown body. The frequency of the musical note is given by the product of the number of revolutions of the disc per second and the number of holes in the disc. The above instruments give only a rough approximation to the frequency. A better instrument is **Cagniard de la Tour's Siren** (Fig. 138). It represents an apparatus in which the wind chest, the disc with holes, and the counter are put together. The wind chest C is a hollow cylindrical metallic box. It is supplied with air from a bellows with a pressure regulator. The fixed top of the chest forms the lower circular disc B of the siren with a number of slanting holes arranged in circles. The upper circular disc D perforated with a similar number of holes slanting in the opposite direction to those in the lower, forms the rotating disc in contact with B. It is mounted upon a vertical axis, the upper end of which terminates in a worm wheel which engages a pair of toothed wheels. Attached to them are two pointers to record the single revolutions and tens of revolutions of the rotating disc. When the stream of air from the lower holes, strikes the sides of the upper holes, it causes the disc to rotate. The speed of the disc is regulated by varying the air pressure. Every time the upper and lower holes come together, small jets of air will emerge from each hole in the upper disc. Of course these small jets issuing simultaneously constitute a single puff. For example, if there are h holes

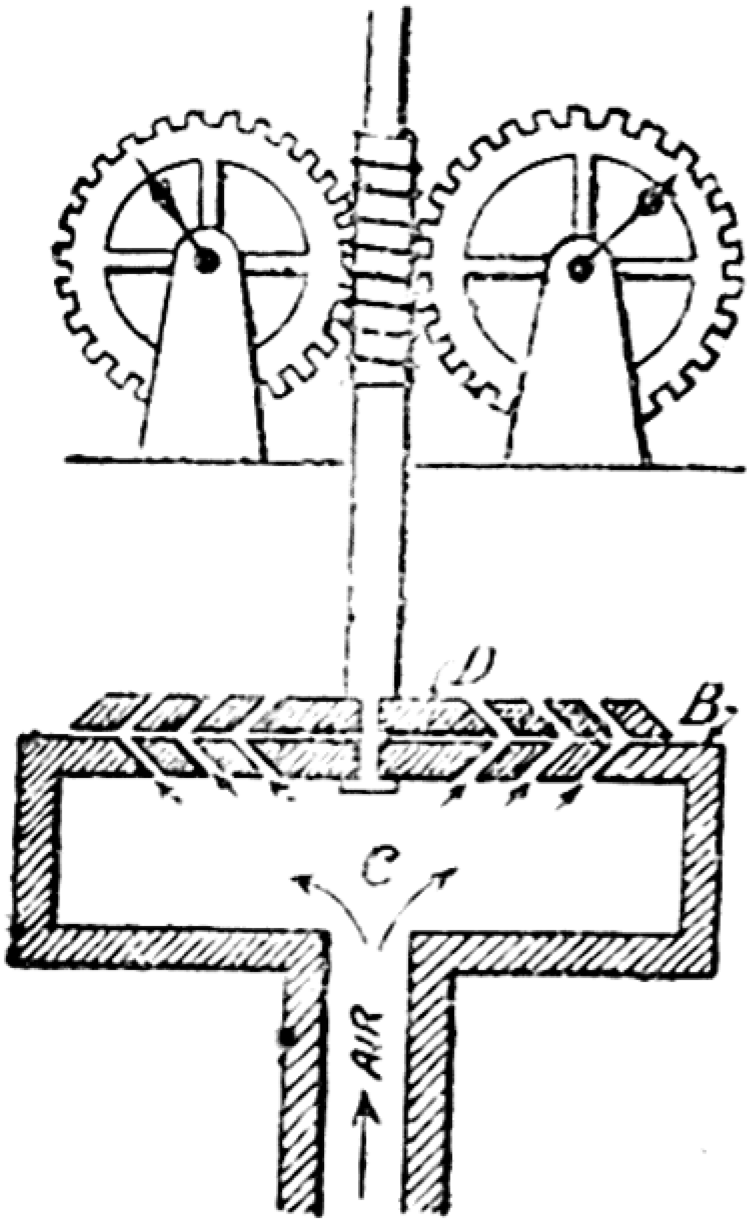


Fig. 138.

in the upper disc, every time the upper and lower holes come together, a single puff made up of h separate jets will be heard. And for each rotation of the upper disc there will be h such puffs. If n is the number of rotations of the disc per second as registered by the counter, there will be nh puffs per second. These successive puffs give rise to a note of frequency nh. To measure the frequency of a source of sound such as a tuning-fork or an organ pipe the siren is regulated in speed by suitable adjustment of pressure until the pitch of the note produced is the same as that of the fork and the frequency is calculated from the relation above.

The air, which is forced into the wind chest of the siren, fulfils the double function of driving the siren and generating the sound. This causes the low notes to be weak and the high ones to be loud. Nowadays the rotating disc is connected to an electric motor whose speed can be easily regulated. In this case we need not have the holes in a slantwise direction.

203. Measurement of Pitch by Sonometer.—In Chapter IV, we saw that the frequency of the fundamental note of a vibrating string is given by $n=\frac{1}{2l}\sqrt{\frac{T}{m}}$, where l is tuning length of the string in cm., T the stretching force in dynes and m the mass per cm. of the string in grams.

A string is stretched on a sonometer box with a known load and a portion of it between two bridges is tuned to unison with the given fork by the method suggested in Art. 51. This is done by adjusting the length of the string until the beats disappear. Then the frequency of the note given by the string is calculated from the above formula. To obtain better results, the following precautionts must be observed :

(1) The pulley in the ordinary sonometer must be quite free from friction, otherwise the tension in the horizontal part of the string will be slightly different from that in the vertical part. In other words, the weights hanging at the free end will not truly represent the tension in the horizontal part. To eliminate this error, the string is so fixed that the whole of it hangs vertically and is held against the bridges by pressing it with a thin piece of wood.

(2) The string must be thin so as not to impair flexibility, otherwise the formula will not be applicable.

204. Measurement of the Absolute Frequency of a Fork by the Falling Plate.—The apparatus consists essentially of a smoked glass plate arranged to fall past the tuning-fork whose frequency is to be determined. The fork F with a light style touching gently against the plate is mounted vertically as shown in Fig. 139 (a) on the block B resting on a base board. To this base board is fixed a vertical beam which supports the smoked plate by a thread hanging from a peg so that by burning the thread the smoked glass plate falls past the fork. The fork is set vibrating by drawing a violin bow across it and the thread is burnt. As the plate falls, the style traces a wavy curve which is the resultant of the S. H. M. in the horizontal direction and a uniformly accelerated motion in the vertical direction. The kind of the trace produced is shown in Fig. 139 (b). Select two consecutive portions AB and BC such that the number of waves in each of them is the same, say a. Evidently the time, t, taken by the plate to fall through the distance AB is the same as that for the distance BC. These distances S_1 and S_2 are measured by means of a reading microscope. If u is the velocity of the plate when the point A passes the style, then we have

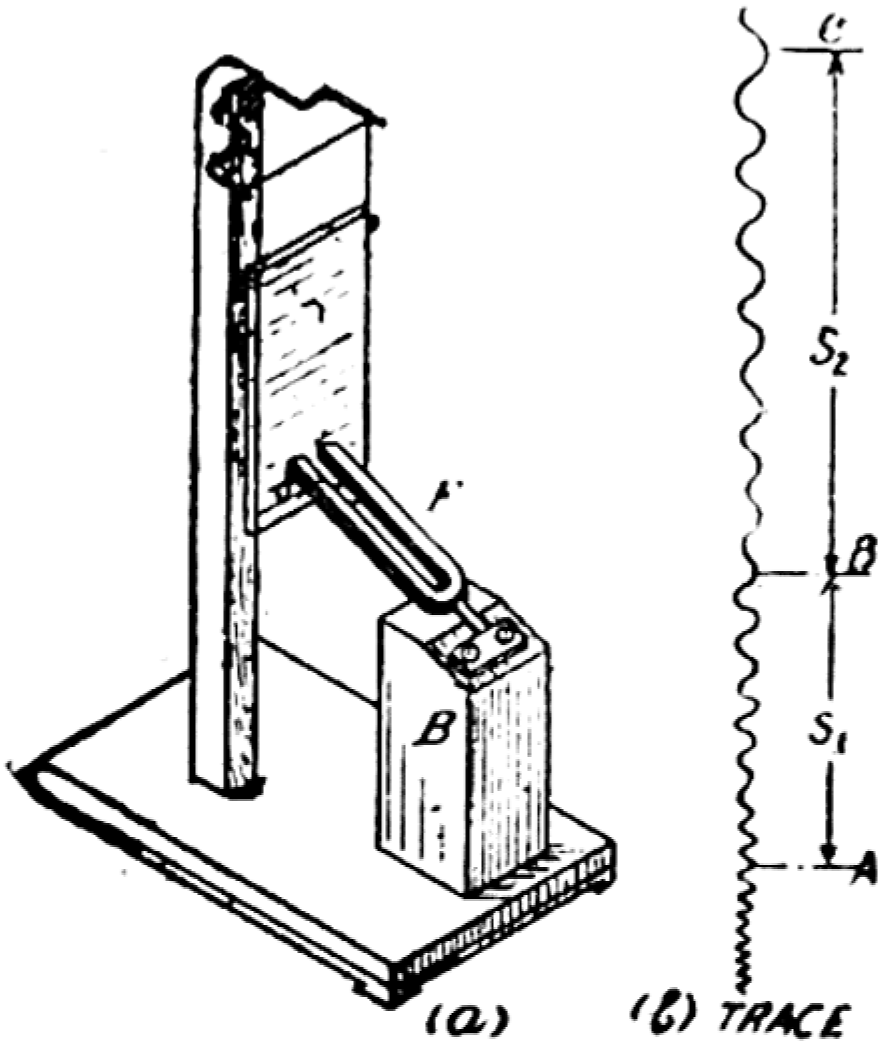


Fig. 139.

$$S_1 = ut + \tfrac{1}{2}gt^2 \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

Also $$S_1 + S_2 = u(2t) + \tfrac{1}{2}g(2t)^2 \quad \ldots \quad \ldots \quad (2)$$

From (1) we have $2S_1 = 2ut + gt^2 \quad \ldots \quad \ldots \quad \ldots \quad (3)$

Subtracting (3) from (2), we get

$$S_2 - S_1 = \tfrac{1}{2}g.4t^2 - gt^2 = 2gt^2 - gt^2 = gt^2$$

$$t = \sqrt{\frac{S_2 - S_1}{g}}.$$

Hence the frequency, n, of the fork

$$=\frac{a}{t}=a\sqrt{\frac{g}{S_2-S_1}} \quad \dots \quad \dots \quad (4)$$

In the above arrangement, there are two sources of error which may be eliminated as under :—

(1) The style attached to one of the prongs increases its inertia consequently the pitch is slightly lowered. For this purpose a second fork is first tuned to the one under test but without the style. Then the number of beats, say b per second, between the second fork and the unknown one provided with a style are counted. Thus, obviously, the frequency, n_0, of the unknown fork without the style = frequency, n, of the fork with the style plus, b, the number of beats per sec.

Hence from (4), we have

$$n_0=n+b=b+a\sqrt{\frac{g}{S_2-S_1}} \quad \dots \quad \dots \quad (5)$$

(2) The *friction of the style* against the plate retards the plate in its fall and so lowers the value of 'g'. Consequently the frequency n, as calculated from equation (4) is a bit higher. To eliminate this error first of all, using another fork of known frequency, the effective value of 'g' is calculated. This value will be a little less than the true 'g'. On substituting this value of g in equation (5), the frequency of the fork under test can be accurately determined.

205. Comparison of Frequencies of two Forks.—In this case the two tuning-forks are clamped on the same block side by side. Light aluminium styles are attached to their respective prongs as in the previous section and held close to the falling smoked plate. The forks are set in vibration, and at the same instant the plate is released. Each style traces a wavy curve upon the smoked plate. Fig. 140 shows traces for a pair of forks. Two horizontal lines AC and BD are then drawn. The traces in between these lines will be made by the forks in the same time. The number of vibrations in AB and CD are counted, and their ratio determined.

This gives us the ratio between the frequencies of the two forks.

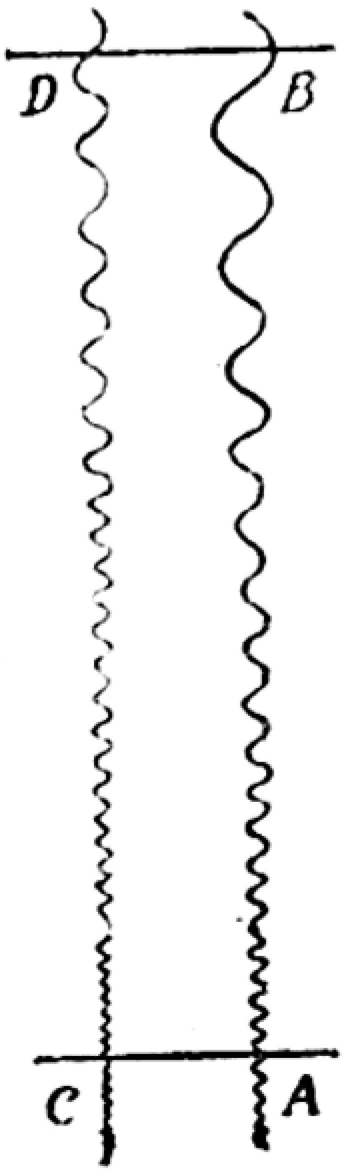


Fig. 140
Traces obtained on a dropping plate.

We can also compare the frequencies of two forks by the following simple **photographic method**. Two tuning-forks are placed side by side, their prongs so arranged as to project into a horizontal slit in a vertical wooden board *B* (Fig. 141). A groove is made behind the board so as to allow a photographic dark slide fall freely downwards when the forks are set in vibration. When an arc light is thrown upon the slit, the shadows of the vibrating forks will be produced on the photographic plate. Two wavy curves like those shown in Fig. 140 are obtained. Counting the number of waves n_1 and n_2 in the same distance for the two curves we have the ratio of the frequencies of the forks $=\frac{n_1}{n_2}$. If one of the forks is a standard one, the frequency of the other can be calculated. Precaution must be observed that the slide may fall with uniform speed. This can easily be managed by connecting the slide by means of a string passing over a pulley to a load dipped in oil.

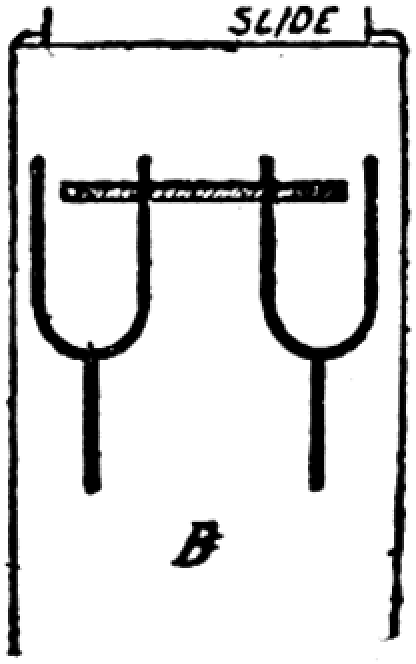


Fig. 141.

A more accurate method depending upon the use of Lissajous' Figures for comparing the pitches of two tuning-forks is explained in Art. 22 Chapter II.

206. Measurement of Pitch by the Revolving Drum Method.—(Chronographic Method). As suggested in Art. 13, when a smoked paper is drawn under a vibrating tuning-fork with a light style attached to one of its prongs, a wavy line is traced on the paper. It is only necessary then to count the number of waves in any interval of time and divide it by the interval to get the frequency of the fork. The form of the apparatus making use of this principle is shown in Fig. 142. It consists of a cylindrical drum *D* which is rotated by a clock work arrangement. A fork *F* with a light aluminium style is adjusted so that the tip of the style touches lightly against a smoked paper wrapped round the drum and traces a wavy curve on it when the fork vibrates. The drum is mounted on an axis with a screw cut on it so that it also travels endways when it rotates to avoid the overlapping of the various turns of the wavy curve. To mark the time intervals we make use of an Induction coil and an accurately timed second's pendulum. *P* and *S* are the primary and secondary of the Induction coil. When the metallic tip of the pendulum *A*, placed in series with the primary coil makes contact with mercury in the cup *C*, a momentary current passes in the primary. This results in a momentary *e.m f.* in the secondary and a spark passes between the style and the cylinder, thus causing a spot on the paper. Now when the drum is rotating and the fork is vibrating, as stated above, a wavy curve will be obtained but marked at intervals with spots or dots showing the ends of seconds (the pendulum being a second's pendulum). The mean number of waves in the spaces between consecutive spots gives the frequency of the fork. The method is very accurate as it is independent of the rate of rotation of the cylinder but merely depends upon the time of vibration of the pendulum which can be determined accurately by the Astronomical method. It should be noted that the pitch determined is that of the fork with the style attached. The style of the fork in rubbing against the paper will necessarily produce some friction, which results in slightly lowering the pitch of the fork.

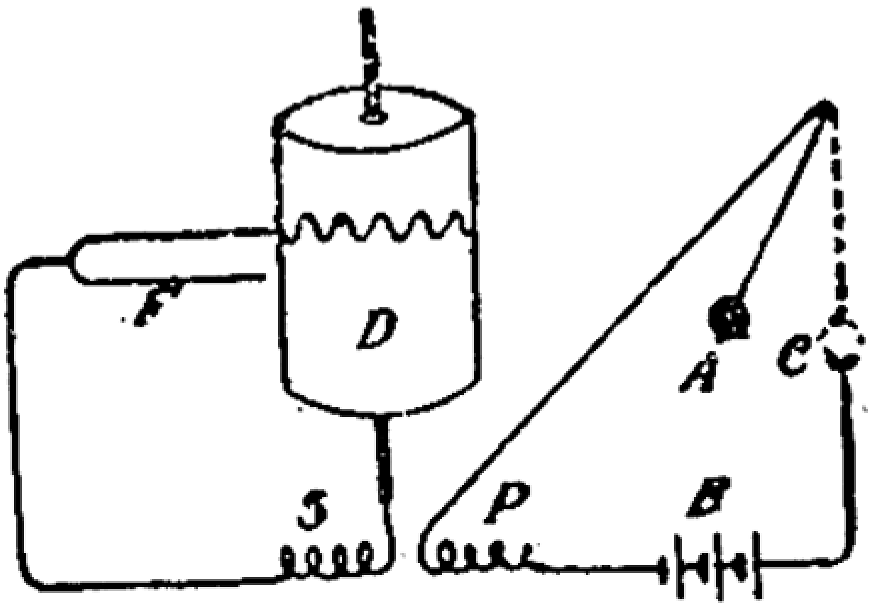


Fig 142.

207. Measurement of Pitch by the Tonometer.—Scheibler devised his Tonometer which consists of a series of tuning-forks extending over an octave, the interval between any two consecutive forks is about four beats per second. This interval can be determined with greatest accuracy. When the beats are very rapid it is difficult to count them. When they are very slow, the exact moment of maximum or minimum intensity is less definite. Now in finding the frequency of any other fork which of course falls within the range of the tonometer we merely compare it with the tonometer forks until we find one with which it beats. The number of beats per second is the difference of their frequencies which is then to be added to or subtracted from the known frequency of the tonometer fork according as the unknown is higher or lower than the known one. To find whether this fork has a frequency higher or lower than that of the known one, load the former by attaching a small piece of wax to one of its prongs. By doing so, its frequency is lowered. Now sound the forks and again count the number of beats per second. If the number of beats per second decreases, this is the faster of the two. But if the frequency of beats increases, obviously it is the lower one.

208. Stroboscopic Method of determining Frequency.—In Fig. 143, *D* is a stroboscope. It is generally a metallic disc provided with one or more circles of equal-sized and evenly-spaced dots or slits. The disc is driven by a motor of adjustable and measureable speed, and is also geared to a counting mechanism. It is arranged to be viewed through a telescope. Between the disc and the telescope is mounted the tuning-fork whose frequency is to be measured.

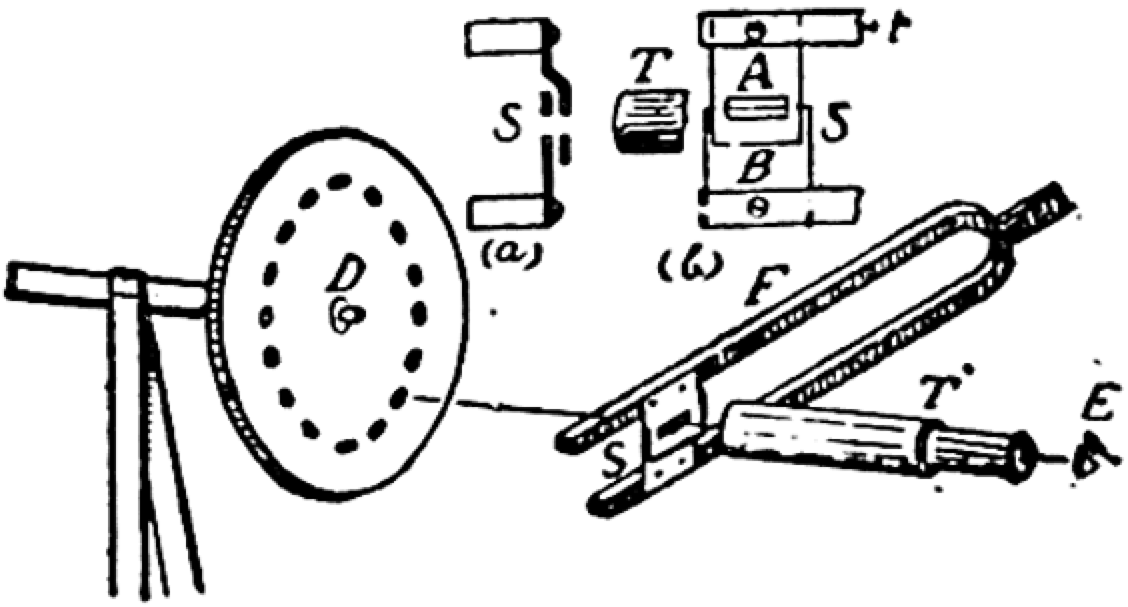


Fig. 143.

Two pieces of light aluminium foil *A* and *B*, shown in section in Fig. 143 (*b*), are attached to the ends of the prongs of the tuning-fork. In the upper piece *A*, a slit is cut, while the lower piece is of such a length that when the prongs are at rest the slit is closed, and when the prongs are separated the slit is open. The figures (*a*) and (*b*) show the case when the slit is open. Thus when the fork is set in vibration, the number of times the slit will be opened and closed is equal to the frequency of the fork. Here, as it is desirable to maintain a tuning-fork in continuous vibration without having to bow or strike it, an electrically driven tuning-fork is used.

To begin with, the slit and the telescope are in a line with the dots on the disc. The disc is set in rotation, and the fork is set vibrating. The speed of the motor can be so regulated that the dots appear stationary when viewed through the slit. This is possible only when the time-period of the fork is exactly equal to the time taken by the dots to move the distance between two consecutive dots. When the disc is driven below this adjustable speed the dots will appear to move slowly backwards and when driven rapidly, the dots will be appear to move forwards*. Thus by careful adjustment, the dots can be kept stationary for a considerable length of time.

If n=the frequency of the fork, its time-period$=\frac{1}{n}$ second.

If m=number of revolutions of the disc as recorded by the revolution counter. and p=the number of dots in the circle on the disc, then the time taken to move the distance between two consecutive dots$=\frac{1}{mp}$ second.

When the dots appear stationary, we have

$$\frac{1}{n}=\frac{1}{mp} \text{ i.e., } n=mp.$$

It should be noticed that if the disc be driven with twice or three times etc. the above speed, the dots will again appear stationary. Then $mp=2n$, $3n$ etc. because each dot would move twice ar thrice etc., the distance between successive dots for two consecutive openings of the slit. Such cases should be carefully distinguished from each other. The results obtained by the stroboscopic method are fairly accurate.

If the frequency of the fork be known, we can also apply this method to accurately determine the frequency of revolution of the disc. It also serves as a delicate method of observing whether the speed of the disc or motor remains constant.

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*Here we have considered a single circle of dots evenly spaced. In case there are more than one concentric circles of dots, it will be noticed that the dots in a circle inside the stationary circle would appear to move forward in the direction of the rotation of the disc and those in an outer circle in the backward direction.

CHAPTER XVII

ACOUSTICAL INSTRUMENTS AND APPLICATIONS

209. There are a number of acoustical instruments which have been devised to record and reproduce sounds. Some of these such as the Bell's telephone, microphones of various types, loud speakers, which involve applications of electricity have also been briefly outlined. For their detailed study some advanced treatises on 'Magnetism and Electricity' and 'Sound' may be consulted. Besides acoustical principles have been applied in the construction of several other instruments which are used in every day life and in industry. In the following sections only broad principles have been discussed.

210. The Phonograph.—When sound waves fall on a thin diaphragm, it oscillates. Conversely, if the diaphragm is made to vibrate as before, similar waves will be generated in air and the original sound will be reproduced. This is the underlying principle of a Phonograph. It was first invented by Edison in 1877. In its original form it consisted of a diaphragm *D* made of mica with a sharp stylus *S* called cutter. The pointed end pressed against a rotating cylinder of hard wax moving along its axis. *M* is a mouth-piece where the sound waves enter and set the diaphragm in vibration. The stylus ploughs its way through the wax, and records a furrow of varying depth in accordance with the varying displacement of the diaphragm. If now the stylus is placed at the commencement of the record and the cylinder again is set in rotation, the diaphragm vibrates in the same way as before and the original sound is reproduced.* The phonograph in its original form has now been altered, the general principle of the instrument remaining the same. The recording is made on a circular disc made of wax which rotates about its centre, the recording point gradually moving inwards. By a suitable arrangement the stylus in this case is made to move laterally and not up and down, thus cutting on the wax disc a wavy lateral groove of uniform depth running spirally from the edge to the centre.

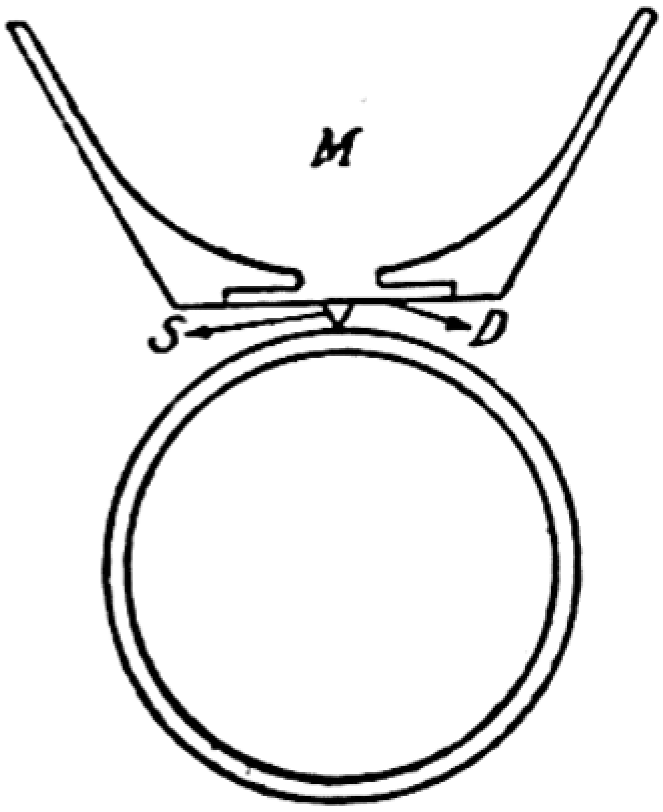

Fig. 144.

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*The phonograph has been successfully used to test the theories of the origin of Vowel sounds; (see Art, 142).

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After the completion of the groove a copper mould of this, called the '*electrotype*,' is made by the electrical process of depositing a thin layer of the metal on the surface of the wax disc which is made conducting by sprinkling graphite on it. This thin copper mould having ridges in place of the corresponding grooves on the original disc is backed by a steel plate to give it strength. Thus the *master record* is ready. Gramophone record discs are made from a mixture of shellac, shredded rags, rotten stone, sulphate of barium and lamp black etc. These ingredients are ground together, and passed between heated rollers, thus giving the 'smooth black plates', which when cold are very hard. To obtain a true copy of the wax record, the master record is then heated and the smooth black plate is squeezed on it in a hydraulic press. This gives us the usual records ready for sale. To reproduce sound, the needle of the *sound box* is made to run in the groove of the record, which causes the diaphragm of the sound box to vibrate, in the same way as during the recording process thereby exactly reproducing the original sound. This is the principle of the gramophone.

211. The Gramophone. Its general appearance is shown in Fig. 145. It is a modification of the Phonograph. The usual sound record is placed on the turntable of the gramophone which is made to rotate about a vertical axis by a simple spring motor inside the case. The most important part of a gramophone is the **sound box** shown in Fig. 146.

It is an air chamber bounded on one side by a corrugated metallic diaphragm made of an alloy of aluminium supported between two solid rubber rings *RR* round its edge.

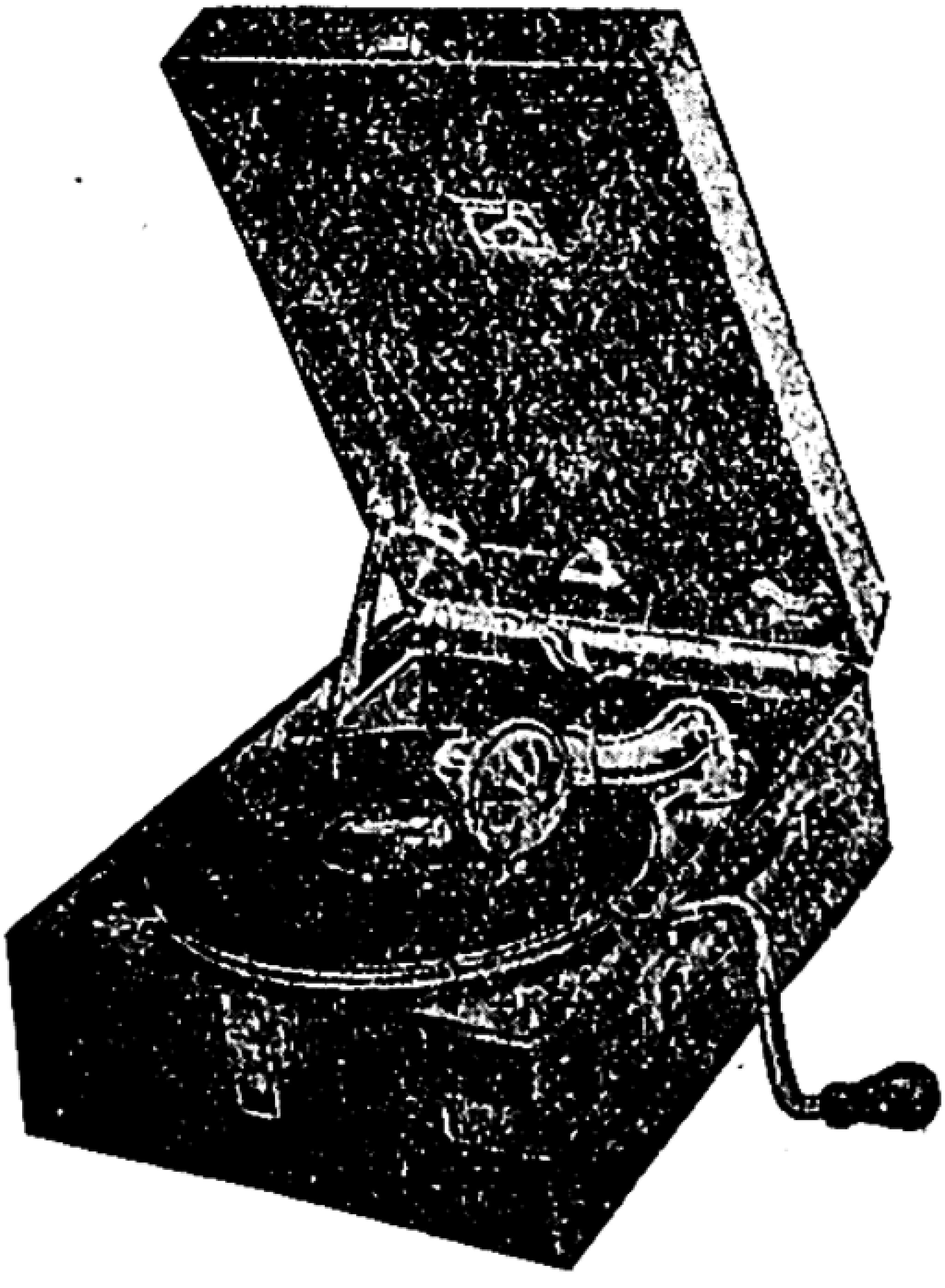


Fig. 145.

The other side is open and coupled to the narrow end of a horn. A bent lever *L* pivoted at *P* has its upper end fixed to the centre of the diaphragm while the lower one ends in a screw *S* by which needle *N* is fastened. To reproduce the sound needle rests on the record which is made to rotate. The wavy groove cut on the record compels the needle to move to and fro laterally in exact imitation of the waves which originally fell on the recording diaphragm and these movements are transmitted through the lever to the mica diaphragm of the sound box. These vibrations of the diaphragm set up waves of compression and rarefaction in the air thus exactly reproducing the original sound. As the upper arm of the lever is much longer than the lower arm, the side wise motion of the needle *N* causes magnified vibrations in the diaphragm.

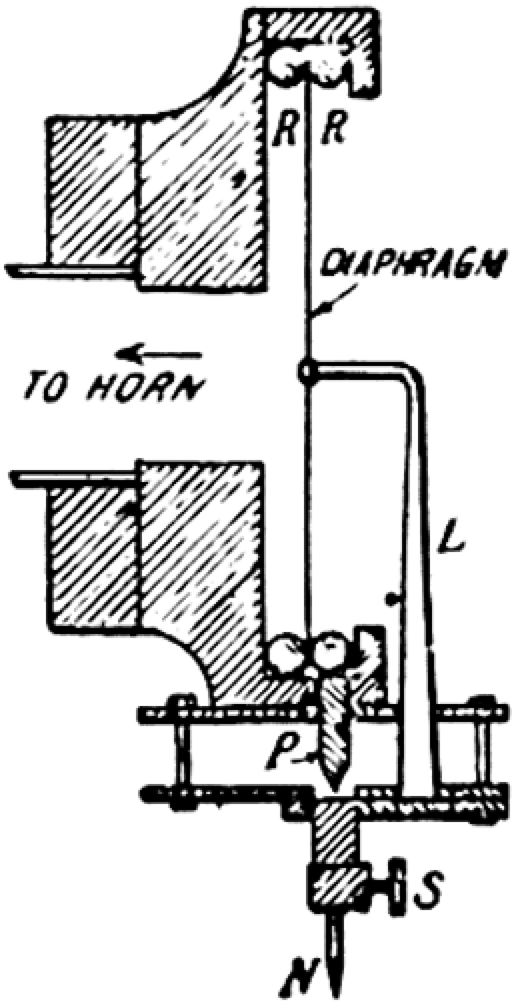


Fig. 146.

Formerly when a horn was used for collecting the energy and concentrating it on a diaphragm, the amount available was very small. Singers and performers had to be huddled together very close to the horn. This caused many difficulties. To overcome them the mechanical energy system has now been replaced by electrical recording (Fig. 147). This goes a great deal to improve the reproduction. The sound is first picked up by a microphone. By using thermionic valves the varying microphonic current, after suitable amplification, is passed on to a coil surrounding a soft iron armature which has a small freedom of motion about a horizontal axis. The soft iron armature which is supported with its upper end between the poles of an electromagnet, moves laterally, in accordance with the fluctuations of the speech current, and the cutter fixed to its lower end traces a spiral groove on the wax disc with lateral irregularity. From this the usual record is prepared as described in the previous section.

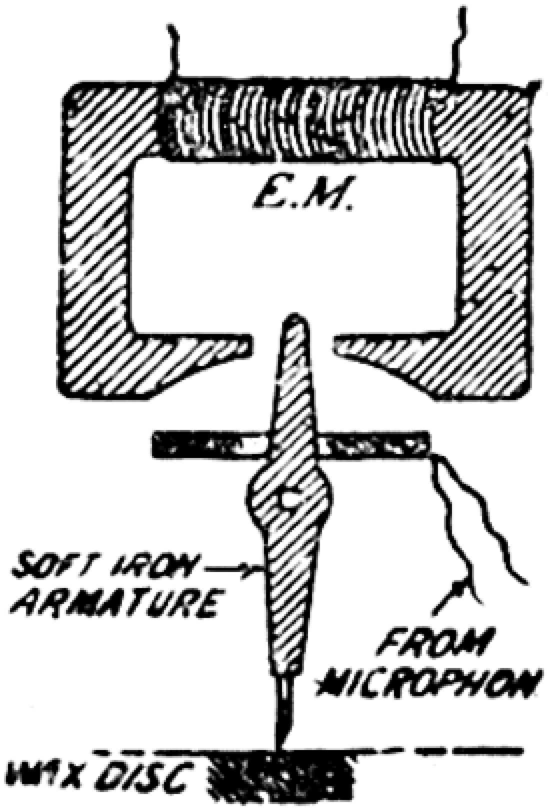


Fig. 147.

Reproduction from a gramophone record can also be made *electrically*. In this case the sound box and the horn are substituted by an 'electric pick up', an amplifier and a loud speaker. The 'pick up' is a microphone actuated by the gramophone needle. The electrical currents thus produced are amplified by thermionic valves and are then delivered to the loudspeaker.

Now we shall describe some of the electrical instruments in wide use for the transmission and reproduction of sound such as the *Microphone*, the *Telephone*, and the *Loud Speaker*.

211a. The Microphone.—The microphone is essentially an arrangement for the conversion of sound vibrations into variations of electrical current. In the telephone communication system it is used as a transmitter and the variations of electrical current thus produced are reconverted into sound by the receiver at a distant end. It is also the first element of a loud speaking equipment or a broad-casting arrangement in which electrical oscillations after proper ampilification are reconverted into sound by the loud speaker. Finally as described in Art. 224 it is the first element of all processes for the electrical recording of sound either mechanically on disc, optically on film or magnetically on steel tape. The general principle of the modern carbon microphone is shown in Fig. 147 (*a*). Carbon granules are enclosed in between two plates one of which is fixed and the other serves as a diaphragm which responds to rapid changes in pressure. The plates are placed in series with a key, a battery and the primary of a transformer, the secondary of which is connected to the telephone receiver. When the key is closed a steady small current flows through the circuit. If a person speaks in front of the diaphragm, it is set into mechanical vibrations thus compressing and decompressing the granules which vary the resistance of the circuit and hence the current. This modulated electrical current passes through *P*, the primary of a transformer and produces by induction a corresponding varying current in *S* its secondary. This amplified current passes through the telephone *T* and excites its diaphragm. The

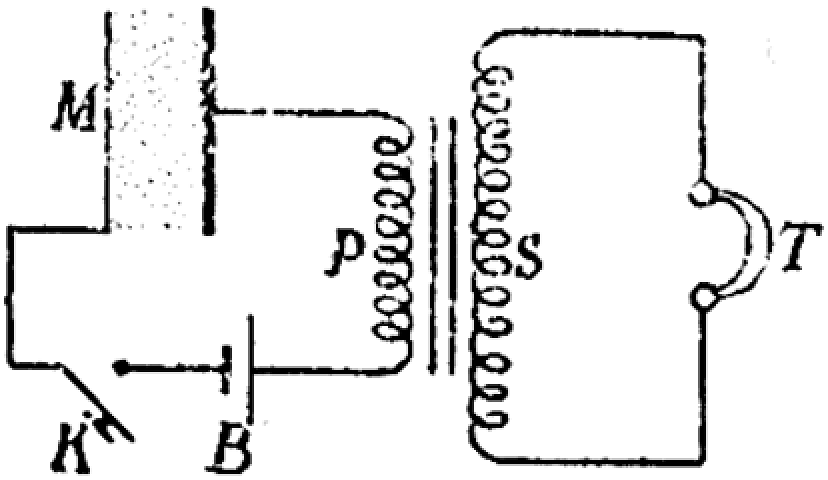
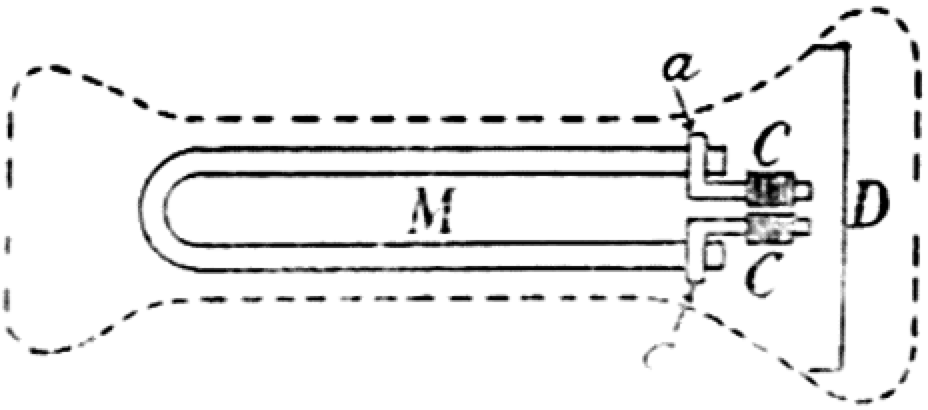


Fig. 147 (*a*).

movements of the latter set the air in corresponding vibrations reproducing the original sound.

The **Electrodynamic Microphone** is another tpye of microphone. It consists of a small coil of wire attached to the back of a freely moving plate. The coil is in the magnetic field between the central pole piece and the peripheral pole piece of a "pot" magnet. The sound waves cause the plate end of the coil to move and varying currents are induced in the coil which are conveyed to the distant end. The instrument possesses a good frequency response and besides being so robust and reliable it is very sensitive. This is largely used in present day Broadcasting.

211b. The Telephone.—The telephone was invented by Graham Bell and with some slight modifications it is still used as a receiver in telephone communication. In a modern form it consists of a permanent horse shoe magnet *M* with its two poles close together. Each pole has a soft iron armature '*a*' attached



Fig, 147 (*b*)

to it and a flat coil of many turns of fine wire is wound round ch armature. The outer ends of the two armatures attract thin soft iron disc *D* situated just in front of them. The rying currents produced by the transmitter at the sending tion and suitably amplified by transformer are sent through e coils C.C. of the receiver and cause corresponding changes in e strength of the magnet of the receiver and thus vary its pull the soft iron disc *D*. The diaphragm of the receiver, therefore vibrates according to the varying current that is, according to the words spoken and these vibrations being communicated to the ear set up sound waves reproducing the original sound in pitch and quality.

211c. The Loud Speaker.—The present day student is familiar with a loud speaker so commonly used in picture houses for the reproduction of sound on a talking film, in loud speaking equipments and as a necessary adjunct of a wireless receiver. On account of the various requirements that a loud speaker must satisfy for faithful sound reproduction, it has under gone numerous modifications since the early days of wireless reception. The commonest and the efficient type of speakers used now-a-days is the moving coil type. When properly matched to the output valve of the receiver or the amplifier it works remarkably well. It is based on the principle that when a variable current is passed through a conductor in a magnetic field, the conductor is acted on by a variable force and if the current is oscillatory the conductor is set in vibration.

The moving part of the apparatus consists of a small coil called 'the voice-coil' wound on a cylindrical strip to which the variable current output of the receiver is led. The coil is free to move in the annular gap between the central pole piece and the peripheral pole piece of a 'pot' magnet designed to produce strong magnetic field in it. The coil is attached to a conical diaphragm made of parchment with circular corrugations and supported round the periphery by a flexible annular strip of leather or rubber. When the variable current passes through the coil in the magnetic field, it causes varying movement of the coil which is connected to the diaphragm. The diaphragm is thus set into-vibrations which are communicated to the air and the sound is reproduced.

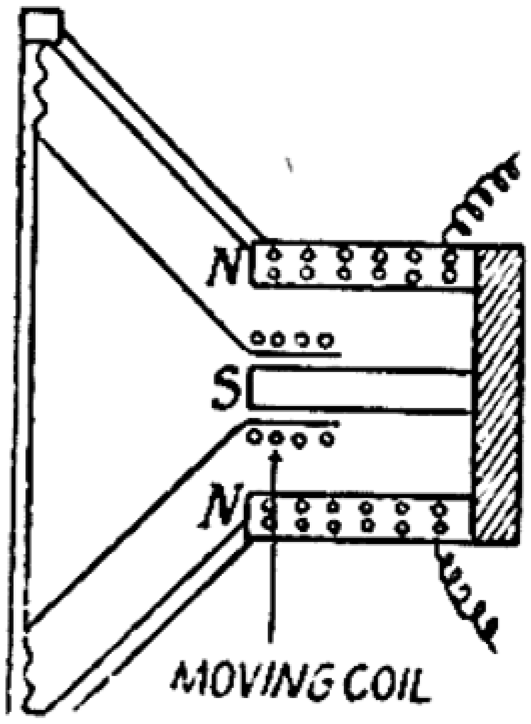


Fig. 147 (c).

The relation between the current and the driving force is linear and force is independent of the position of coil in the gap for considerable movement. When suitably designed a fairly uniform response from 80 p.p.s. to 1000 p.p.s. is secured. It is capable of radiating large power without appreciable asymmetric distortion.

In one type the magnet producing the field is a permanent magnet and these are known as '*Permanent magnet moving coil speakers*', in another type current is taken from the D.C.

mains to magnetise the magnet and these are spoken of as '*Electrodynamic moving coil speakers*'. The latter type has a powerful output.

212. The Phonodeik.—With this apparatus sound waves can be projected on a screen and demonstrated to a large audience. By its use it is possible to photograph the wave forms of various musical instruments or of the human voice. Its principle is the same as that of a gramophone. It consists of a thin diaphragm *D* made of mica (Fig. 148) fixed to a metal frame *F*, and supported between two solid rubber rings *RR* round its edge similar to those in the sound box of a gramophone. To the centre of the disc is tightly screwed a steel stylus *S* flattened at the other end which presses a rolling thin needle placed over a platform *P*. A concave mirror of long focal length is attached to the needle as shown in Fig. 148 (*a*). Having constructed such a sound box, it is fixed to the 'tone-arm' at the narrow end of a horn. When speech is delivered in front of the horn, sound waves fall on the diaphragm and cause it to vibrate. This makes the needle to roll backwards and forwards. Consequently, the mirror is tilted, and a beam of light directed towards this mirror is reflected to a revolving mirror which in turn throws it on to a distant screen. A curve is traced on the screen which is characteristic of the original sound. If a permanent trace is required, a photographic plate sliding horizontally gives the record of the wavy curves as shown in Fig. 149 (*a*) and (*b*) which represent the vowel sounds '*oo*' and '*ee*' as in *mood* and in *feed* respectively. The overtones in the two curves may be clearly marked.

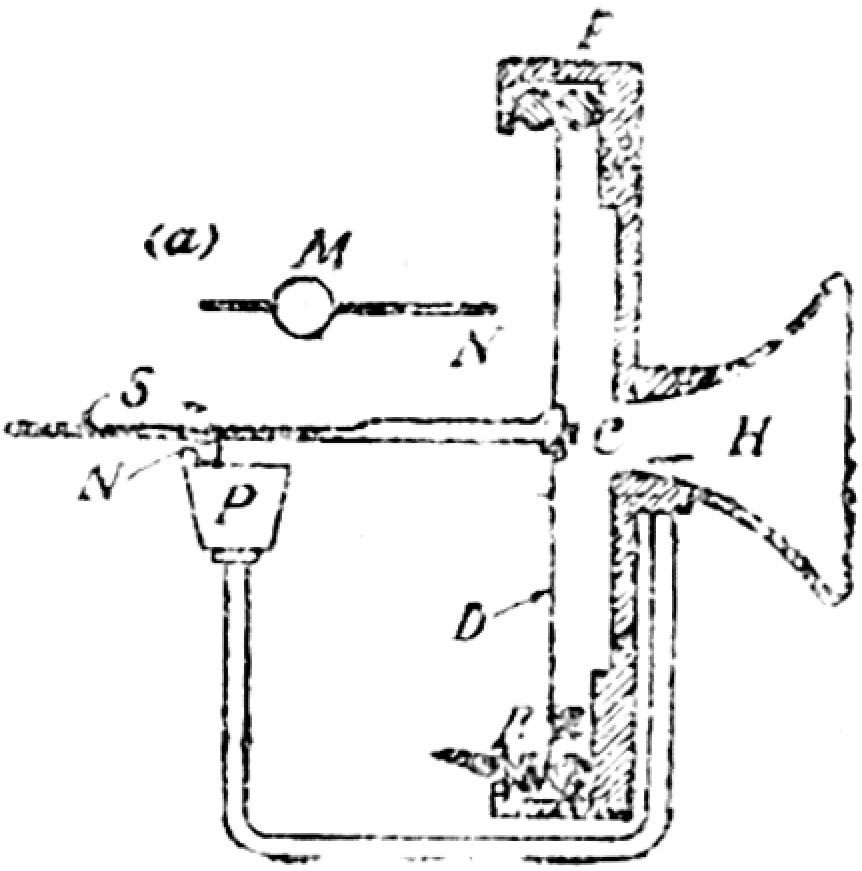
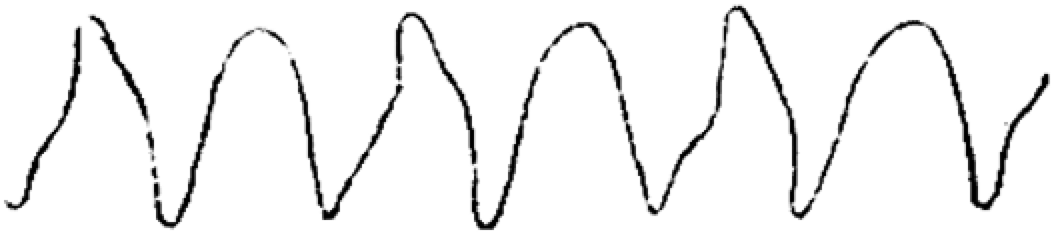
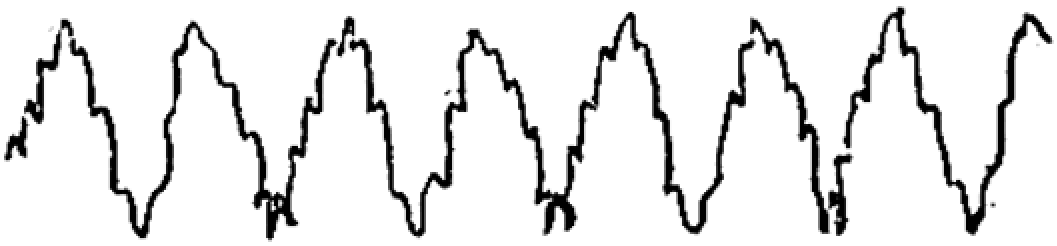


Fig. 148.

213. The Phonoscope.—The phonodeik was developed by Prof. D. C. Miller. Earlier Dr. J. Erskine-Murray devised a phonoscope. The sound box represented a conical tube *ABCD*. The smaller end is covered by a thin membrane* which may be somewhat slack. To its centre a light support is fixed. Attached to it is a wire *F* with a small light mirror *M* hinged on it but quite free to move without shake. As before when sound is directed into the conical tube, flutter of the membrance rocks the mirror about its axis. A beam of light reflected at the mirror *M*, and then at the revolving mirror *R*, traces a wavy curve on the screen. If necessary the wave form can be photographed as before. By one of the experimental methods described above, wave forms of various musical sounds can be studied, and thus the quality of the notes determined.



(a)



(b)

Fig. 149.

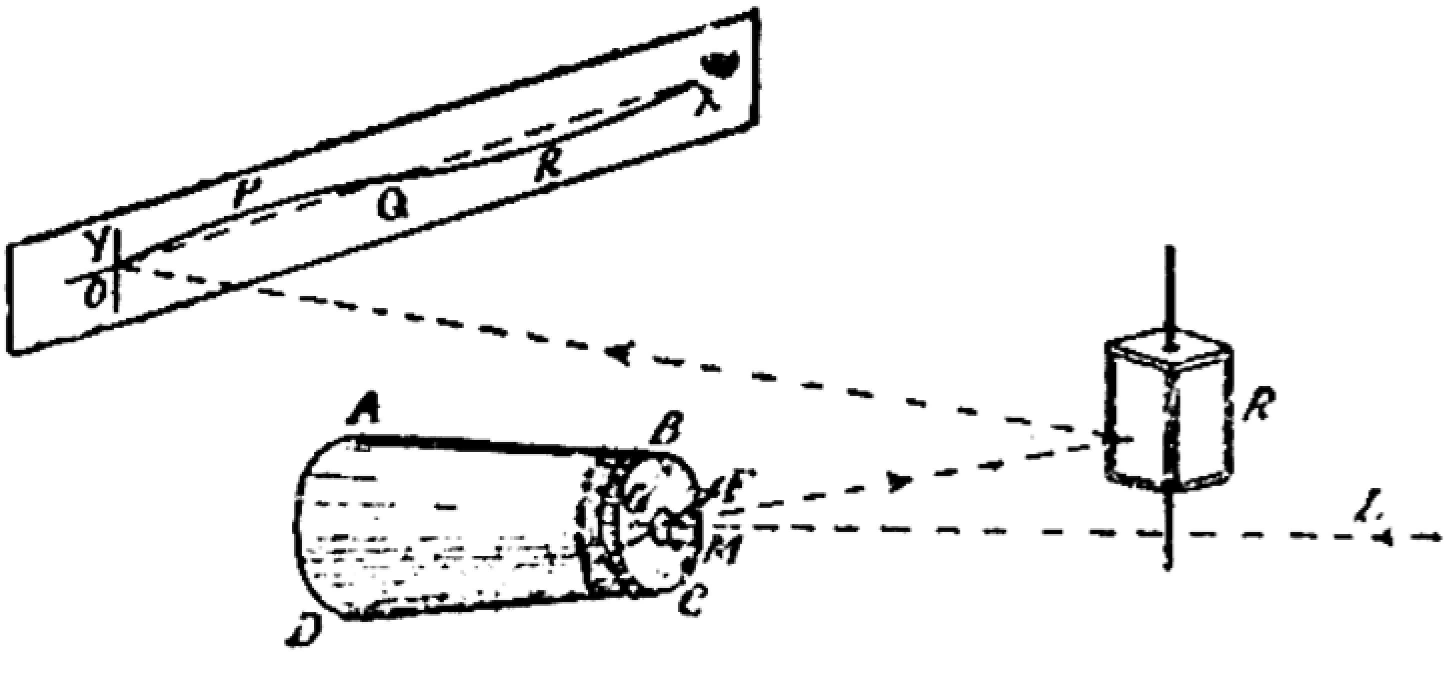


Fig. 150.

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*The diameter of the memberane should be chosen with reference to the pitch of the original sound.

214. The Seismograph.—It is an instrument used for recording earthquakes. When a quake arrives at a place, everything on its surface moves in the same manner as the earth does. Hence to record these movements, we should have a body which is relatively at rest with respect to the earth *i. e.*, a body with a large time-period. This is only possible when the inertia of the body is very large. For this purpose a massive weight W is screwed to the end of a rod CW (Fig. 151) called, the 'boom'. The other end of the boom is fixed at C at right angles to the axis AB which is inclined to the vertical at a very small angle about 6′. Under the circumstances when some motion of the support occurs, the weight does not immediately follow its movement. D represents a cylinder, over which a paper film is rolled steadily. On the arrival of the quake, the earth oscillates. Along with it, the support, the axis and the cylinder also oscillate, and the pointer of the boom which moves only slightly draws a trace on the paper plane. The record obtained is due to the failure of the weight to follow the motion of the support. It indicates relative movement between the massive weight (W) and its support.

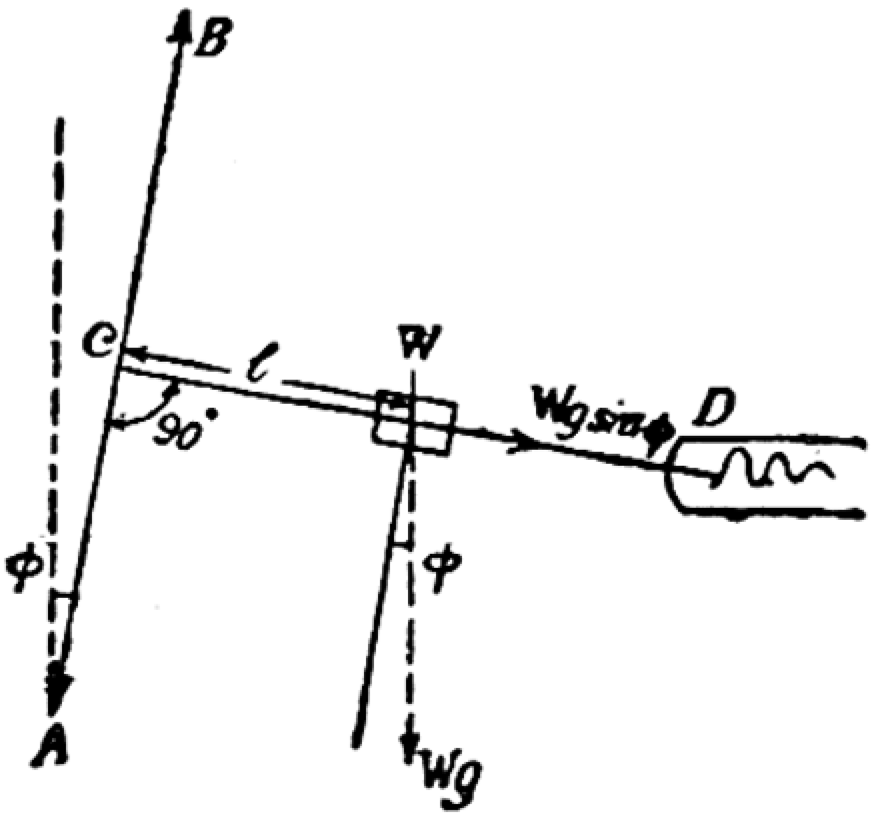

Fig. 151.

Really speaking, the boom with the massive weight W at its one end acts as it were a horizontal pendulum of length l with the point of suspension at the other end of the boom. Force along CW in the position of rest is $Wg \sin \phi$. On being slightly displaced, the restoring force is $Wg \sin \phi . \sin \theta$, where θ is the angular displacement.

$\therefore$ The restoring couple$=Wg \sin \phi \sin \theta \, l = Wgl\phi\theta$.

[$\because$ angles ϕ and θ are small.]

Hence the period of vibration of the system is given by

$$T=2\pi\sqrt{\frac{I}{Wgl\phi}}$$

where I is the rotational inertia of the system about AB.

As I is large and ϕ is small, *so T is very large* satisfying the condition mentioned above. The seismographs are also employed to pick up sound waves. They are similar in construction to those used for recording earthquakes (see Art. 215).

215. Prospecting for Oil.—The best way to know what is under the ground is to drill a hole in it. But the drilling business to find out whether oil is present or not, is often very expensive. Within the past few decades a number of physical methods have been used for studying the structure of the upper layers of the earth's crust in search for minerals and oil. These methods depend upon the gravitational, electrical and magnetic properties of the medium underneath. Sometimes we employ compressional waves produced by artificial explosions for the purpose. These compressional waves are sound waves, and the instruments used for detecting them are *Seismographs* described in the previous section. We do not directly aim to locate the oil itself but seek certain geological structure, such as 'salt dome', with which oil is commonly associated.

Fig. 152 represents a simple case. AB is the level of the ground. CD is the upper surface of the 'salt dome'. A charge of dynamite is exploded at E and compressional waves travel outwards in all directions. Velocity of these waves in the upper medium (say clay and sand) is greater than that in air, and in the lower medium (salt dome) is still greater*. When these waves reach the salt dome they are partly reflected and partly refracted. As the intensity of the reflected waves is very feeble,

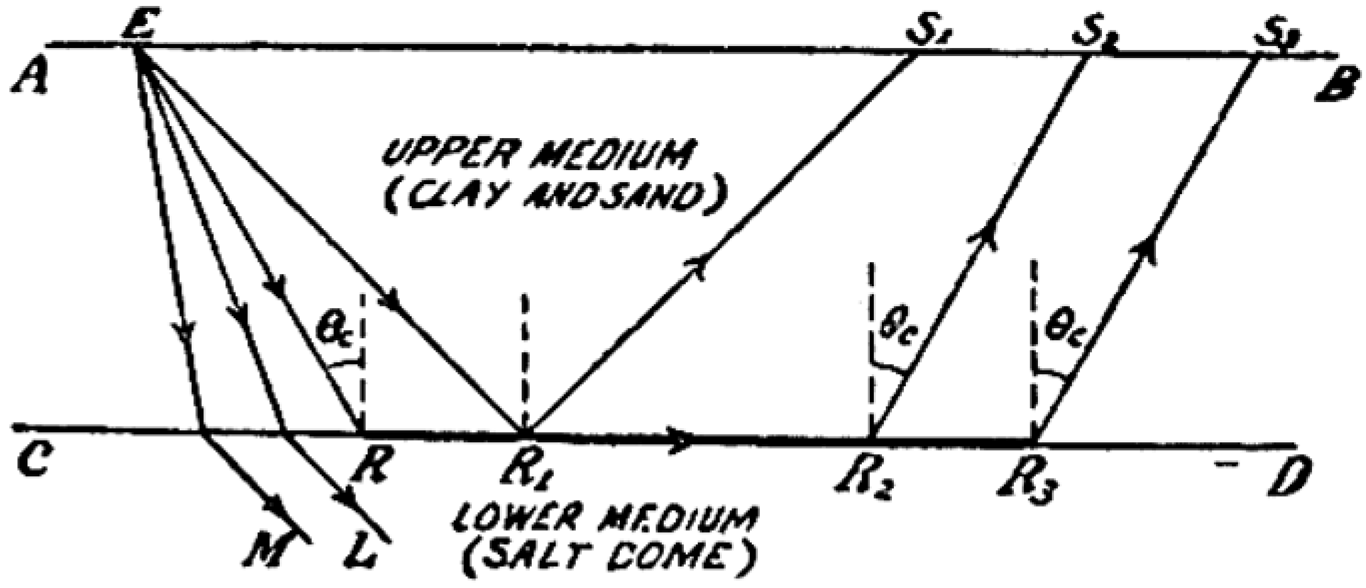

Fig. 152.

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*In a salt dome the velocity (V_2) is 5300 metres/sec. while in sand and clay its value (V_1) is 2000 metres/sec. $\therefore$ $\sin\theta_c=\frac{V_1}{V_2}=\frac{2000}{5300}=\frac{20}{53}$.

refracted rays (RR_1R_2 etc.), are made use of. It is found that a certain ray ER incident at the critical angle θ_c on the surface CD, travels parallel to the stratum, and a fraction of it goes back to the surface AB by diffraction making the same angle with the normal to the stratum. It travels in the directions as shown by the lines R_2S_2, R_3S_3, and may be picked up by seismographs placed at S_2, S_3. The time interval between the arrival of the direct wave along $ES_1S_2S_3$ and the diffracted wave along R_2S_2. R_3S_3, etc., is noted. When the direct distance ES_2 is small the direct wave reaches first and the diffracted wave later, but as the distance of the point of observation S_2, increases ; the interval decreases, the reason being that velocity of sound in the lower medium is greater than that in the upper medium. With a sufficient number of seismographs placed at different distances, it is possible to determine the distance where both these disturbances (the direct and the refracted compressional waves) arrive at the same time. And knowing the speed of the compressional waves both in the salt dome and in the stratum (clay and sand) lying above, θ_c is determined, (see foot note p. 295). Then it is possible to calculate out the depth of the salt dome and drilling can be done with much probability of striking oil than if the seismic methods had not been employed.

216. Detection of Aircraft by Sound.—We owe our power of locating sound to our two ears, but the distance between them being small, difference of intensity at them will not be perceptible unless the source is very close to the listener. The difference in phase at the two ears is, however, not small. Experiment shows that for the lower pitches our judgement of direction comes from the difference in *phase* of the sound at the two ears. Such phase differences produce certain time differences in the impulses in the nerve fibres going from the two ears to the brain and these time differences apparently give a clue as to the direction from which the sound comes. When the sound comes from the front or the back, phase difference is zero. It is greatest when it comes from one side of the listener. If we could increase the distance between the ears, difference of phase effect could be increased. Such a separation of the ears was made use of during the War of 1914—1918 for the location of Aircraft.

Two conical trumpets lying a few feet apart, have their axes parallel and are fixed on a framework which rotates about a vertical axis. The small ends of the trumpets are connected by rubber tubes to the two ears of the listener. If a source of sound moves past before such an arrangement at rest with the axes of trumpets horizontal, the listener can judge with great accuracy the instant when the source comes exactly in the front.

In order to locate an invisible aircraft during the night or in foggy weather, we make use of the sound of the engine, the propellers etc. The trumpets are moved round on their vertical axis, until the listener judges that they are pointing to the source of sound. Any small movement of the trumpets from this position makes the sound seem to move far to the left or to the right. This arrangement does not give us the *position* of the source of sound but tells us only the *direction* from which the sound is reaching the observer. In case the sound does not reach the observer directly but is heard after many a reflection or refraction, then the direction given by the above arrangement is not true. The method illustrated above for locating the direction of sound by means of two ears is known as *Binaural Method*.

In an improved type of the sound locator we have two pairs of trumpets, and each pair is connected by tubes to the ears of the two observers. One observer can swivel the trumpets sideways, and the other can tilt them up or down. These movements are carried on till a position is reached when the sound is heard equally in both ears. Now the axis of the trumpets will be pointing towards the aeroplane which of course is moving swiftly all the time. In the **modern British sound locater** (Fig. 153) paraboloid reflectors replace the trumpets, and microphones are used to assist the listeners. Electrical filters are used to cut out the unwanted sounds, and the listeners receive the sound through telephones.

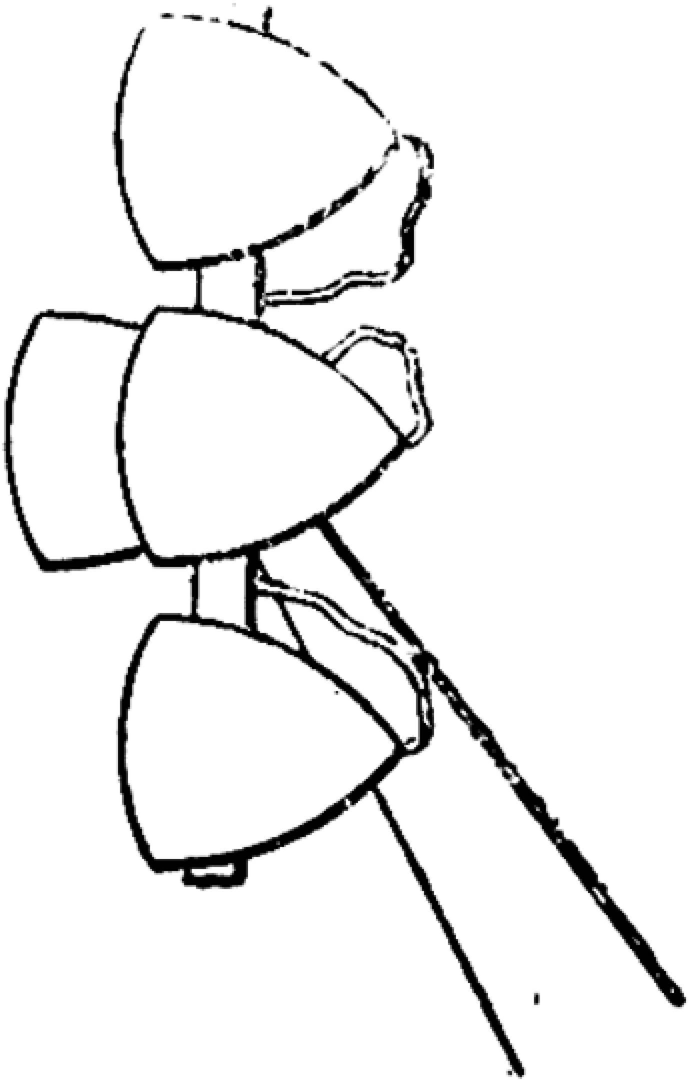

Fig. 153.

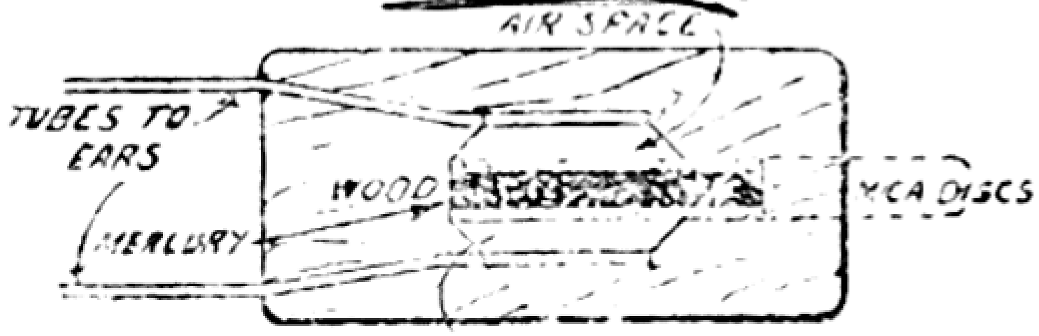

Fig. 154.

217. The Geophone—Its Application to Mining.—The Geophone was invented by a French engineer, and was used for the first time in the World War (1914—18) for the detection of the enemy tunnels being driven in our lines. The instrument (Fig. 154) consists of a small circular wooden box about 3″ in diameter and 2″ in depth. In the centre it contains a mass of mercury enclosed between two mica discs. Above and below the mica discs there are two small air chambers, each of which can be connected to the ear by a rubber tube.

For *detecting* the sound of mining only, the two air spaces of a single geophone are separately connected to the two ears by rubber tubes. When the box is placed on the ground, any sound waves travelling past the box will set it vibrating but the inertia of the mercury prevents any great movement of the mica discs. This will alternately compress and rarefy the air in the air spaces, and waves of sound will be transmitted along the tubes to the ear.

To find the *direction* from which the sound is coming, two geophones are placed on the ground. One air gap of each is closed with a cork and each of the remaining gaps is connected to one ear. The geophones are then moved about on the ground and their positions are found for which the intensity of sound, received by each ear is the same. That will be the case when the sound seems to come straight from the front (enemy being known to be in front). The direction perpendicular to the line joining these two positions is the direction from which the sound is coming.

218. The Hydrophone : Location of Sounds in Water.—Hydrophone (Figs. 155—56) is an instrument which was largely used in the FirstGreat War to detect sound waves beneath the surface of water. It not only detects the presence of a submarine but also determines its position and direction from an observing ship. It consists of a large metallic ring carrying a steel diaphragm to the centre of which is attached a small watertight chamber with a carbon microphone inside it (Fig. 155). Sound waves under water set the diaphragm vibrating. These vibrations are imparted to the microphone and consequently sound will be heard in a telephone receiver connected with the microphone. Such a hydrophone will be silent to sound waves when they are parallel to the plane of the diaphragm because in this position waves strike on both sides of the diaphragm in the same phase, thus causing no vibration, Fig. 156 (*c*). But it will act as bi-directional when it is set at right angles to the direction of the sound waves. In this position the instrument is sensitive to sounds both coming from the front and the back, hence the true direction cannot be located. To overcome this difficulty, one side of the diaphragm is screened by a solid plate or 'baffle' of some non-resonant material. Fig. 156 (*a*) shows the instrument without the baffle and Fig. 156 (*b*) shows it with the baffle. This latter type is most sensitive to sounds in front only Fig. 156 (*e*), while it will remain deaf to sounds coming from the back Fig. 156 (*d*). Thus it will now serve as a uni-directional type.

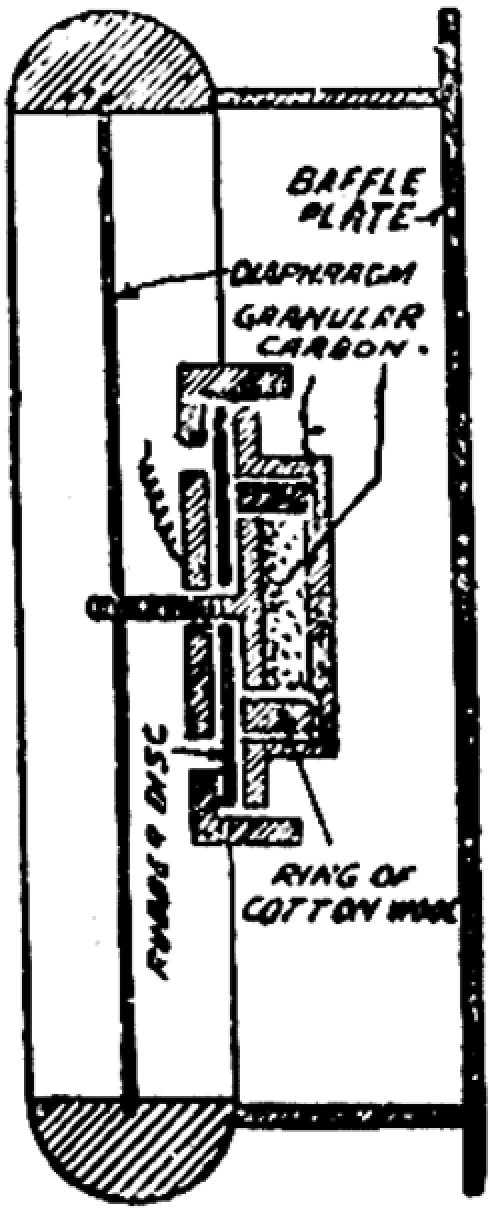

Fig. 155.

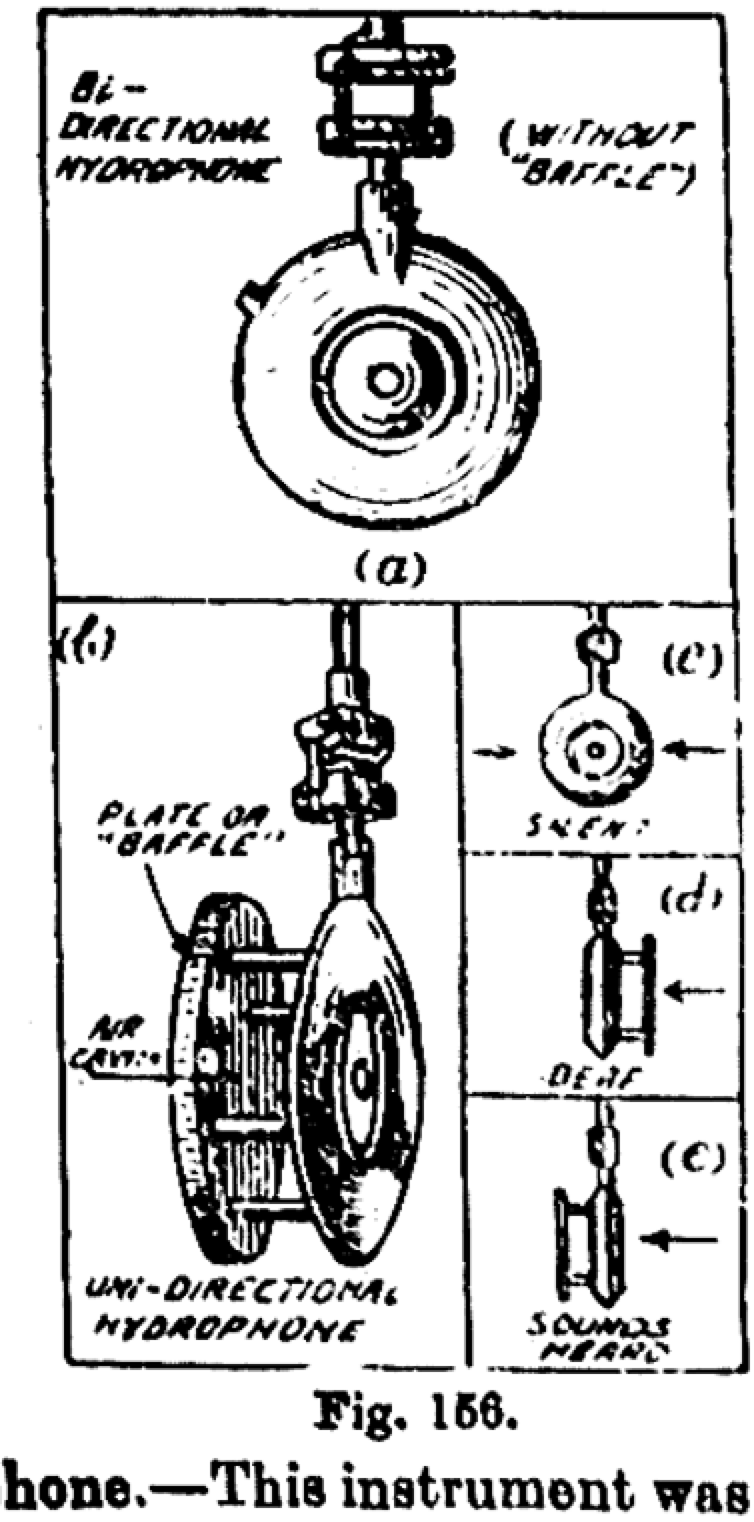

Fig. 156.

219. The Hot Wire Microphone.—This instrument was devised

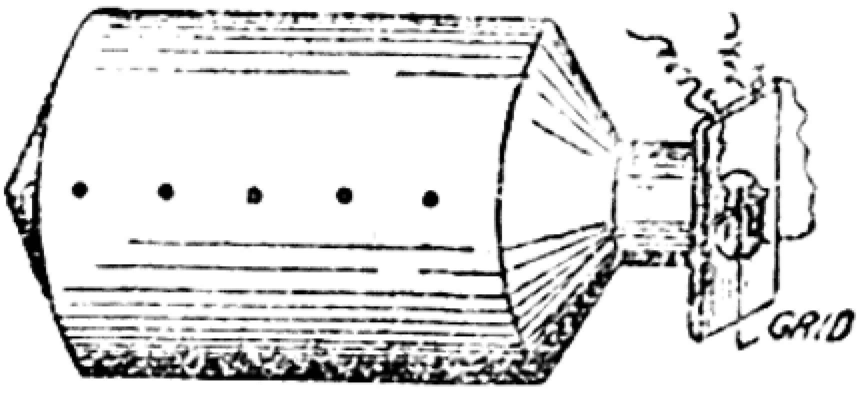

Fig. 157. The hot wire microphone.

by W. S. Tucker for sound ranging. The underlying principle was the effect of change of temperature upon the resistance of a metallic wire. The instrument (Fig. 157), consists of a metal cylinder with a wide neck, to act as a resonator. A 'grid' of thin plantinum wire is stretched across the open mouth of the cylinder. Its ends terminate in binding screws and are carefully insulated from the body of the cylinder. This wire is heated to a steady temperature by means of an electric current and is placed in one of the arms of a Wheatstone bridge. When compressional waves due to the firing of a distant gun fall upon the mouth, they set up oscillations of air in the resonating vessel, and cool the grid momentarily. As a consequence its electrical resistance is reduced. This change in resistance is at once indicated by the deflection of a spot of light reflected from the mirror attached to the Wheatstone bridge galvanometer.

220. Sound Ranging.—The report of a hidden gun is recorded from three different stations with the help of Hot-wire Microphones. Imagine a gun at *P* and the **microphones situated at** *A*, *B*, and *C*. These microphones are connected to special type of galvanometers kept at a distant central station. The spots of light reflected from the galvanometer mirrors are received on a photographic film wrapped round a cylinder rotating at a constant speed. They will trace three parallel straight lines on the film. The instant of arrival of the sound waves at the microphones is given by a sudden kink in their respective lines. Knowing the speed of the moving film, the time interval between any two kinks can be accurately determined. This will be the time required by sound to travel the distance between, wave-fronts reaching the two microphones. Supposing the sound is first heard at *A*, then *BQ* will represent the distance travelled by the sound waves in

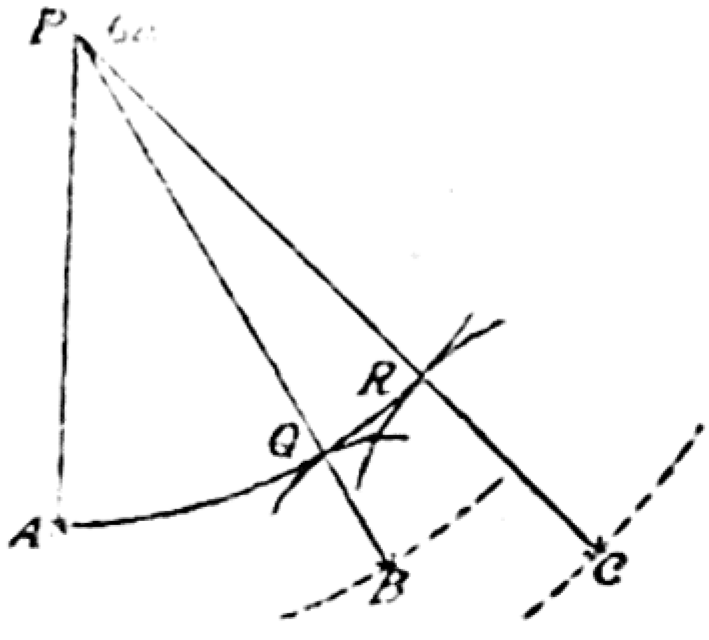

Fig. 158.

between the wave-fronts reaching the microphones at A and B during the time interval as recorded between two kinks on the corresponding lines on the photographic film. Let the sound heard at A be t_1 and t_2 seconds sooner than it arrives at B and C. Then with centres B and C and radii vt_1 and vt_2, where v represents the velocity of sound in air, draw circles round B and C. Then the gun will be at the centre of a circle AQR which passes through A and touches circles drawn round B and C at Q and R. The whole construction is made on a map and the position of the gun is located. The circle AQR evidently represents the wave-front when the station A has received the report. The wave-front at this instant lies at distance vt_1 and vt_2 from the other two stations B and C. Graphically we may proceed as follows: Supposing the sound arrives at the microphone A half a second sooner than it arrives at the microphone B. If 1130 ft./sec. be taken as the velocity of sound, then the gun must be 565 ft. more distant from B than it is from A. Draw arcs round A and B, the

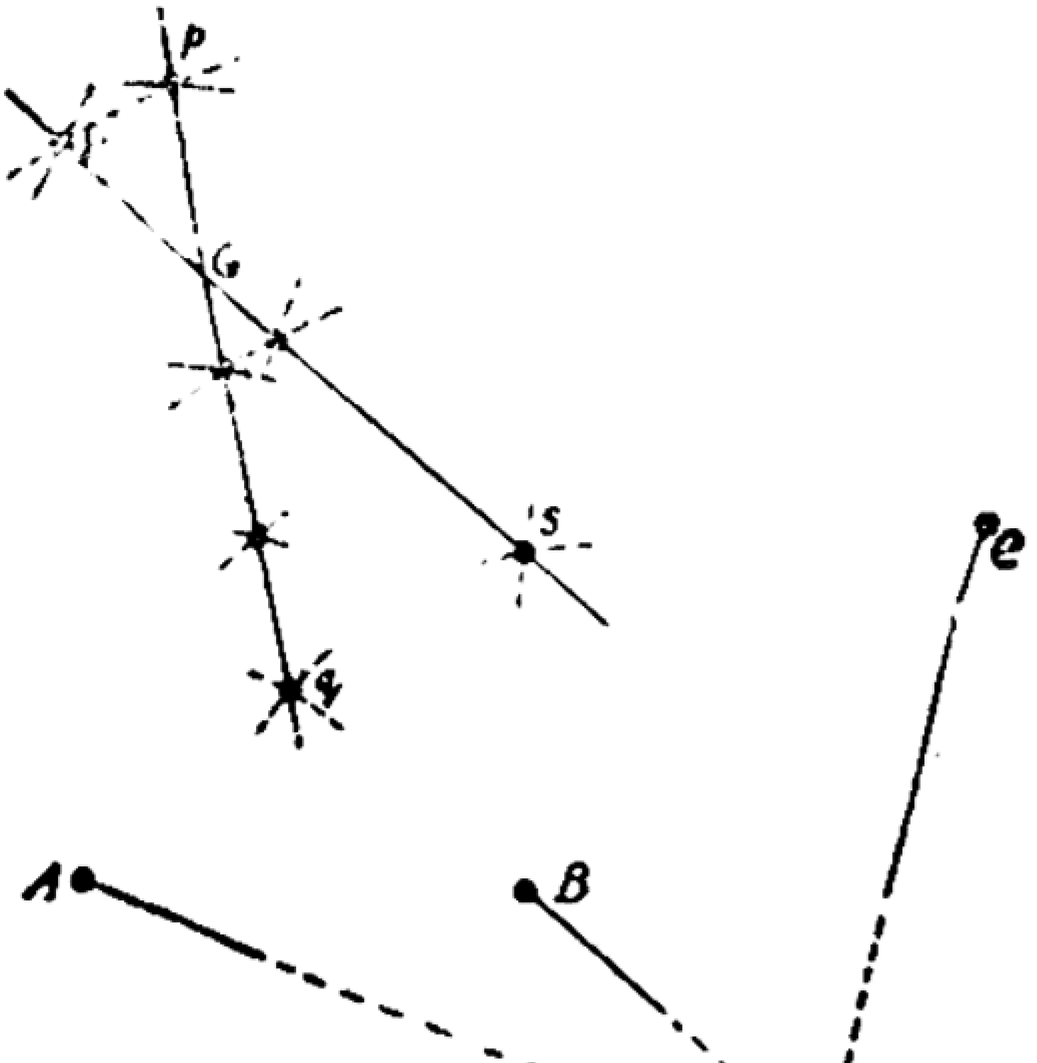

Fig. 159.

radii of the latter being 565 ft. greater. The point of intersection of each pair gives a possible position of the gun. Join these points by a line pq, then the gun will be somewhere on this line.

Again suppose sound at C is received $\frac{1}{2}$ second later than that at B. Draw arcs round B and C, the radii of the latter being 565 ft. greater. As before get the points of intersection. Then the gun must be somewhere on the line rs. The actual position is thus obtained at G, the point of intersection of the lines pq and rs.

In a similar way we can send information to a ship desiring to know its true position. This is possible in case there is at some distance on the shore a 'wireless station' with sound ranging equipment with hydrophones which could be submerged in the sea. Firstly by wireless telegraphy, the ship will inform the wireless station on shore that she desired her position. Soon she will drop a 'depth charge,' the sound waves of which will reach the sound ranging equipment on theshore. Afterwards the land station will intimate the information to the ship by wireless telegraphy as to its true position.

221. Ultrasonics.—When an explosion takes place, various types of waves are associated with it. One type has a very low frequency even below the '*audible limit*'. Such sounds are called *Infra* sounds or '*Infra-sonics*'. The other is the ordinary audible sound. Hot wire microphones can detect both these types of waves. There is a third type in which the frequencies are beyond the 'limits of hearing'. These are called *Supersonic* sounds or '*Ultra-sonics*'. Their frequency is 20,000 or upwards. The Galton whistle is very suitable for generating ultrasonics with constant amplitude and frequencies upto 100 kilocycles. High frequency vibrations have also been obtained with the help of certain crystals like quartz, tourmaline and Rochelle salt.

If pressures are applied to the opposite faces of a quartz crystal cut so that the faces are perpendicular to the optic axis* of the crystal, the faces will develop equal and opposite charges with consequent differences of potential. But when the crystal is under tension the signs of charges are reversed. This phenomenon is known as *Piezo-electric Effect*. The converse effect is also true—that voltages applied to the faces of the crystal produced corresponding changes in its dimensions. When such a crystal plate is placed in an electric-field in the direction of thickness, it expands or contracts to an extent depending upon the strength and direction of the electric field applied. When alternating potential differences are applied the crystal contracts and expands alternately and elastic vibrations are set up in the crystal. When the frequency of such alternating voltages coincides with one of the possible modes of vibration of the crystal plate, a large resonant vibration will occur.

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*See Chapter on polarisation in a Text Book of Light.

Such vibrations may be regarded as longitudinal and the possible frequencies of vibration are given by the relation,

$$N=\frac{p}{2l}\sqrt{\frac{E}{\rho}},$$

in which $p=1, 2, 3$ etc., l=thickness, E the value of Young's modulus for the particular axis chosen and ρ the density of the material.

For quartz the average velocity $\left(v=\sqrt{\frac{E}{\rho}}\right)$ may be taken as $5{\cdot}5\times10^5$ cm. per sec. Thus a plate of quartz 5 cm. long and 0·5 cm. thick will consequently have frequencies multiples of $5{\cdot}5\times10^4$ and $5{\cdot}5\times10^5$ cycles per second. Thus if such a crystal is made to vibrate by means of alternating voltages of suitable frequency, *ultra sonic* waves will be generated.

Fig. 160 shows a simple Piezo-electric generator, for the production of ultrasonic waves. Q is a crystal of quartz placed within two metallic foils AB and CD The combination forms a parallel plate condenser with crystal as a dielectric. The plates are connected to the terminals of a coil which is inductively coupled to the oscillating circuit of a three electrode valve. If the natural frequency of oscillations of the valve circuit is equal to the frequency of that crystal mode which it is desired to excite, resonance occurs and the crystal will undergo linear expansions and contractions at exactly the same rate. In other words the crystal is set vigorously into mechanical vibrations. The arrangement may be regarded as an electrically maintained mechanical vibrator emitting sound waves of definite frequency or as a mechanically controlled valve oscillator generating alternating current of

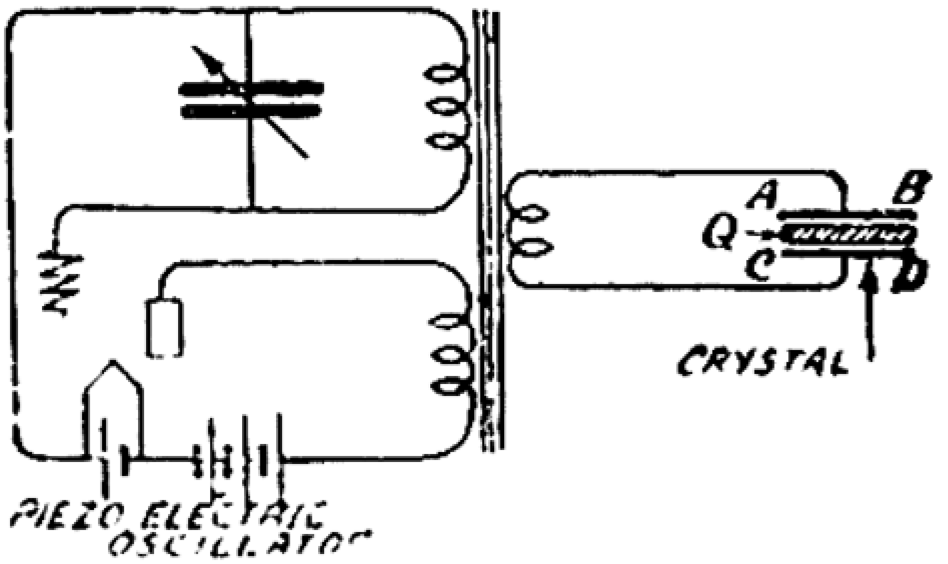

Fig. 160.

this frequency. With a moderate size quartz crystal ultrasonic waves of frequencies of the order of 55000 cycles/sec. can be produced. In order to detect ultrasonics Kundt's tube, sensitive flames and thermal detectors are used. Langevin used the quartz crystal mentioned above for their detection. Electric charges are produced on the pair of faces that are perpendicular to those that receive the 'ultrasonics'.

222. Applications of Ultrasonics: —

(1) **Velocity of Sound in Liquids.**—Ultrasonics have been used for determining the velocity of sound in liquids which are available in small quantities. Such determinations reveal many Physico-chemical characteristics of the medium such as compressibility, chemical structure, concentration, specific heat, etc.

(2) **Elastic Symmetries of Crystals.**—A liquid or solid traversed by an ultrasonic wave system acts on Light like a diffraction grating, the lines of compression being lines of greater density. If ultrasonic waves are thus applied to anisotropic crystals, they give rise to interference images which reveal at one glance the elastic symmetries of the crystal. Bergmann who shows some beautiful pictures of this type in his book on 'Ultrasonics' emphasizes that one and the same sample yields atonce a complete system of all constants in a way not obtainable in any previous method.

(3) **Sound signalling—Detection of submerged submarines, icebergs, depth of sea, etc.**—Langevin used a special type of quartz crystal transmitter. He generated ultrasonics of 38,000 cycles per sec. This could easily be done by taking a quartz of 2 mm. thickness covered with two steel plates of 25 cm. diameter. Ultrasonic beam is sent from such a transmitter and the instant of its generation is indicated by the movement of a spot of a light on a scale. These waves go normally downwards and come back after reflection from the sea bed. During the interval the quartz is connected to the receiving circuit and acts as the detector. The reflected beam causes vibrations, and again the spot of light is thrown on the same scale. The time interval between the two throws is accurately measured by a mechanism similar to that used in

sound ranging equipment. And knowing the velocity of propagation of the 'ultrasonics' in water, the depth is calculated. Thus by the combination of these apparatuses (Quartz crystal used as a transmitter and receiver), it is even possible to signal to a distant ship, determine the presence of a submerged submarine or a hidden iceberg. The principle of sending out ultrasonic waves and measuring the reflection time is used in industry for finding flaws in metal blocks or the level of a liquid in a metal tank.

Recently ultrasonic waves have been employed industrially for washing clothes especially silken fabrics and for several other purposes.

(4) **Chemical Effects.**—Intense 'ultrasonics' change immiscible liquids such as water and oil, or water and mercury, into very stable emulsions. Iron and aluminium hydroxides are liquefied by them in the same way as by shaking.

(5) **Biological Effects.**—It was also found that smaller animals such as frogs, fish etc. were killed or lamed when placed in the ultrasonic field. Yeast cells when exposed to ultrasonic radiation, lose their power of reproduction.

223. **Heat-maintained Sounds.**—Under suitable circumstances an intermittent supply of heat to a body may cause a periodic vibration of a part or whole of it, and the body may produce a musical note of a definite pitch.

(1) **Trevelyan Rocker.**—Fig. 161 (*a*) shows a Trevelyan Rocker which consists of a prism of brass or copper attached to a handle ending in a ball. It has a groove running throughout an edge of the prism. When the prism stands on this edge, it really rests on two edges of the groove which serve as two parallel ridges. When the rocker is heated and placed on a

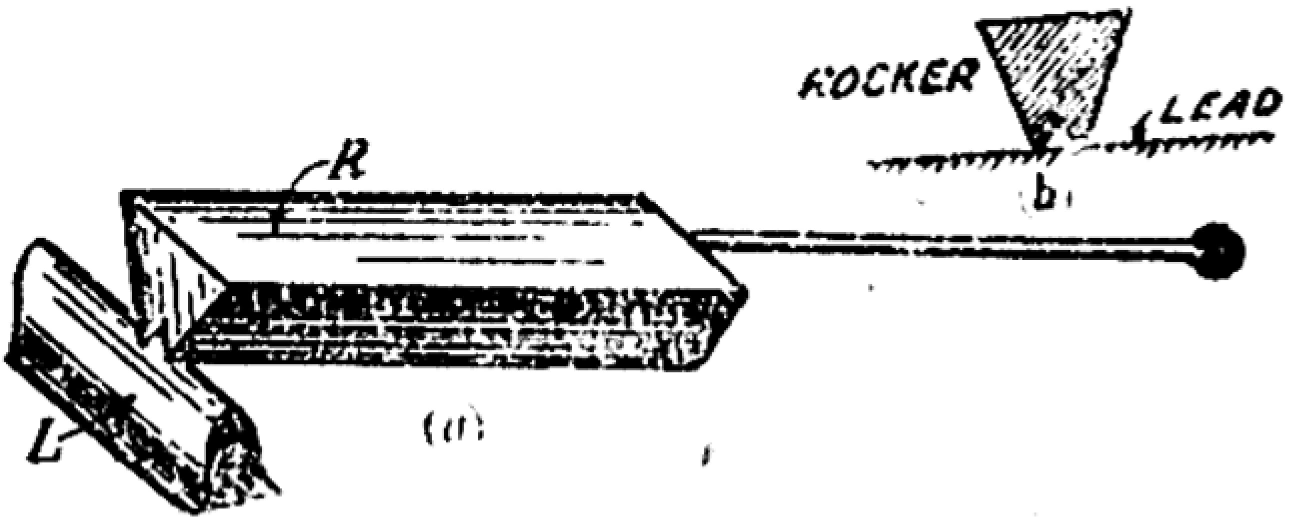

Fig. 161.

lead block, and is given a smart tap *about a ridge*, it continues to rock *about the ridges*, and a musical note is produced. If a tiny mirror is attached to the handle of the rocker, and a strong beam of light is thrown on it, the rocking can be observed by making the reflected beam fall upon a rotating mirror.

When the prism is slightly pressed while it is rocking, pitch of the note is raised. What happens is that when one ridge is more heavily in contact with the block than the other, heat will flow from the rocker to the lead more rapidly *via* this ridge. This heat will cause the lead to expand immediately under this ridge, a hump is produced which tilts the prism over to the second ridge. Heat now flows from the prism to the lead *via* the second ridge so that the lead now expands under it producing a hump as before and causes the prism to tilt back on to the first ridge. In this way as the heat flows from the rocker to the lead *via* one ridge or the other, a hump is produced periodically about one ridge and then about the other, the rocker tilting alternately about each ridge. This periodic tilting causes a thump on the table and these thumps merge into a musical note, the energy being supplied by the heat of the rocker.

The maintenance of the vibrations can be accounted for by assuming that there is a slight lag between the contact and expansion. Otherwise, the approach of the rocker would be checked and the motion discouraged to an extent equal to its encouragement in its return motion and no balance left over for the maintenance of vibrations.

(2) **Singing Flame.**—On gradually lowering a pipe open at both ends (Fig. 162) of about 3.5 cm. diameter and 30 cm. in length over a small jet of coal gas, we find that when the jet is near the midpoint of the pipe, a musical note is produced. The flame actually throbs and sings within the pipe, and is called a '*Singing flame*'. The pitch of the note $= \frac{v}{2l}$, where l is the length of the open pipe and v is the velocity of sound in air. It is found that to obtain a musical note,

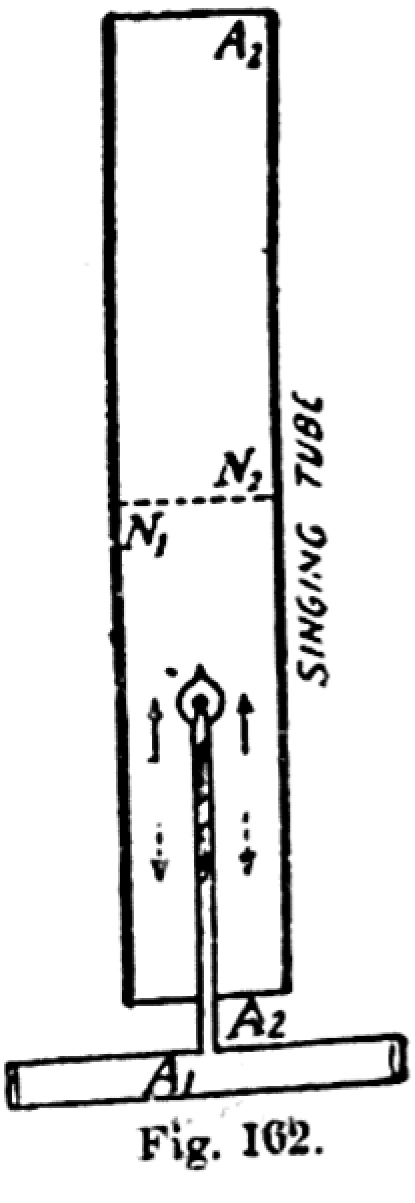

Fig. 162.

the length of the jet tube must be less than half the whole length of the sounding tube. If we examine the flame in a rotating mirror without inserting it in the pipe a continuous band as shown in Fig. 163 (*a*) is obtained, indicating that the flame is burning steadily. When the flame is introduced into the pipe, on examination it shows a tooth-like structure, as in Fig. 163 (*b*) thus indicating that it burns intermittently and its period is the same as that of the note. The constancy of the pitch of the note given out by the flame is due to the stationary vibrations of the air column inside the pipe, which react on the jet tube and set up stationary waves in it. The effect of these variations in the jet tube is that the emission of the gas, instead of being uniform, is intermittent so that the size of the flame and hence also the supply of heat to the air near the flame is intermittent. Now when a column of air is in vibration and the rise of temperature takes place at the moment of greatest compression, the force with which air tends to expand and go away from the nodal point to regain its normal pressure, is increased and so the amplitude of the air particles is increased. If, on the other hand, supply of heat occurs at the instant of greatest rarefactions, this will tend to resist the return of air to its condition of rest and will, therefore, tend to stop the vibrations. Hence to maintain the vibrations of air column the periodic increase in the size of the flame must occur at the instant when the air near the flame is, owing to the natural vibrations in the air column of the surrounding pipe at its maximum compression, that is, periods of column of gas in the jet tube and of the column of air in the pipe, must be commensurate. It is evident, therefore that the length of the jet tube and the position of the flame bear an important part in the phenomenon. In Fig. 162; the nodes and antinodes for the supply tube are denoted by N_1 and A_1

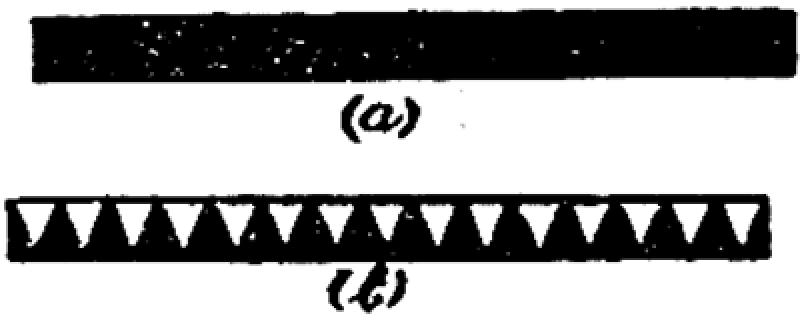

Fig. 163.

and those for the singing tube by N_2 and A_2 respectively.

224. Talking Motion Pictures.—The commerical development of talkies began about the year 1928. In the earlier methods the sound was reproduced from a gramophone record by means of an electric pick up'. It was then absolutely necessary for the record to run at such a speed as to keep step with the pictures—a condition often very difficult to realise in practice. Nowadays, the sound is recorded in the form of a "sound track" $\frac{1}{10}$th inch wide near one edge of the film itself as shown in Fig. 164 (*a*) and (*b*). These are two different types of sound records. In (*a*) the edge of the blackened part of the sound track is the *vibration curve*. In (*b*) the varying density of the blackening on the film gives the vibration curve. The former is an image of a slit of variable area, while the latter represents the image of a slit of constant area but of variable density. Below we describe briefly the process of taking these photographs of sound.

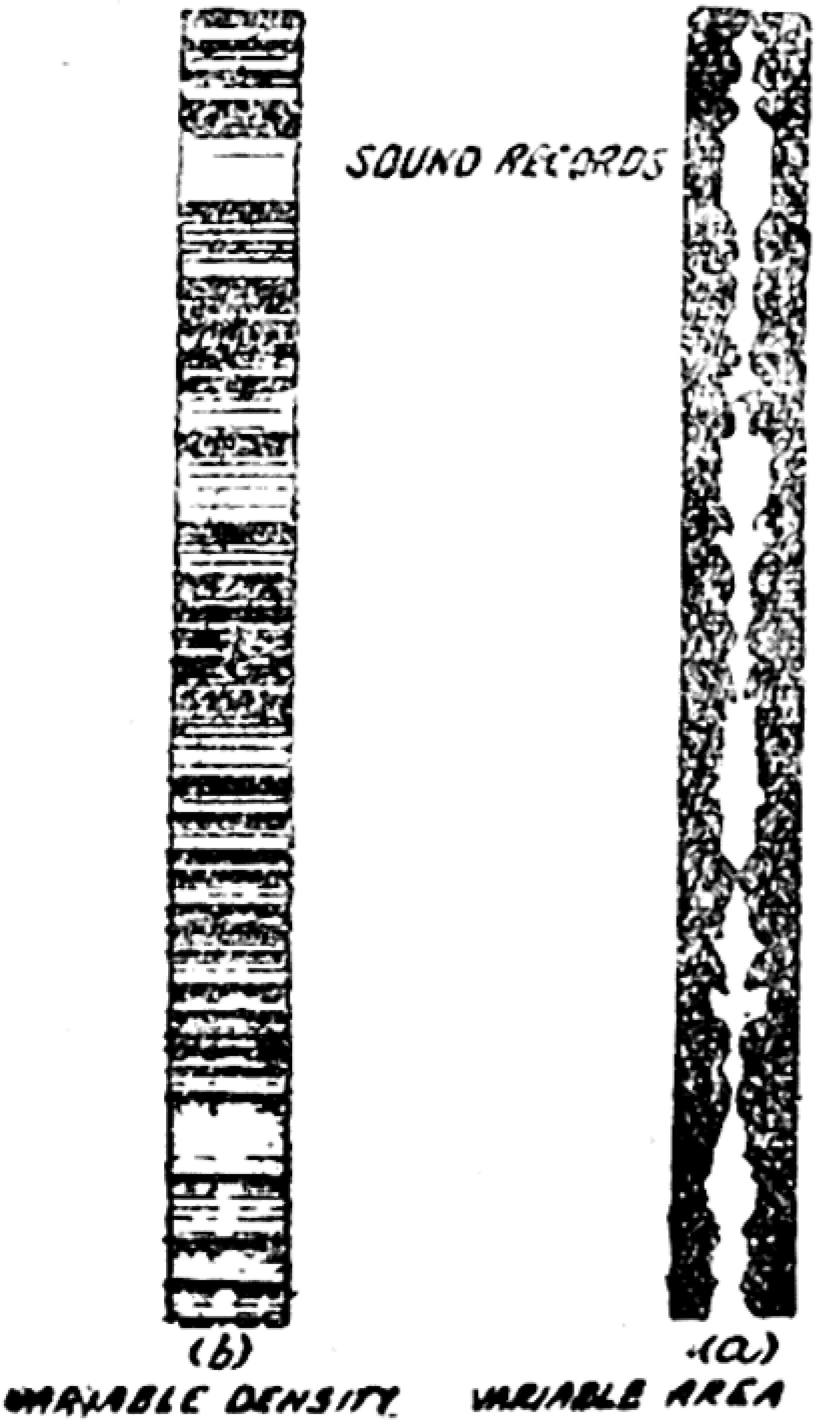

Fig. 164.

(1) **'Variable area' method.**—Here light from a source of constant intensity (Fig. 165) after reflection from the mirror of an oscillograph, fall on a narrow slit of a given width placed in front of a film. When the amplified speech current passes through the loop of the oscillograph, the mirror is tilted in accordance with the vibrations of the speech current. Consequently the slit is either fully or partially illuminated by the reflected light resulting in an illuminated strip on the film of variable area.

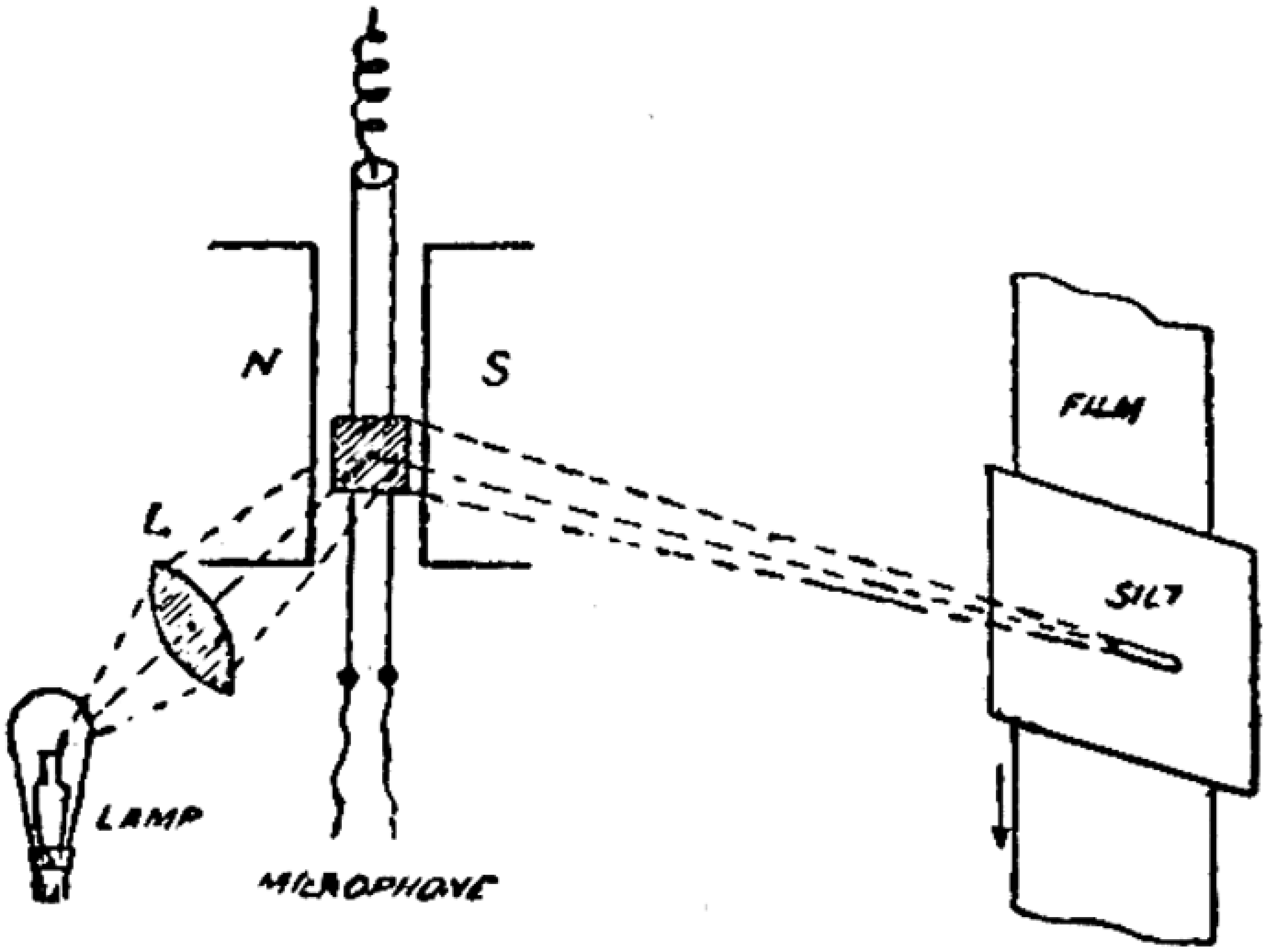

Fig. 165.

(2) **'Variable density' Method.**—In this case a special type of vacuum tube called an 'Aeo' lamp containing a little quantity of helium gas, a plate and a filament like that of a 'diode valve', illuminates a slit in front of the film (Fig. 166). Intensity of the 'Aeo' lamp varies in accordance with the speech current, resulting in an image of the slit with variable intensity.

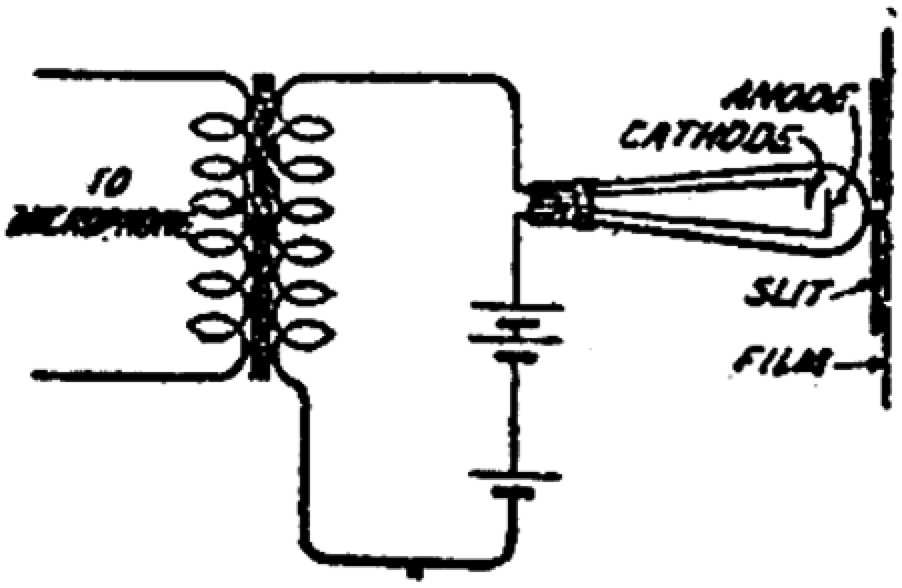

Fig. 166.

225. Reproduction of Sound—To reproduce the sound from the film record, light from a point source after being focussed by a lens system into an extremely narrow line of about $\frac{1}{100}$ of an inch in width, is made to illuminate the sound track. The length of the bright line is equal to the width of the track. As the film runs past this narrow beam, it varies the amount of light that falls on a 'photo-electric cell, (electric eye), and so produces corresponding fluctuations of the electric current through the cell. After amplification the varying electric current is carried to the loudspeaker (Fig. 167).

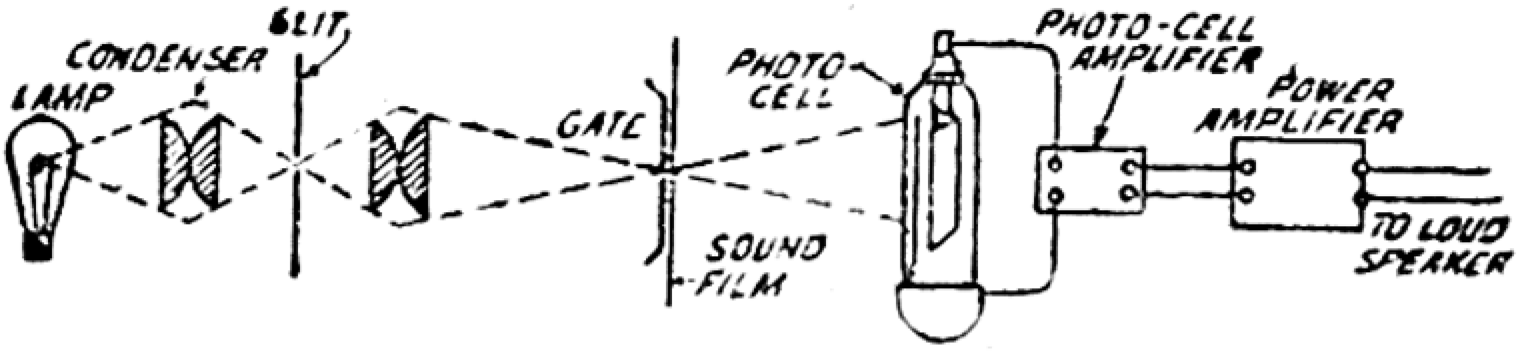

Fig. 167.

In actual practice the picture gate and the point where the corresponding sound record is illuminated, are not in the same horizontal plane, but they are one above the other. In a typical machine this distance is $14^{1}/_{2}''$, and therefore the sound record is printed $14^{1}/_{2}''$ in advance of the corresponding picture record. Thus while shooting a picture, it will not be advisable to take the sound as well as the picture record side by side. Sound may be recorded in a room different from the studio where the pictures are recorded. Due care must be taken to run the two films exactly at the same speed by separate motors connected electrically in a manner so as to give the same speed of rotation.

In the modern type of cinema halls, four loudspeakers are used generally. Two of them are installed in the line of the stage and point towards the balconies. The other two are kept in a line with the upper edge of the screen and point towards the audience in the body of the hall.

226. Architectural Acoustics.—It was noticed (Art. 99) that a sharp clap is converted into a musical sound on account of successive and regular reflections from a row of palings. Similarly, where sound waves are produced in a big hall they spread out, scatter, and strike the surfaces of the various objects in the hall such as ceiling, walls, floor, etc. Some of the sound energy is reflected and some absorbed. The reflected portions travel back into the hall, and reunite to form objectionable echoes and they produce indistinctness and interfere with good hearing. In other words the sound from the source does not cease immediately after it is directly received but is lengthened out owing to the energy received by continuous reflections from successive portions of the walls etc. Thus the original wave on breaking up, wanders about for some time in the air of the hall, and even when the direct pulse has ceased, some of the energy is still on its way from the source to the walls and thence to the ear, and hence the prolongation of sound. This persistence of sound is called *Reverberation*. The time during which sound persists is called 'reverberation time' of sound in the hall measured from the time that the original sound is produced, or in the case of a continuous note from the time that the source stopped sounding. Obtaining the right amount of reverberation is the secret of good Acoustics.

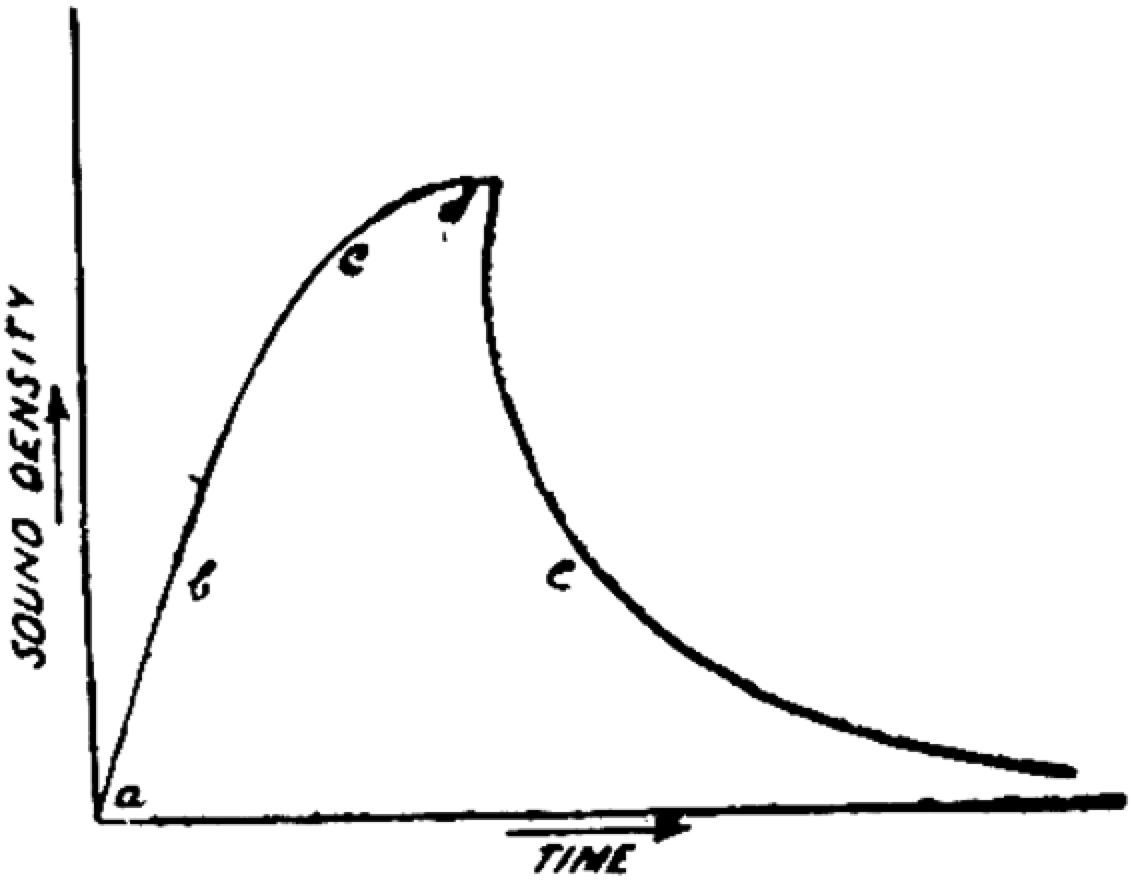

Fig. 168.
Growth and fall of *sound* density.]

Let us first see how sound density in a hall varies with time, counting time from the instant the sound first reaches the ear. The curve shown in Fig. 168 illustrates the growth and fall of sound density for a spoken syllable within an auditorium. It is exactly similar to the growth and decay curve of the electric current in an inductive circuit. At '*a*' the direct sound energy just reaches the ear. At '*b*' the original sound being still emitted, the sound intensity is being reinforced by reflections, the losses due to absorption at the surfaces being yet less than reinforcements. At '*c*' these losses are almost equal to the amonnt of reinforcement, till at '*d*' the intensity on listener's ears reaches a steady value. If now the original sound ceases, the intensity of sound falls off as shown by the part '*e*' of the curve and hence the prolongation of sound. If the absorption is large, the intensity falls off quickly and the reverberation time will be small. On the other hand if the absorption is small, the reflections will continue for a longer interval making the sound persist for an appreciable time. This makes the reverberation time large.

For obtaining good acoustic properties we should have the following conditions :—

(1) Each separate syllable should produce sufficient energy in every part of the hall.

(2) Sound of each syllable should soon decay so that the succeeding syllable may be heard distinctly.

(3) There should not be present echoes more than necessary for maintaining continuity.

(4) There should be no undesirable focussing of sound not there should be present zones of silence or regions of poor audibility anywhere in the hall.

(5) There should not be any unpleasant reinforcement of any of the overtones, thus resulting in the change of tonal quality.

So in the designing of big halls, firstly due consideration should be given to prevent bad echoes and secondly the right amount of reverberation should be provided.

W. C. Sabine (1868-1919) an American Physicist who studied the whole subject of architectural acoustics scientifically established that the standard period* of reverberation is given by

$$t=\frac{0\cdot 05V}{\Sigma S_1a_1}$$

where V is the volume of the hall in cu. ft. and S_1, S_2 etc. being different surfaces in square feet of absorbing coefficients a_1, a_2, etc. The *absorbing coefficient* of any surface is the ratio of its absorption of sound to that of an equal area of a perfect absorber such as an open window. The *unit* of absorption of sound is a square foot of open window. Some acoustic absorption coefficients for ordinary vocal frequencies are as follows :—

Open window	1·00
Brick wall	0·03
Window glass	0·025
Carpets and rugs	0·15-0·3
Curtain	0·15
Inch thick hair felt ...	0·78
Audience per person ...	0·5
Wooden chair	0·17
Upholstered chair	0·3
Cushion	0·2
Celotex	0·36

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*It is the time between the cutting off of a sound to the time at which the sound becomes inaudible. *viz.*, its intensity falls to one millionth of its original intensity.

As reverberation is due to repeated reflections, the remedy lies in increased absorption which may be brought about by having,

(1) a few open windows,

(2) maps or pictures hanging from walls,

(3) heavy curtains

(4) a good audience (one listener is equivalent to 5 sq. ft. of an open window).

(5) no curved walls or corners bounded by two walls, firstly to avoid concentration of sound at one place and secondly to avoid 'dead' spaces.

(6) walls covered with absorbent materials such as celotex, perforated cardboards, felt, asbestos, coarse cloth, etc., or walls engraved and roughened with decorative materials. Porous bodies by providing small capillaries absorb a good deal of sound energy incident on them and serve very well as sound suppressors.

(7) upholstered seats which will be responsible for absorption and would avoid approximately the presence and absence of the audience.

It should be noted that by decreasing time of reverberation, steady value of sound also decreases, hence one would not like the sound energy to be too small to be audible, *i.e.*, to make the room 'dead'. So the time of reverberation should not be decreased beyond a certain optimum value as decided by experience.

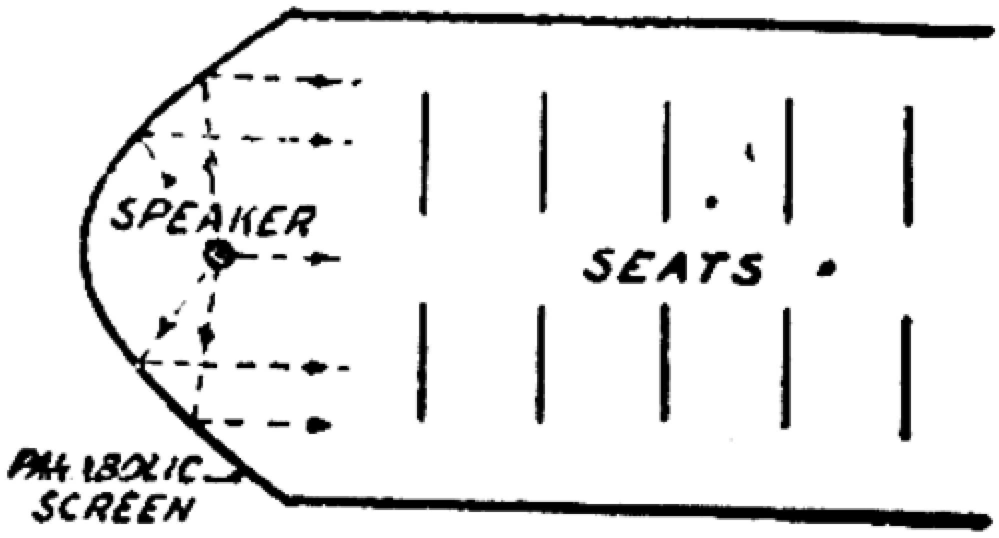

Fig. 169.

General plan of the hall with seats should be somewhat like as shown in Fig. 169.

It follows directly from above that in case of a room possessing bad acoustical properties, the person should speak slowly so as to use as little volume of sound just to fill the room. The desirable period of reverberation in seconds for halls with full audience is given by the empirical formula,

$t=0{\cdot}75+0{\cdot}0175\sqrt[3]{V}$, where V is the volume of the hall in cubic ft.

It has been estimated that 't' for halls of 10,000 cubic ft. capacity is 1·03 seconds for speech where each syllable is wanted to be heard as distinctly as possible.

For purposes of music, this may be slightly increased.

QUESTIONS

CHAPTER I

1. What is meant by a simple Harmonic vibration ? How are the displacement and acceleration related in such a vibration ?

2. When a mass supported at the end of a spiral spring is slightly displaced, it will execute S. H.M. Calculate the periodic time.

3. What is the nature of the force that will make a particle move in S. H. M. ? What reasons are there for believing that the motions that give rise to sound are simple harmonic ?

4. What is S. H. M. ? Show how it may be represented in a diagram of time and displacement.

5. Demonstrate experiments to show that it is the mechanical vibration that produces sound.

6. Deduce the differential equation for the Simple Harmonic motion of a particle.

7. Show that the energy of a particle in 'S. H. M.' is directly proportional to the square of the amplitude and inversely to the square of the time-period.

8. What is meant by Simple Harmonic motion ?

A vertical tube of uniform cross-section contains water upto a height of 30 cm. Show, that if water on one side is depressed and then released, its motion up and down the tube is Simple Harmonic, and calculate its period. (*D. U. 1933*)

Sol. Let l be the total length of the liquid (say water) contained in a U tube of uniform cross-section. Let it be sucked up by y cm. on one side say to A'. The column will be depressed on the other side also by y cm. The difference in height on the two sides becomes $2y$ cm. The force acting on the liquid to bring it back i.e., B' to B is the weight of the column $A'C$ i.e., $2y\rho ag$, where a is the cross-section and ρ the density of water. Since the total mass of water is ρal, the acceleration f at this particular instant is given by

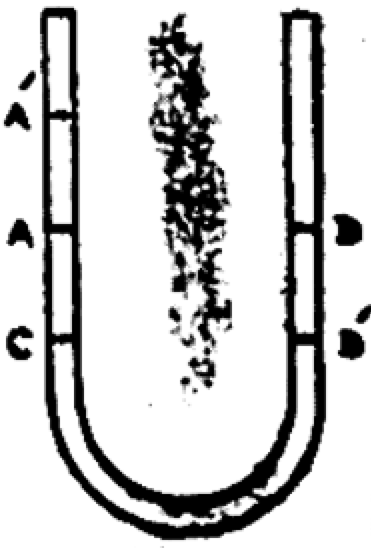

Fig. 170.

$$f=\frac{2y\rho ag}{\rho al}.$$

$$=\frac{2g}{l}y.$$

We, therefore, see that acceleration is proportional to displacement y *i.e.*, the motion is Simple Harmonic and the periodic time

$$T=2\pi\sqrt{\frac{l}{2g}}.$$

Here $\frac{l}{2}=30$ cm.

$$T=2\pi\sqrt{\frac{30}{981}}\text{ seconds.}$$

9. Show that when a particle is moving in simple harmonic motion, its velocity at a distance $\frac{\sqrt{3}}{2}$ of its amplitude from the central position is half its velocity in the central position.

10. A body of mass 10 grams executes simple harmonic vibrations. The force acting on the body when the displacement is 6 cm. is 40 gram wt. Find the period. If the maximum velocity is 400 cm./sec. find the amplitude and the maximum acceleration $g=981$ cm./sec.^{2}

Ans. T=·2457 sec. $a=15{\cdot}64$ cm. $f=10230$ cm./sec.^{2}

11. A weight-less spring is hung vertically and loaded with a mass of 2 oz. and a force of 2 lbs. stretches it 16 in. Find its period and frequency of oscillation. *Ans.* 0·32 sec. 3·13 sec^{-1}

12. A mass of 10 gm. executes S.H.M on the end of a spring completing 2 oscillations every second. The amplitude of the motion is 8 cm. Find the maximum velocity and maximum acceleration. Also calculate the stiffness of the spring and the total energy of the vibrating mass.

Ans. $v_{max}=32\pi$ cm./sec. ; $f_{max}=1280$ cm./sec.^{2} ;
F=1600 dynes ; E=$5{\cdot}12\times10^4$ ergs.

13. An unknown mass, when hung on a light spiral spring stretches it 20 cm. Determine the period of oscillation of the mass if it is displaced from its neutral position and then released. *Ans.* ·8976 seconds.

14. A particle of mass m, free to move in the xy plane is subject to a force whose components are $F_x = -kx$, $F_y = -ky$, where k is a constant. The particle is released when $t=0$ at the point (2, 3). Show that its subsequent motion is S. H. M. along the line $2y=3x$.

15. Show that the energy of a vibrating body is proportional to the square of the amplitude of vibration. A particle of mass 5 grams is vibrating 15 times per second with an amplitude of 8 cm. Find (1) its maximum velocity; (2) its energy. *Ans.* 240π cm./sec.; $14{\cdot}4 \times 10^5$ ergs.

16. The amplitude of a S.H.M. is 2·5 cm. and frequency 64/sec. If the mass of the body executing it be 1·6 grams, find the Kinetic Energy when at the middle of an oscillation.

Ans. $8{\cdot}2 \times 10^5$ ergs. approx.

CHAPTER II

1. Explain the 'Periodic motion' and S. H. M. Find graphically or mathematically the resultant of two S. H. M.'s at right angles to each other, which have the same period and and a phase of $\frac{\pi}{4}$.

2. Show graphically or otherwise that the resultant motion of a particle acted upon by two S.H.M.'s of equal periods and amplitudes along two directions at right angles, but differing in phase by $\frac{\pi}{2}$, is circular.

3. Two S. H. M.'s having the same period but differing in phase are acting in the same direction on a particle. Show that the resultant motion is S. H M. and deduce expressions for the resulting amplitude and phase.

4. Give a general explanation of the manner in which Lissajous' figures may be produced and observed and how they may be practically utilised in acoustical determinations.

5. Calculate the resultant of two rectangular S. H. M.'s whose amplitudes as well as periods are in the ratio of 1 : 2, and the phase difference is 90°. (*P. U. 1941*)

6. Find the resultant of two mutually perpendicular S.H.M.'s which agree in period, but differ in phase. Consider

the important cases for phase differences varying from 0 to 2π.

7. What are Lissajous' figures ? Give an optical method of observing them.

What type of figure will you get when you compound two simple harmonic vibrations whose amplitudes as well as periods are in the ratio of 1 : 2 and the phase difference is 90°.

What will happen if the frequencies are in the ratio of 100 : 201. *(P. U.)*

8. Two Simple Harmonic vibrations P and Q of equal periods and differing in phase by $\frac{\pi}{2}$ are impressed on the same particle. The amplitudes of P and Q are 4 in. and 6 in. respectively. Find the amplitude of the resulting vibration and the phase difference from P. *Ans.* 7·211 in. ; 0·5882 radian.

9. Lissajous' figures are produced with two tuning forks whose frequencies are approximately in the ratio of 2 : 1. It takes 5 seconds to go through a cycle of changes. On loading slightly the fork of higher pitch, the period of cycle is reduced to 3 seconds. If the frequency of the lower fork is 400, find the frequency of the other fork before and after loading.

Ans. $799\frac{4}{5}$ before $799\frac{2}{3}$ after loading.

10 Show that two simple harmonic motions at right angles to each other, simultaneously impressed on a body may give it an elliptical motion, which under certain circumstances becomes circular.

CHAPTER III

1. Distinguish clearly between transverse and longitudinal waves.

Which of these cannot exist in a gas and why ?

2. Distinguish between wave motion and vibration. How is the velocity of a wave related to its frequency ? Show how the wavelength of a wave in air of given frequency changes with temperature.

3. Explain the meaning of the expression 'velocity of propagation of a longitudinal wave.' Represent graphically the state of disturbance at any time at any place during the propagation of such a wave. Give in detail the mechanical process involved.

4. Starting with the general equation for the harmonic wave disturbance, derive the wave equation.

5. Show how a transverse wave may be represented by means of a curve, and show also how the velocity of any particle may be found from this curve.

6. How a longitudinal wave may be represented by means of a sine curve ? Distinguish between a displacement curve and a compression curve.

7. What reasons are there for believing that sound is conveyed by wave motion ?

8. Deduce the standard equation for wave motion in the form

$$y = a \sin \frac{2\pi}{\lambda}(vt - x)$$

Prove with its help that Particle velocity is equal to wave velocity × slope of the curve.

9. A body vibrating with a constant frequency sends waves 18 cm. long in a medium *M* and 24 cm. long in a medium *N*. The velocity of the waves in *M* is 90 m./sec. Find the velocity of waves in *N*. *Ans* : 120 m/sec.

10. An electrically maintained tuning fork of frequency 76 is sending out plane waves of amplitude 0·15 cm. Find (*a*) the energy radiation per c.c. and (*b*) energy current per sq. cm /sec. (Speed of sound = 340 m./sec. and density of air = 0·001293 gm. per c.c.)

Ans. (*a*) 3·36 ergs per c.c. (*b*) $114{\cdot}24 \times 10^3$ ergs per sq. cm./sec.

11. Find the displacement of an air particle 3·5 metres from the origin of disturbance at $t = 0{\cdot}05$ second, when a wave of amplitude 0·2 mm. and frequency 500 p. p. s. travels along it with velocity 350 m/sec. *Ans*. = Zero.

12. A simple harmonic wave train travelling in the positive x direction has an amplitude of 2 cm. frequency of 75 vibrations /sec. and velocity of propagation 45 m./sec. Calculate the displacement, the particle velocity and the particle acceleration at $x = 135$ cm. from the origin at $t = 3$ sec.

Ans. $y = 2$ cm. ; $v =$ zero cm./sec. $f = 45 \times 10^4$ cm./sec.

CHAPTER IV

1. Show that the velocity of waves in a stretched string is given by the expression $\sqrt{\frac{T}{\rho}}$, where T is the tension and ρ the mass of unit length of the string. Hence deduce an expression for the frequency of a vibrating string.

2. Work out completely the laws of transverse vibrations of a string, and show how the diameters of two wires of the same material can be compared with the help of a sonometer.

3. Describe and explain Melde's Experiment. In an experiment it was found that the string vibrated in 5 loops when 10 grams were placed in the scale pan. What mass must be placed in the pan to make the string vibrate in 7 loops? (Neglect the wt. of the scale pan.) *Ans.* 5·1 gms. (*P. U. 1933*)

4. Describe how the result for the expression $v=\sqrt{\frac{T}{m}}$ can be verified experimentally.

5. Indicate how the laws of vibrations of a stretched string can be verified with Melde's apparatus.

6. Explain the production of nodes and loops in case of strings.

7. State the laws of vibration of strings, and describe experiments to verify them.

A wire of length 70 cm. and mass 2·18 gm. is stretched by means of a load of 16 kilograms. Calculate the frequency of the fundamental vibration. g=981 cm./sec^2. *Ans.* 160·3.

8. A wire 75 cm. long and of mass 1 gm. is making 256 vibrations/sec. under a tension supplied by a brass weight hanging vertically. On immersing the weight in water the vibrating length of the wire has to be shortened by 4·45 cm. to regain its original pitch. What is the density of the brass? *Ans.* 8·7 gm./c.c.

9. Show by means of diagrams the possible modes of vibration of a stretched string.

10. Write down the equation of motion of a stretched string and find an expression for the velocity of a transverse wave travelling along the string. (*P. U. 1935*)

11. Obtain an expression for the velocity of transverse vibrations along a stretched string, and thence deduce the frequency of a string vibrating in p segments. (*P. U. 1937*)

12. Prove that the velocity of waves on a stretched string is $\sqrt{\frac{T}{\rho}}$. Hence find the period of vibration of a string when giving out its fundamental note. If an addition of 25 lbs. to the tension of a string raises its pitch a fifth, what was original tension. *Ans.* T=20 lbs. (*P. U. 1946*)

13. The *E* string of a violin is 36 cm. long and has a mass of 0·15 gm With what tension must it be stretched if tuned to 660 vibrations for its fundamental. *Ans.* 9·59 kgm.

14. Calculate the tension to give standing waves with three loops in a string 90 cm. long, weighing 0·4 gm. and hanging from a fork vibrating 150 times per second. *Ans.* 91·7 gm. wt.

15. Calculate the velocity of the transverse wave in a copper wire 1 mm^2 crossection under the tension produced by 1 kgm wt. (S. G. of copper=8·93). *Ans.* 33·12 m./sec.

16. If a string 80 ft. long weighs ·05 lb., with what force in pounds must it be stretched in order that a transverse wave would travel its length in 0·1/sec. *Ans.* 12·5 lbs wt.

17. Four violin strings all of the same length and material but of diameters in the ratios 5 : 4 : 3 : 2 are to be stretched so that each gives a note a true fifth above the preceding. Compare the stretching forces necessary. *Ans.* 400 : 576 : 729 : 729

18. A steel wire 36 in. long and 0·02 in. in diameter is in unison with a fork of pitch 200. A second steel wire 24 in long and 0·025 in. in diameter is stretched with the same force. Find the frequency of its fundamental note. *Ans.* 240.

19. In a Melde's experiment a string vibrated in 3 loops when 8 grams were placed in the scale pan. What mass must be placed in the scale pan to make the string vibrate in 5 loops (neglect the weight of the scale pan). *Ans.* $2\frac{22}{25}$ grams.

20. In performing Melde's experiment in the longitudinal mode of vibration the string divides into 4 segments with a weight of 40 grams. What weight must be carried to cause the string to vibrate in 5 and 6 segments ? Explain what effect will be produced by turning the fork so that the plane of vibration of the fork is at right angles to the string.

Ans. $25\frac{3}{5}$ gm ; $17\frac{7}{9}$ gm ; Tension must be increased four times to produce the same number of loops.

21. Two strings are stretched side by side on a monochord tuned to g and g'. If either string is sounded and then silenced, the other will be found to be sounding g'. Explain this.

CHAPTER V

1. Explain as far as you can the mode of propagation of sound through air.

2. Obtain an expression for the velocity of sound in a gas. How does the velocity depend on humidity, temperature and pressure ? (*P. U. 1948*)

3. What is Newton's expression for the velocity of sound in a gas ? How did Laplace modify this expression, and why ?

4. Derive an expression for the velocity of sound in a gas, and hence show that the change in the velocity of sound in air is about 0·6 metre per degree (centigrade) change in temperature. (Velocity in air at 0°C=330 metres/sec.)

5. Derive an expression for the velocity of sound in a gas in terms of its pressure and density. Prove that $V=c\sqrt{\frac{\gamma}{3}}$, where V is the velocity of sound in a gas, c the root mean square velocity of its molecules, and γ the ratio of the two specific heats.

6. Derive an expression for the velocity of sound in a liquid and describe an experiment to determine it. (*P. U. 1942*)

7. Calculate the velocity of sound in air at 0°C and pressure of 760 mm. of mercury ; given that the corresponding density of air is 0·001293 gm./c.c. $g=981$ cm./sec^2. and S. G. of mercury is 13·6. *Ans.* 331·2 metres/sec.

8. If the velocity of sound in Hydrogen at 0°C is 4200 ft. per second, what will be velocity of sound (at the same temperature) in the mixture of two parts by volume of Hydrogen to one of Oxygen ? (The density of Oxygen is 16 times that of Hydrogen). *Ans.* 171·4 ft./sec. (*C. U.*)

9. Find the temperature at which sound travels in

hydrogen with the same velocity as in oxygen at 1000°C.
Ans. -193°C.

10. At what temperature is the velocity of sound in oxygen equal to its velocity in nitrogen at 30°C given that the atomic weights of the gases are 16 and 14 respectively. *Ans.* -8°C

11. Calculate the velocity of sound in air at 20°C and $1 \cdot 1 \times 10^6$ dynes per cm.^{2} pressure given that density of air at 0°C and 10^6 dynes per cm.^{2} pressure is 0·00122 gm. per cm^3. ($\gamma = 1 \cdot 41$.)
Ans. 352·2 m./sec.

12. The velocity of sound in a tube of air is found to be 1090 ft./sec. at 0°C. Calculate the velocity in the same tube when the temperature is raised to 30°C ; (*a*) if the air is not allowed to escape from the tube and (*b*) if the pressure is kept constant.
Ans. (*a*) 1150 ft./sec. (*b*) 1150 ft./sec.

CHAPTER VI

1. Obtain an expression for the resultant of two wave-trains travelling along the same direction, with the same velocity and amplitude, but of slightly different frequencies, and explain its meaning.

2. Distinguish between beats and combination tones.

3. A tuning fork *A*, of frequency 256 cycles/sec. produces 4 beats per sec. with another tuning fork *B*. When the prongs of the fork *A* are loaded with one gram wt., the number of beats is one per sec., and when loaded with 2 grams wt., the number of beats becomes two per second. What is the frequency of the fork *B* ? Explain why the number of beats increases under the greater load. (*P. U.*)

4. Describe and explain how combination tones are produced.

5. Investigate mathematically the phenomenon of 'beats'. Differentiate carefully between the phenomena of 'beats' and that of interference. Give examples of each. (*P. U. 1944*)

6. What are combination tones ? What are the theories of their origin ? Show how two tuning-forks can be tuned to an exact octave by making use of the existence of combination tones. (*P. U. 1946*)

7. Explain and illustrate by a diagram the occurrence of beats when two forks of nearly the same pitch are sounded

together. How would you determine which fork is vibrating faster ? (*C. U.*)

8. Write short notes on the following :—

(*a*) Interference of sound waves, (*b*) Beats, (*c*) Combination tones and (*d*) Silence zones in the neighbourhood of a fog siren.

How can the existence of these phenomena be shown ? (*C. U.*)

9. Describe some methods of experimentally demonstrating the phenomenon of interference.

10. A tuning fork is struck and then slowly rotated about its stem. Explain the changes in the sound heard during one complete rotation of the fork.

11. Two tuning forks whose pitches differ by three vibrations/sec. are set in vibration simultaneously. Describe and explain the character of the resulting sound.

12. Two tuning forks make 10 beats in 3 seconds when sounded simultaneously. One fork makes 384 vibrations per second and the beats cease when the other fork is filed a bit at the end of the prong. What is the frequency of the second fork ? *Ans.* $380\frac{2}{3}$.

13. Two tuning forks *A* and *B* give 4 beats per second. On loading *A* slightly we get 3 beats in 5 seconds, what is the frequency of *A* before and after loading of that of *B* is 256.
Ans. 260 before loading and 256·3 after loading.

CHAPTER VII

1. Distinguish clearly between progressive and stationary waves.

2. Explain the phenomenon of stationary waves when two trains of exactly similar waves are travelling along the the same string from opposite directions. Obtain an expression for the distance between two nodes.

3. Give the theory of the formation of stationary waves. Discuss their formation in a pipe which is closed at one end and in one which is open at both ends. (*P. U. 1943*)

4. Explain the manner in which reflection of a transverse wave occurs when the wave reaches a fixed point in the string. Show, with the help of diagrams how stationary waves are formed in a string. (*P. U. 1934*)

5. Show how stationary vibrations are produced in air ? What are their properties ?

Explain, in a general way, how reflection of sound takes place at the open end of a tube.

6. Give a general account of the distribution of energy in a progressive wave and a stationary vibration. (*P. U. 1947*)

CHAPTER VIII

1. Describe with illustrative examples the phenomenon of the reflection and refraction of sound. (*P. U. 1941*)

2. Give some examples of shadows in sound. Explain why sound shadows are generally not so well marked as those of light ? (*C.U.*)

3. Explain the production of echoes.

A gun is fired on a sea shore in front of a line of cliffs. A man standing 300 ft. away from the gun and equidistant from the cliffs notices that the echo takes twice as long to reach him as does the direct report. Find by calculation or graphically the distance of the gun from the cliffs. (*C. U.*)

Sol. *LK* is a line of cliffs. G is the position of the Gun, *O* that of the observer such that *OL*=*OG*=300 ft.

To obtain the path of the sound reflected from the cliff drop $GK \perp$ to LK and produce it to *N* so that $KN = GK$. Then *N* is the acoustical image of *G*. Join *NO* intersecting *LK* in *M*. Join *GM*. Then obviously *GMO* is the path of the reflected sound. It is required to find the distance *GK*. Let it be x.

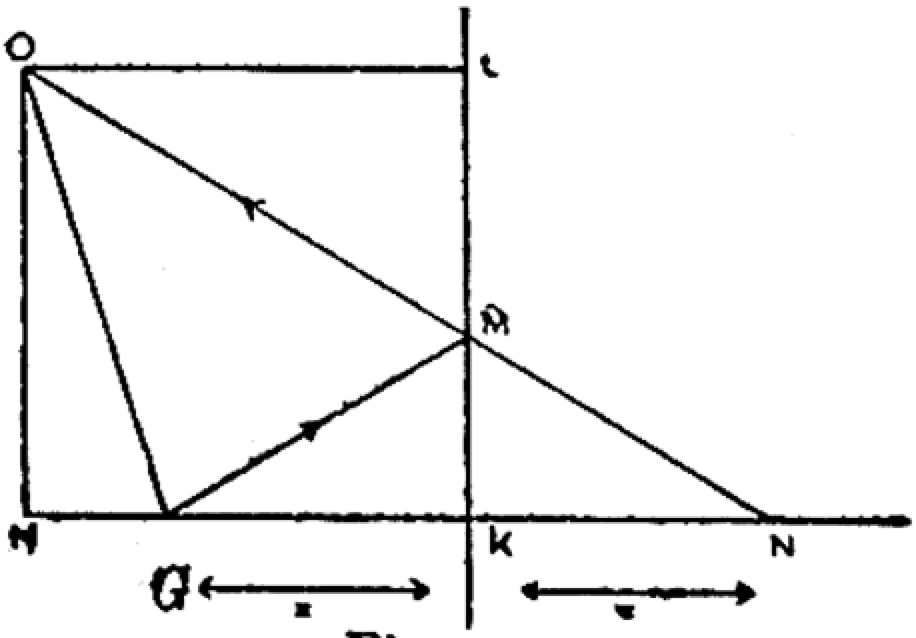

Fig. 171.

Since the time taken by the direct report is twice that of the echo

$$GM + MO = 2OG = 600 \text{ ft.}$$

Also $GM = NM \therefore NM + MO$ or $NO = 600$ ft.

From O drop $OH \perp KG$ produced.

In the right ∠d. triangle OHG

$$OH^2 = OG^2 - HG^2 = OG^2 - (HK - GK)^2$$
$$= (300)^2 - (300 - x)^2 \ldots\ldots\ldots (i)$$

In a right ∠d. triangle OHN

$$OH^2 = NO^2 - HN^2 = NO^2 - (HK + KN)^2$$
$$= (600)^2 - (300 + x)^2 \ldots\ldots\ldots (ii)$$

From (*i*) and (*ii*) we have

$$(300)^2 - (300 - x)^2 = (600)^2 - (300 + x)^2$$

or $$(300 + x)^2 - (300 - x)^2 = (600)^2 - (300)^2$$

or $$600 \times 2x = 900 \times 300$$

$$\therefore \quad x = \frac{900 \times 300}{600 \times 2}$$
$$= 225 \text{ ft.}$$

4. Explain why a whisper at some point in a large room can sometimes be heard distinctly at some other point.

5. How do you prove directly the laws of reflection of sound.

6. Discuss and explain the laws of refraction and reflection of sound.

7. Describe an experiment by which the refraction of sound can be demonstrated.

8. How do you account for the following :—

(*a*) Sounds travelling with the wind are better heard than those travelling against it.

(*b*) Sounds of explosion are heard up to a distance of nearly 50 miles, followed by a zone of silence and again heard at a distance of about 100 miles. (*P. U. 1936*)

9. Compressional wave impulses are sent to the bottom of sea from the ship and the echo is heard after 1·45 seconds. Calculate the depth of the sea assuming that mean temperature of water is 4°C and its bulk modulus $2{\cdot}2 \times 10^{10}$ dynes/sq. cm.

Ans. 1073 metres.

10. A person standing at the end of a row of pailings 9 inches apart emits a sharp sound which is echoed from the pailings. What is the frequency of the note resulting from the series of echoes ? Velocity of Sound = 1120 ft./sec.

Ans. 747 approx.

11. Two men half a mile apart stand at the same distance from a vertical hillock. One of them fires a shot and the other hears the echo 3 seconds after hearing the direct sound. Find the distance of the men from the hillock assuming the velocity to be 1120 ft. per second. How will the interval between two sounds be affected on a hot or cold day ? *Ans*. 898 yds ;

12. A train is approaching a tunnel surmounted by a cliff and the driver sends a short whistle when one mile away. The echo reaches him 9 seconds later. Find the speed of the train ? Velocity of Sound = 1120 ft./sec.

Ans. $36\frac{4}{11}$ miles/per hour.

13. A person drops a stone in a well and hears the splash $4\frac{8}{35}$ seconds later. How deep is the well ? Take the velocity of sound 1120 ft./sec. *Ans*. 256 ft.

CHAPTER IX

1. Explain :—Free vibration, forced vibration and resonance, giving two examples of each.

How is the principle of resonance used to analyse a complex note ?

2. Describe with illustrative examples, the phenomena of forced vibrations and resonance. How are vibrations maintained in a flue organ pipe ? *(P. U. 1940)*

3. Explain the meanings of and distinguish between Free, Forced and Resonant vibrations, giving examples.

State in general terms the effect of a periodic force in producing vibrations in an elastic body. *(P. U. 1945)*

4. Discuss the phenomenon known as 'the sharpness of resonance', and explain upon what factor it depends.

5. Explain why is it that when the handle of a vibrating tuning-fork is pressed against a thin wooden board the intensity of sound is greatly increased.

6. Describe and explain the phenomenon of resonance.

Why is it that a tuning-fork will resound to another tuning-fork only when the tuning is exact while a column of air can resound even when imperfectly tuned. *(P. U. 1934)*

7. Discuss the action of the sound board to which a vibrating string is attached by two pins.

Indicate how Melde's experiment illustrates the difference between the mode of vibration of the pins and that of the sound board. *(P. U. 1945)*

8. Explain carefully what is meant by forced vibration. How does the amplitude of a forced vibration depend upon the frequency of the vibrating body.

9. Describe a Helmholtz's resonator and find an expression for its frequency.

10. Explain the terms : damped oscillations, coefficient of damping, mechanical impedance, velocity amplitude and resonance catastrophe.

11. Give mathematical theory of the phenomena of forced vibrations and resonance.

12. Distinguish between *free* and *forced* vibrations. State the conditions necessary for the production of the forced vibrations. *(P. U. 1950)*

CHAPTER X

1. Distinguish between noises and musical sounds. What are important characteristics of a musical sound ? Explain clearly the difference between each.

2. Define pitch and quality of a musical note. Explain how they are represented in the wave-curve.

3. Explain clearly the chief characteristics of a musical sound, and show how can they be determined accurately.

4. Explain Doppler's effect due to source and listener in motion.

5. How do you show that, in the absence of wind, a given speed of approach of source raises the apparent pitch more than the same speed of the listener.

6. Show that if the source moves away with the velocity of sound from an observer who is at rest, the frequency of the note heard is halved.

7. A locomotive whistle emitting 2000 waves/sec. is moving towards you at the rate of 60 miles/hr. on a day when the thermometer stands at 15°C. Calculate the apparent pitch of the whistle (velocity of sound at 0°C=1090 ft./sec.). *Ans.* 2157.

8. A railway engine while whistling moves past an observer with a speed of 30 miles per hr. Find the ratio of the pitches when the source is first approaching and then receding (velocity of sound in air=1088 ft./sec). *Ans.* 1·08.

9. Find the velocity of the car with respect to a stationary observer if the latter observes a drop in pitch of 100 vibrations per sec. when the car passes him. The car emits a note with a frequency of 1000 vibrations/sec. and the velocity of sound is 1142 ft./sec. *Ans.* 57·1 ft./sec.

10. A spectrum line, of wavelength 4×10^{-5} cm. in the spectrum of light from a star is found to be displaced from its normal position towards the red end of the spectrum by an amount equivalent to 10^{-8} cm. What velocity of the star in the line of sight would account for this? (Velocity of light $=3 \times 10^5$ km./sec.) *Ans.* A velocity of recession of 75 km./sec.

11. How is it possible to calculate the velocity of sound in air from a projectile?

12. Give the arrangements for driving a tuning-fork electrically, explaining clearly how the vibrations are maintained.

13. State Fourier's theorem and apply it to analyse a compound curve.

14. Give Helmholtz's analysis of complex sound by the use of his resonators in order to test the validity of Fourier's theorem.

15. How can you detect the presence of harmonics in a musical note and what are their functions?

16. Explain the chracteristics of the vocal organ in man.

17. State briefly the theories as to the nature of vowel sounds.

18. What is that enables us to distinguish between the sounds of the different vowels when we hear them, and how do these differences arise?

19. Describe briefly the anatomy of the ear and physical principles involved as regards its power of analysing complex vibrations.

20. Discuss fully how the apparent pitch of a sound is affected by (*a*) motion of source, and (*b*) motion of observer.

A train approaches a stationary observer, the velocity of the train being $\frac{1}{20}$th of the velocity of sound, and a sharp blast is blown with the whistle of the engine at equal intervals of a second. Find the interval between the successive blasts as heard by the observer. *Ans.* 0·95 sec. (*P. U. 1946*)

21. A man standing by a railway track notices the change of pitch of the note due to the whistle of an engine as it passes him. If the frequency of the whistle be 256 vibrations/sec. and the velocity of the engine be $\frac{1}{20}$th of that of sound, what will be the frequencies of the notes heard by the man before and after the engine passes him? (*C. U.*)

Ans. 269·47 ; 243·8.

22. If a vibrating fork is rapidly moved towards a wall, beats may be heard between the direct and reflected sounds. Account for these, and calculate their frequency, if the fork makes 512 vibrations per sec. and approaches the wall with a velocity of 300 cms./sec. The velocity of sound may be taken as 330 metres/sec. (*P. U. 1932*)

Ans. Frequency of beats=9·3/sec.

23. The whistle of an engine moving at 30 miles/hr. is heard by a motorist driving at 15 miles/hr. and estimated to have a pitch of 500. What must be the actual pitch of the whistle to the nearest whole number when,

(*a*) the two are moving in opposite directions but approaching each other.

(*b*) the two are moving in opposite directions but away from each other.

(*c*) the two are moving in the same direction, the motorist being behind the engine.

(*d*) the two are moving in the same direction, the motorist being in front of the engine?

(Velocity of sound=1200 ft./sec.) (*P. U. 1937*)

Ans. (*a*) 473/sec. (*b*) 528/sec.
(*c*) 508/sec. (*d*) 491/sec.

[**Note** :—Take velocity of sound 1120 ft./sec. unless otherwise stated.]

24. Describe Doppler effect in sound. Show how the frequency of a note is affected by the motion of an observer towards or away from the source.

A motor-car travelling 30 miles per hour, is proceeding towards a high building. If the frequency of the note produced by the horn is 600, what would be the pitch of the note heard by the driver after reflection from the building ?

Ans. 649·2. (*Pb. U. 1949*)

25. How does the intensity of a sound depend upon the amplitude of vibration of the source ? What is a decibel ?

26. The intensity level of a sound is increased by 10 decibels. If the original intensity was 1^{-6} watt/cm.^{2} What is the new intensity ? *Ans.* 10^{-5} watts/cm^2.

27. What is Helmholtz's theory of the quality of a musical sound ? Explain carefully how did he verify his conclusions experimentally ?

28. Two sounds have intensities of 20 and 2000 μ watts/cm^2 respectively. How many decibels is the one sound above the other ? *Ans.* 20 decibels.

29. Two persons A and B some distance apart both blow whistles of pitch 300. An observer in between them and travelling towards A hears 4 beats/sec. How fast is he travelling ? *Ans.* $7\frac{7}{15}$ ft./sec.

30. Calculate the amplitude of vibrations in air for the threshold of audibility and the threshold of feeling assuming pressure amplitudes as 2×10^{-4} dyne/cm.^{2} and 200 dynes/cm.^{2} respectively.

(Take $n=1000$ p.p.s. and velocity of sound$=340$ m./sec.)
$\rho=0{\cdot}00122$ gm. per c.c.,

Ans. $2{\cdot}25\times10^{-8}$ cm. ; $2{\cdot}25\times10^{-2}$ cm.

31. Calculate the intensity of sound for plane waves with a pressure amplitude 10 dynes/cm.^{2} and the number of decibels it is above the threshold of audibility and the amount of energy passing through each square metre of a wave front p. per minute. Velocity of sound$=340$ m./sec, $\rho=0{\cdot}00122$ gm. per c.c.

Ans. 93·8 ; $7{\cdot}23\times10^3$ ergs.

32. A train is approaching a vertical cliff at 60 miles per

hour. The driver sounds a whistle of pitch 500. What will be the pitch of the echo as heard (1) by the engine driver (2) by an observer who is stationary.

Ans. (1) 585·2 (2) 542·6.

33. A train travelling at the rate of 40 miles per hour blows a whistle, the pitch of which is 384. What will be the pitch of the note heard by a listener in a train travelling towards the first at the rate of 60 miles an hour ? *Ans.* 437.

34. An Engine which blows a whistle of pitch 400 moves past a stationary observer with a velocity of 60 miles per hour. Find the difference between the pitches of the notes heard on approach and recession. *Ans.* 63·2.

35. An engine, travelling at 60 miles per hour passes an observer at rest. If the frequency of the note heard is 580 per second when the engine sounds its whistle while moving towards the observer, what is the frequency of the note which will be heard when the engine is receding ?
(Velocity of Sound in air is 1100 ft./sec.) *Ans.* 494.

36. State Ohm's law regarding the Physical basis of pure tones.

Explain carefully Helmholtz's synthetic method of verifying the quality of a note.

37. Compare the resolving power of the ear with that of the eye and state the limitations of the former.

38. Define the terms Intensity of sound and pressure amplitude. How are they related ?

Calculate the *pressure amplitude* and *Intensity* of a sound wave in air having a frequency 1000 p.p.s., amlpitude $2{\cdot}25 \times 10^{-8}$ cm. given that the density of air$=0{\cdot}00122$ gm per c.c. and velocity of sound 340 metres/sec.

Ans. pressure amplitude$=2 \times 10^{-4}$ dyne/cm.^{2}
Intensity$=48{\cdot}2 \times 10^{-11}$ ergs per sq. cm. per sec.

39. Describe the human voice organ as a musical instrument.

40. What constitutes a musical sound ? What experiments would you perform to demonstrate the Physical nature of the intensity, the pitch and the quality of a musical sound ?

(*P. U. 1949*)

CHAPTER XI

1. Describe an organ flue pipe. If two such pipes of the same length are sounded, the one open and the other closed, how do the notes differ from each other ?

2. Show clearly the relation of harmonics which are obtainable from closed and open organ pipes.

3. Distinguish between overtones and harmonics. Give a few instances of overtones that are not harmonic.

4. How can the existence of nodes and antinodes in a sounding organ pipe be demonstrated ?

5. Describe the manometric flame apparatus. What information does it give as to the nature of a sound ?

6. An open organ pipe produces 8 beats/sec., when sounded with a tuning-fork of 256 vibrations/sec. the fork giving the lower tone. How much must the length of the pipe be altered to bring it into unison with the fork ?
Velocity of sound=344 m./sec.
Ans. Increase of length by 2·03 cm.

7. How can the existence of nodes and antinodes in a sounding organ pipe be demonstrated ?

8. Explain fully the theory of reflection from the open and closed end of a pipe.

9. What is meant by 'end correction' ? Discuss its effect on the pitch and quality of a note given out by the pipes.

10. Describe and explain the various methods of tuning organ pipes.

11. A closed brass pipe emits a note of frequency 512 at 0°C. If the coefficient of linear expansion of brass is $1{\cdot}87 \times 10^{-5}$ per °C, determine the pitch of the note at 20°C. *Ans.* 530·5

12. Two organ pipes give 6 beats per second when sounded together in air at 10°C. How many beats will be produced when temperature rises to 24°C ? *Ans.* 6·15 per sec.

13. Describe the action of a mouth piece of an organ flute pipe, and show that an open end pipe will produce sounds richer in overtones than a closed one. (*P. U. 1937*)

14. A tuning fork of frequency 440 vibrations per second is to be mounted on a wooden box (with one end open) to reinforce its sound. How long should be the sounding box ?
Ans. $\frac{7}{11}$ ft.

15. Determine the length of an organ pipe that is closed at one end if it is to have a frequency of 264/sec. Also calculate the frequencies of its first two overtones ?

Ans. $1\frac{2}{33}$ ft. ; 792 ; 1320.

16. In a velocity determination experiment with a resonance tube, the first and second resonance columns with a tuning fork of frequency 512 per second are 6·2 in. and 19·2 in. Calculate the velocity of sound. *Ans.* $1109\frac{1}{3}$ ft./sec.

17. Four beats per second are heard between the fundamental notes of two open organ pipes when sounded together. If the length of one of them be 5 ft. find that of the other. Take velocity of sound = 1120 ft./sec. *Ans.* $4\frac{24}{29}$ ft. or $5\frac{5}{27}$ ft.

18. Explain carefully the effect of the correction for the open end on the relative frequencies of the overtones of a pipe.

Why the note of a wide pipe contains fewer overtones than a note of a narrow pipe ?

19. A closed end pipe 1 ft. long resounds to the lowest note produced by an open pipe 2 ft. long and 4″ in diameter. What is its diameter ? *Ans.* 2″.

20. Two open organ pipes 2 ft. and 4 ft. long when sounded together produce 3 beats per second. If the diameter of the shorter pipe be 4 in., calculate that of the longer. Take velocity of sound = 1135·2 ft. /sec. *Ans.* 6″.

CHAPTER XII

1. Consider in general terms possible modes of vibration in case of a rod fixed at one end, the other end being free.

2. Explain with diagrams the nature of the vibrations of a tuning-fork. What special features make it a valuable instrument in the scientific study of sound.

3. Consider the modes of vibration of bells.

4. State how the presence of overtones in the sound of (*a*) a closed organ pipe and (*b*) a bell, may be observed and their pitches determined. Explain clearly the difference in the nature of the tones in the two cases.

5. In what way would you expect the rise of temperature to affect the frequency of a tuning fork and why ?

6. Obtain the relation for the period of vibration of an elastic body deduced by the method of dimensions.

A wooden rod 5 ft. long clamped in the middle and rubbed with a resined piece of leather is in unison with the fundamental of an open organ pipe 11 inches long. Find the velocity of sound in the rod, temp. being 20°C. *Ans.* 6109 ft./sec.

8. Two tuning forks have the same proportions and are made of the same material. One of the forks is a fourth higher than the other. What is the ratio of their weights ?

9. What are Chladni's figures ? Describe the modes of vibrations of plates both circular and rectangular when clamped in the centre.

10. Describe the change in the position of nodes which progressively occurs as a bar is bent more and more from the straight form to a U-shaped one.

CHAPTER XIII

1. Explain the sensation produced when two notes sounded together are in unison, and then the frequency of one of them in gradually increased until it is an octave above the other.

2. What are concord and discord ? Discuss the relationship between the frequencies of two notes which cause them to be discordant when sounded simultaneously.

3. Give an account of Helmholtz's theory of consonance. Two tuning forks nearly an octave apart and free from overtones give beats when sounded together. What is the cause of beats ? (*P. U. 1947*)

4. Compare the consonances of an octave 2 : 1, fifth 3 : 2, and a fourth 4 : 3. Give a brief account of Helmholtz's theory of consonance and dissonance.

5. A major tone is given (1) on two hautboys and (2) one note on a clarinet and one on a hautboy. Explain the different degrees of dissonance.

6. Explain the terms 'Consonance' and 'Dissonance'. Briefly describe the conditions under which concords and discords arise between two notes of different frequencies.

CHAPTER XIV

1. Explain what you understand by the musical interval between two notes ? What intervals are used in the diatonic

scale ? What is temperament, and why is the tempered scale used in keyed instruments.

2. Distinguish between the diatonic scale and scale of equal temperament and show how each has been built up. (*C. U.*)

3. What is mean tone temperament ? Show how it has been built up.

4. Write a short essay on musical scales.

5. What is a diatonic scale ? Explain its musical basis. How does the scale used in harmonium differ ftom it ?

6. Explain what is meant by a musical interval. What is the interval between two steel strings of equal length, and diameter when stretched with 2 kgm. and 4 5 kgm. *Ans.* a fifth.

7. What is the interval of a fifth, a fourth, a major third and a semitone ? What is the difference between the intervals of a major and minor tone ?

Ans. 3 : 2 ; 4 : 3 ; 5 : 4 ; 16 : 15 ; 80 : 81.

8. What do you understand by temperament ? Describe the scale of equal temperament and discuss its merits and demerits relative to the major diatonic scale.

9. Assuming the frequency of the note *C* to be 264, find the frequency of the note *F* on the diatonic scale and on the scale of equal temperament. *Ans.* 352 ; 352·4.

10. The frequency of the middle *C* of a musical instrument is 256, find the frequency of *A*, upper *A* and lower *A*.

(For an equally tempered scale.) *Ans.* 430·4 ; 860·8 ; 215·2

11. Assuming that the strings of a violin are exactly of the same length, diameter and material and they emit notes g, d^1, a^1, e^2 when bowed. Compare the tensions in the strings given that the frequency of the note *C* is 256.

Ans. Frequency ratios : 384 : 576 : 854 : 1280

Tension ratios : 1 : 2·25 : 4·94 : 11·11.

12. What is a Comma ? Show that the fourth and the major third of the equal temperament are sharper than the true fourth and third by $\frac{1}{11}$ and $\frac{7}{11}$ comma respectively.

13. Show that on the Scale of Equal temperament (*i*) three major thirds make an octave (*ii*) the fifth is flatter by $\frac{1}{11}$ of a comma than true fifth.

14. Show that an equal temperament minor third is about $\frac{8}{11}$ comma flat by using the fact that four equal temperament minor thirds make an octave.

15. The pitch of the whistle of an Engine falls from C to B on the major diatonic scale as it passes a stationary listener. Assuming velocity of sound to be 1120 ft./sec. find the speed of the engine *Ans.* $50\frac{10}{11}$ miles per hour.

16. Two aeroplanes are approaching each other at the rate of 200 miles per hour. To a listener in one aeroplane, the pitch of the whistle of the other appears to be that of note F in the scale of key note 264. Find the actual pitch of the whistle if velocity of sound is 1120 ft./sec. *Ans.* 206 approx :

CHAPTER XV

1. What are the essential parts of a musical instrument regarded from the scientific point of view ? Explain their action and importance with reference to a flute.

2. What are the main classes of musical instruments ? Describe briefly their chief distinguishing characterics.

3. Describe the principle and working of : (1) a sitar (2) a pianoforte. In what essential respects do a sitar and a pianoforte differ.

4. Explain the principle and use of Helmholtz vibration Microscope. How has it been used for the study of the motion of a violin string ?

5. Explain the action of a violin bow and describe the modes of vibration of a string of a violin.

6. Explain in a general way how the quality of the note of a pianoforte is affected by ; (1) the point struck, (2) the shape of the hammer and (3) the hardness of the hammer.

7. How is the sound produced in a brass instrument ? Discuss some features common to them all.

8. Write note on the following :—

(1) Harmonium, (2) Veena
(3) Clarinet, (4) Tabla
(5) Human voice organ.

9. Describe how organ pipes with out reeds speak and account for the different qualities of tone of various pipes.

10. Discuss the arrangement and tuning of organ pipes with or without pipes.

11. How do you account for the difference of quality of the notes emitted by a pianoforte, a flute and a tuning fork, the nominal pitch being the same in each case.

CHAPTER XVI

1. Mention the different factors or sources of error that influence the velocity of sound as determined by open air observations, and explain how they are corrected in order to obtain an absolute value.

2. Describe an arrangement used in a modern outfit for the determination of the velocity of sound in open air.

3. Describe Colladon and Sturm's method to determine the velocity of sound in water.

4. Explain the theory and the method of finding the velocity of sound in air by the resonance column.

5. Calculate the velocity of sound in dry air at 0°C and 760 mm. pressure from the following data, supposing the density of moist air inside to be ·00120 gm. per c.c.

Length of air column for the 1st resonance=33 cm.
" " " " " 2nd " =101·6 cm,
Temp. of air inside the tube=30°C
Barometric height (corrected)=760 mm.
Frequency of the fork=256
Density of dry air at 0°C=·001293 gm.
Ans. 335·1 metres per sec.

6. Describe in detail an experimental arrangement (Kundt's apparatus) for comparing the velocity of sound in gases. Why is it unsuitable for finding the absolute velocity of sound in a gas ?

7. Give some acoustical methods of finding the coefficient of longitudinal elasticity.

8. How do you use Kundt's apparatus for finding the ratio of the specific heats of a gas ? What other determinations could be made with the help of Kundt's tube ?

9. Give briefly two methods by which the velocity of sound in carbon dioxide could be found.

10. Explain the stroboscopic method of finding the frequency of a tuning-fork.

11. How may stroboscope be used to adjust the rate of revolution of an axle to uniformity ?

12. A stroboscopic disc spaced with 16 spots was rotated and adjusted to agreement with a vibrating fork so that the spots appeared stationary. It made 3072 revolutions in 192 seconds Find the frequency of the fork. *Ans.* 256.

13. Give, with full details, the falling plate method for finding the absolute frequency of a tuning fork. Discuss any possible sources of error, and suggest how you would correct them.

14. Describe the application of the chronograph for determining the pitch of a tuning fork.

15. What evidence is there that notes of different pitch travel with the same velocity.

16. Describe Cagniard de la Tour's siren and you would use it to determine the frequency of a fork.

17. Give a method to determine directly the ratio of the frequencies of two tuning-forks ?

18. How do you use the Lissajous' figures to compare the frequencies of two forks ?

19. Explain with examples how the stroboscopic methods may be used for the observation of rapid periodic motion. Describe some forms of a stroboscope. *(P. U. 1943)*

20. What is Scheibler's Tonometer and how is it used for finding the frequency of a tuning-fork ?

21. Describe Hebb's Telephone method for determining the velocity of sound in air.

CHAPTER XVII

1. Explain the principle of phonograph and how it has been used to test the theories of the origin of vowel sounds.

2. Describe briefly the modern method of recording and reproducing music electrically.

3. Describe the principle of a Phonodeik and Dr. Erskine-Murray's Phonoscope. How are they used in studying the wave forms of various musical sounds.

4. Describe the construction and operation of a gramophone.

5. Write notes on :—

Seismograph and prospecting for oil.

6. Write notes an (1) Binaural method and its use in sound locating and (2) Geophone and its application to mining.

7. Explain the construction and use of a hydrophone.

8. Write short notes on :—

(*i*) Hot wire microphone (*ii*) Sound ranging (*iii*) Singing flames.

9. What are ultrasonic waves ? How are they produced and detected, and to what uses are they put ?

10. Describe experiments illustrating the maintenance of vibrations by heat. Explain in a general way the mode of their maintenance.

11. Give a short account of the production and exhibition of sound motion pictures

12. Explain clearly what causes reverberation in a hall and how it can be minimised.

13. Compare the actions of the phonograph and the telephone.

14. Describe (*a*) some form of Carbon transmitter (*b*) a loud speaker.

15. A lecture hall with a volume of 450,00 cu. ft. is found to have a reverberation time of 1·5 seconds. What is the total absorbing power of all the surfaces in the hall ? If the area of the sound absorbing surface is 8000 sq. ft. calculate the average absorption coefficient. Total absoring power=1500

Ans. Average absorption coefficient=0·19 approx :

16. What is piezo-electric effect ? How has it been employed for the production of supersonics.

APPENDIX

208a. The Phonic Wheel. It is an electrical application of the principle underlying the theory of the stroboscope. An electric current is rendered intermittent by a fork and is passed through two electromagnets E_1E_2 [Fig. 143 (*a*)]. These magnets are placed at opposite ends of a diameter of a soft-iron toothed wheel (W) and nearly touching the teeth of the wheel. When the wheel is set in motion at such a speed that every time the magnets are energized one of the teeth is just to pass them, the wheel will receive a series of impulses of the same frequency as the fork. Thus the wheel is kept moving at a constant speed determined by the frequency of the fork. If the wheel has p number of teeth, it will rotate once for every p vibrations of the fork. Then time required for p vibrations of the fork $= \frac{p}{n}$ seconds, where n is the frequency of the fork. This is the time required for one revolution of the wheel. The wheel is mechanically geared to a device which rings a bell every, say, fifty revolutions. From this the time for one revolution of the wheel is determined. Thus knowing p and time of one revolution, n can be calculated. This method is also free of the disadvantage of the stroboscopic experimental arrangement where the loading of the prongs of the fork by aluminium foils lowers its frequency.

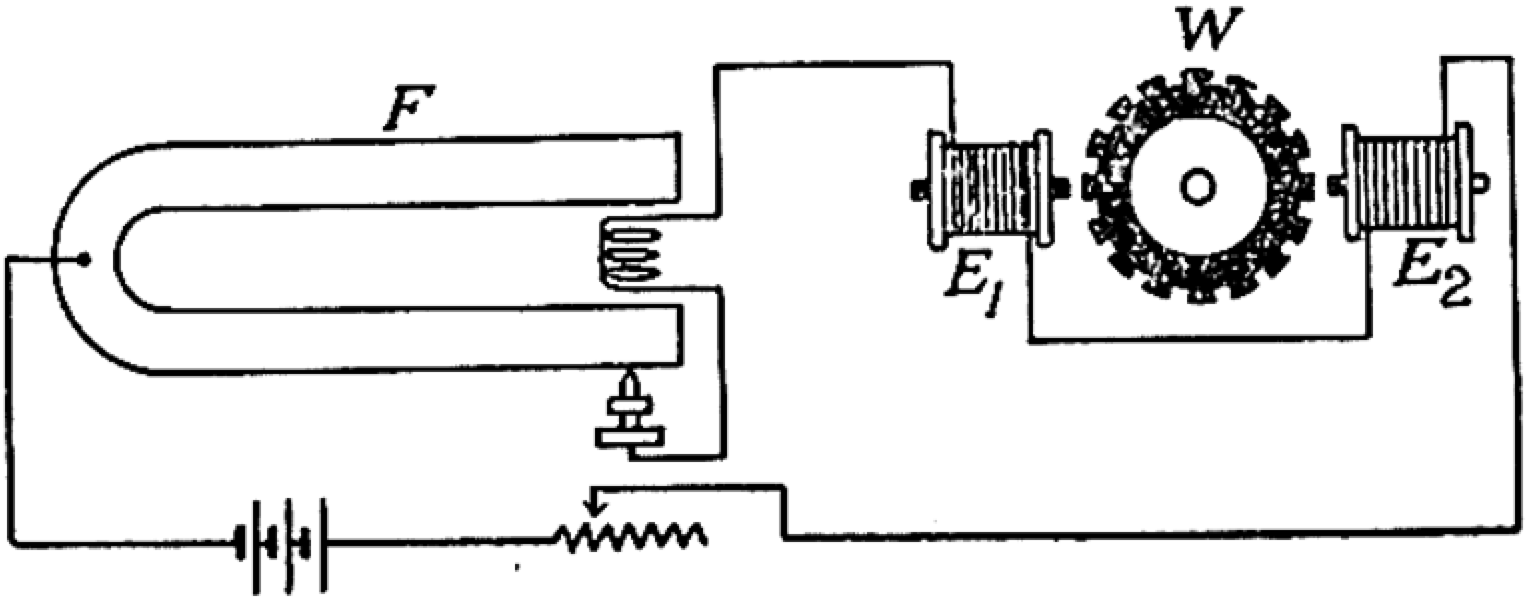

Fig. 143 (*a*).

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